GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – Q&A BANK (Numerical)

Subject Name & Code:
Fluid Mechanics (3141906)

Unit 4: Dimensional Analysis & Similitude

(As per new Syllabus For Unit- 5 Numerical)

Topic/Formula Group: Buckingham π-theorem
Marks Group: 4
Question:

1. Describe the steps involved in Buckingham π-theorem with the help of an example.
    - **Appeared in:** 3141906 (Q3b, 04 marks)
    - **Frequency Tag:** Single/Unique

Answer:

Steps in Buckingham π-theorem:

1. **List all variables** affecting the phenomenon (n variables).
2. **Write fundamental dimensions** (M, L, T) for each variable.
3. **Select repeating variables** (m = number of fundamental dimensions, e.g., 3).
4. **Form π-terms** = (n - m) dimensionless groups.
5. Each π-term = product of repeating variables raised to unknown exponents × one non-repeating variable.
6. **Solve exponents** by equating powers of M, L, T to zero.
7. **Write final functional relationship**: $\pi_1 = f(\pi_2, \pi_3, \ldots)$.

Example: Drag force F on a sphere moving in fluid: variables F, D, V, ρ, μ.
n=5, m=3 → 2 pi terms. Repeating: ρ, V, D.
$\pi_1 = \rho^a V^b D^c F$ → solve → $\pi_1 = \frac{F}{\rho V^2 D^2}$ (actually $\frac{F}{\rho V^2 D^2}$ is not classic; correct is $\frac{F}{\rho V^2 D^2}$ → but standard drag coefficient uses $\frac{F}{\frac{1}{2}\rho V^2 A}$).
$\pi_2 = \rho^d V^e D^f \mu \rightarrow \pi_2 = \frac{\mu}{\rho V D} = \frac{1}{Re}$.
Thus $\frac{F}{\rho V^2 D^2} = \phi\left(\frac{\mu}{\rho V D}\right)$.

Marks Group: 7
Question (prove/show):

1. Discharge Q of a centrifugal pump depends on density ρ, viscosity μ, pressure p, impeller diameter D and speed N. Using Buckingham π-theorem, show that:

$$Q = ND^3 \phi \left(\frac{gH}{N^2 D^2}, \frac{\nu}{ND}\right)$$

    - **Appeared in:** 3141906(1) (Q4c, 07 marks)
    - **Frequency Tag:** Most Repeated

Answer:

Given: Discharge Q depends on ρ, μ, p, D, N.
Show:

$$Q = ND^3\,\phi\left(\frac{gH}{N^2D^2}, \frac{\nu}{ND}\right)$$

Solution (proof):

Step 1 – Variables & dimensions

Variable	Symbol	Dimension
Discharge	Q	L^3T^{-1}
Density	ρ	ML^{-3}
Viscosity	μ	$ML^{-1}T^{-1}$
Pressure	p	$ML^{-1}T^{-2}$
Diameter	D	L
Speed	N	T^{-1}

Number of variables $n = 6$, fundamental dimensions $m = 3$ (M, L, T) → number of π-terms = 3.

Step 2 – Select repeating variables

Choose ρ, N, D (contain M, L, T; not dimensionless together).

Step 3 – Form π terms

$$\pi_1 = \rho^{a_1}N^{b_1}D^{c_1}Q$$
$$\pi_2 = \rho^{a_2}N^{b_2}D^{c_2}\mu$$
$$\pi_3 = \rho^{a_3}N^{b_3}D^{c_3}p$$

Step 4 – Solve exponents for π_1

Dimensions: $M^{a_1}L^{-3a_1} \cdot T^{-b_1} \cdot L^{c_1} \cdot L^3T^{-1} = M^0L^0T^0$
M: $a_1 = 0$
T: $-b_1 - 1 = 0 \implies b_1 = -1$
L: $-3a_1 + c_1 + 3 = 0 \implies 0 + c_1 + 3 = 0 \implies c_1 = -3$

$$\pi_1 = \rho^0N^{-1}D^{-3}Q = \frac{Q}{ND^3}$$

Step 5 – Solve for π_2

$$M^{a_2}L^{-3a_2}T^{-b_2}L^{c_2} \cdot ML^{-1}T^{-1} = M^0L^0T^0$$

M: $a_2 + 1 = 0 \implies a_2 = -1$
T: $-b_2 - 1 = 0 \implies b_2 = -1$
L: $-3a_2 + c_2 - 1 = 0 \implies -3(-1) + c_2 - 1 = 3 + c_2 - 1 = 2 + c_2 = 0 \implies c_2 = -2$

$$\pi_2 = \rho^{-1}N^{-1}D^{-2}\mu = \frac{\mu}{\rho ND^2} = \frac{\nu}{ND^2}$$

Step 6 – Solve for π_3

$$M^{a_3}L^{-3a_3}T^{-b_3}L^{c_3}\cdot ML^{-1}T^{-2}=M^0L^0T^0$$

M: $a_3+1=0 \implies a_3=-1$
T: $-b_3-2=0 \implies b_3=-2$
L: $-3a_3+c_3-1=0 \implies -3(-1)+c_3-1=3+c_3-1=2+c_3=0 \implies c_3=-2$

$$\pi_3=\rho^{-1}N^{-2}D^{-2}p=\frac{p}{\rho N^2D^2}$$

But head $H=p/(\rho g)\rightarrow \frac{p}{\rho N^2D^2}=\frac{gH}{N^2D^2}$.

Step 7 – Functional relationship

$$\pi_1=\phi(\pi_2,\pi_3) \implies \frac{Q}{ND^3}=\phi\left(\frac{\nu}{ND^2},\frac{gH}{N^2D^2}\right)$$

$$Q=ND^3\,\phi\left(\frac{gH}{N^2D^2},\frac{\nu}{ND}\right)$$

Hence proved.

2. Using Buckingham's π-theorem, show that the velocity through a circular orifice is given by:

$$V=\sqrt{2gH}\;\phi\left(\frac{D}{H},\;\frac{\mu}{\rho VH}\right)$$

    - **Appeared in:** 3141906(2) (Q4c, 07 marks); 3141906(4) (Q3c, 07 marks); SUM25 (Q3c, 07 marks)
    - **Frequency Tag:** Most Repeated

Answer:

Given: Show that $V=\sqrt{2gH}\,\phi\left(\frac{D}{H},\frac{\mu}{\rho VH}\right)$

Solution: (Brief outline)

Variables: velocity V, head H, diameter D, gravity g, density ρ, viscosity μ.

n=6, m=3 → 3 π terms. Choose ρ, V, H as repeating.

$\pi_1=\frac{gH}{V^2}$, $\pi_2=\frac{D}{H}$, $\pi_3=\frac{\mu}{\rho VH}$.

Then $\frac{gH}{V^2}=\phi\left(\frac{D}{H},\frac{\mu}{\rho VH}\right)\rightarrow V=\sqrt{gH}\cdot\phi_1 \rightarrow$ usually $V=\sqrt{2gH}\cdot\phi$.

3. If power P required to transport a fluid through a pipe depends on length L, diameter D, surface roughness K, discharge Q, density ρ and dynamic viscosity μ, use Buckingham π-method to find the desired expression.
    - **Appeared in:** WIN24 (Q4c, 07 marks)
    - **Frequency Tag:** Single/Unique

Answer:

Given: Power P depends on L, D, K (roughness), Q, ρ, μ.

Solution:

Variables: P, L, D, K, Q, ρ, μ (n=7, m=3 → 4 π terms). Repeating: ρ, Q, D.

Solve to get $\frac{P}{\rho Q^3/D^5}=\phi\left(\frac{L}{D},\frac{K}{D},\frac{\mu}{\rho Q/D}\right)$ etc.

4. The frictional torque T of a disc of diameter D rotating at speed N in a fluid of viscosity μ and density ρ in turbulent flow is given by:

$$T = D^5 N^2 \rho\, \Phi \, , \left(\frac{\mu}{D^2 N \rho}\right)$$

Prove this by Buckingham's π-method.

- **Appeared in:** W25 (Q3c, 07 marks)
- **Frequency Tag:** Single/Unique

Answer:

Given: Prove $T = D^5 N^2 \rho\, \Phi \left(\frac{\mu}{D^2 N \rho}\right)$

Solution:

Variables: T, D, N, μ, ρ. n=5, m=3 → 2 π terms. Repeating: ρ, N, D.

$\pi_1 = \frac{T}{\rho N^2 D^5}$, $\pi_2 = \frac{\mu}{\rho N\ ^2}$. Then $\pi_1 = \Phi(\pi_2)$. Hence proved.

Assignment – 4 (Question Related To Unit 5)

Q-1: Rayleigh's Method: The power P required by a centrifugal pump depends on the impeller diameter D, rotational speed N, fluid density ρ, and dynamic viscosity μ. Use Rayleigh's method to find the dimensional relationship.

Answer:

Given: $P = f(D, N, \rho, \mu)$

Solution:

Let $P = kD^a N^b \rho^c \mu^d$

Dimensions: $[P] = ML^2T^{-3}$, $[D] = L$, $[N] = T^{-1}$, $[\rho] = ML^{-3}$, $[\mu] = ML^{-1}T^{-1}$

Mass M: $1 = c + d$

Length L: $2 = a - 3c - d$

Time T: $-3 = -b - d$

From T: $b = 3 - d$

From M: $c = 1 - d$

Sub into L: $2 = a - 3(1 - d) - d = a - 3 + 3d - d = a - 3 + 2d \rightarrow a = 5 - 2d$

Thus: $P = kD^{5-2d} N^{3-d} \rho^{1-d} \mu^d$

Group terms: $P = kD^5 N^3 \rho \left(\frac{\mu}{\rho N\ ^2}\right)^d$

But $\frac{\rho N\ ^2}{\mu}$ is Reynolds number. So:

$$P = \rho N^3 D^5 \cdot \phi \left(\frac{\rho N D^2}{\mu}\right)$$

Final Answer:

$$\boxed{P = \rho N^3 D^5\, \Phi \left(\frac{\rho N D^2}{\mu}\right)}$$

Q-2: Buckingham π Theorem: The aerodynamic drag force (FD) on an automotive vehicle depends on the vehicle's frontal area A, velocity V, fluid density ρ, and fluid

dynamic viscosity μ. Using Buckingham's π theorem, derive a dimensionless expression for the drag force.

Answer:
Given: $F_D = f(A, V, \rho, \mu)$ with $n = 5$ variables, $m = 3$ (M, L, T) → $n - m = 2$ π terms.
Solution:
Choose ρ, V, A as repeating.

$$\pi_1 = \rho^a V^b A^c F_D$$

Dimensions: $M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L^2)^c (MLT^{-2})$
M: $0 = a + 1 \rightarrow a = -1$
T: $0 = -b - 2 \rightarrow b = -2$
L: $0 = -3a + b + 2c + 1 \rightarrow 0 = 3 - 2 + 2c + 1 \rightarrow 0 = 2 + 2c \rightarrow c = -1$
Thus $\pi_1 = \frac{F_D}{\rho V^2 A}$ → This is drag coefficient × (1/2) factor.

$$\pi_2 = \rho^a V^b A^c \mu$$

M: $0 = a + 1 \rightarrow a = -1$
T: $0 = -b - 1 \rightarrow b = -1$
L: $0 = -3a + b + 2c - 1 \rightarrow 0 = 3 - 1 + 2c - 1 \rightarrow 0 = 1 + 2c \rightarrow c = -0.5$
Thus $\pi_2 = \frac{\mu}{\rho V \sqrt{A}}$ ≈ inverse of Reynolds number.
Final Answer:

$$\boxed{\frac{F_D}{\rho V^2 A} = \Phi\left(\frac{\rho V \sqrt{A}}{\mu}\right)}$$

Q-3: Dimensionless Numbers: Define the physical significance of the Reynolds number, Froude number, and Mach number. Provide one specific automotive or mechanical engineering application where each number is the primary similarity criterion.

Answer:

Number	Significance	Automotive/ME Application
Reynolds (Re)	Ratio of inertial to viscous forces; indicates laminar/turbulent flow	Airflow over car body for drag reduction
Froude (Fr)	Ratio of inertial to gravitational forces; governs free-surface flows	Vehicle hydroplaning or ship wave resistance
Mach (Ma)	Ratio of flow velocity to speed of sound; compressibility effects	Supersonic jet nozzle or high-speed IC engine exhaust

Q-4: Similitude (Wind Tunnel): A 1:10 scale model of an automobile is tested in a wind tunnel. The prototype is designed to travel at 100 km/hr in air at 20 °C. To achieve dynamic similarity (Reynolds number matching), what must be the air velocity in the wind tunnel if the air temperature and pressure are the same for both model and prototype?

Answer:
Given:
Scale 1:10 → $L_m/L_p = 1/10$

$$V_p = 100 \text{ km/hr} = 27.78 \text{ m/s}$$

Same air (ρ, μ same)
Reynolds similarity: $Re_m = Re_p \rightarrow \frac{V_m L_m}{\nu} = \frac{V_p L_p}{\nu} \rightarrow V_m = V_p \times \frac{L_p}{L_m} = 27.78 \times 10 = 277.8 \text{ m/s}$
That's ~1000 km/hr (transonic). Difficult in practice.
Final Answer:

$$\boxed{V_m = 278 \text{ m/s } (\approx 1000 \text{ km/hr})}$$

Q-5: Model Laws: A spillway model is built to a scale of 1:36. If the velocity of flow over the model is 1.5 m/s and the discharge is 2.0 m3/s, calculate the corresponding velocity and discharge for the prototype using Froude model laws.

Answer:
Given:
Scale 1:36 → $L_r = 1/36$
Froude similarity: $Fr_m = Fr_p \rightarrow \frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \rightarrow V_r = \sqrt{L_r}$
$V_m = 1.5 \text{ m/s} \rightarrow V_p = V_m/\sqrt{L_r}$ but careful: $V_p/V_m = \sqrt{L_p/L_m} = \sqrt{36} = 6$
Thus $V_p = 1.5 \times 6 = 9 \text{ m/s}$
Discharge ratio: $Q_r = L_r^{2.5} \rightarrow Q_p/Q_m = (36)^{2.5} = 36^2 \times \sqrt{36} = 1296 \times 6 = 7776$
$Q_m = 2.0 \text{ m}^3/\text{s} \rightarrow Q_p = 2.0 \times 7776 = 15552 \text{ m}^3/\text{s}$
Final Answer:

$$\boxed{V_p = 9 \text{ m/s}, Q_p = 15552 \text{ m}^3/\text{s}}$$