

# GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – Q&A BANK (Numerical)

**Subject Name & Code:**  
**Fluid Mechanics (3141906)**

## Unit 3: Laminar & Turbulent Flow Analysis

(As per new Syllabus For Unit- 6&7 Numerical)

**Topic/Formula Group: Chezy's Formula (Friction loss)**

**Marks Group: 3**

**Numerical Question:**

- Determine the head lost due to friction in a pipe. Diameter and length of pipe are 250 mm and 60 m respectively. Velocity of flowing water inside pipe = 2.5 m/s. Chezy's constant = 60. Use Chezy's formula  $V = C\sqrt{mi}$ .
  - Appeared in:** 3141906(2) (Q5a, 03 marks)
  - Frequency Tag:** Single/Unique
  - Input Variables:**  $D = 250 \text{ mm} = 0.25 \text{ m}$ ,  $L = 60 \text{ m}$ ,  $V = 2.5 \text{ m/s}$ ,  $C = 60$

**Answer:**

**Given:**

Diameter  $D = 250 \text{ mm} = 0.25 \text{ m}$

Length  $L = 60 \text{ m}$

Velocity  $V = 2.5 \text{ m/s}$

Chezy's constant  $C = 60$

**To find:** Head lost due to friction  $h_f$

**Formula:** Chezy's formula:  $V = C\sqrt{mi}$

Where hydraulic mean depth  $m = D/4$  (for circular pipe),  $i = h_f/L$

**Solution:**

$$m = \frac{D}{4} = \frac{0.25}{4} = 0.0625 \text{ m}$$

$$V = C \sqrt{m \cdot \frac{h_f}{L}} \Rightarrow 2.5 = 60 \sqrt{0.0625 \times \frac{h_f}{60}}$$

$$\frac{2.5}{60} = \sqrt{0.0625 \times \frac{h_f}{60}}$$

$$0.04167 = \sqrt{0.0625 \times \frac{h_f}{60}}$$

Square both sides:

$$0.001736 = 0.0625 \times \frac{h_f}{60}$$

$$0.001736 = \frac{0.0625h_f}{60}$$

$$0.001736 \times 60 = 0.0625h_f$$

$$0.10416 = 0.0625h_f$$

$$h_f = \frac{0.10416}{0.0625} = 1.666 \text{ m}$$

**Final Answer:**

$$h_f = 1.667 \text{ m}$$

**Topic/Formula Group: Pipe Sizing (Darcy-Weisbach / Friction loss)**

**Marks Group: 7**

**Numerical Question:**

- Water is to be supplied to the inhabitants of a college hostel through a supply main. Data: distance = 4000 m; inhabitants = 3000; consumption per inhabitant per day = 180 litres; head loss due to friction = 18 m; friction coefficient  $f = 0.007$ . Half of the daily supply is pumped in 8 hours. Determine the size of the supply main.
  - Appeared in:** 3141906(5) (Q3c, 07 marks)
  - Frequency Tag:** Most Repeated
  - Input Variables:**  $L = 4000$  m, population = 3000, per capita consumption = 180 L/day,  $h_f = 18$  m,  $f = 0.007$ , pumping time = 8 hrs (for half daily supply)

**Answer:**

Length  $L = 4000$  m

Population = 3000, consumption per inhabitant per day = 180 litres

Head loss due to friction  $h_f = 18$  m

Friction coefficient  $f = 0.007$  (Darcy friction factor)

Half of daily supply pumped in 8 hours.

**To find:** Size of supply main (diameter  $D$ )

**Solution:**

**Step 1 – Daily total supply**

Total per day =  $3000 \times 180 = 540,000$  litres =  $540 \text{ m}^3$

**Step 2 – Pumping rate**

Half daily supply =  $270 \text{ m}^3$  pumped in 8 hours.

Flow rate  $Q = \frac{270}{8 \times 3600} = \frac{270}{28800} = 0.009375 \text{ m}^3/\text{s} = 9.375 \text{ L/s}$

**Step 3 – Darcy-Weisbach equation**

$$h_f = \frac{fLQ^2}{12.1D^5} \text{ (for pipe flow, in consistent units)}$$

Actually standard:  $h_f = \frac{fL}{2gD} V^2$ , with  $V = \frac{4Q}{\pi D^2}$

Substitute:

$$h_f = \frac{fL}{2gD} \cdot \left( \frac{16Q^2}{\pi^2 D^4} \right) = \frac{8fLQ^2}{g\pi^2 D^5}$$

Using  $g=9.81$ ,  $\pi^2 \approx 9.8696$ :

$$h_f = \frac{8fLQ^2}{9.81 \times 9.8696 D^5} = \frac{8fLQ^2}{96.86 D^5} \approx \frac{fLQ^2}{12.1 D^5}$$

$$\text{So } D^5 = \frac{fLQ^2}{12.1 h_f}$$

**Step 4 – Compute**

$$f = 0.007, L = 4000, Q = 0.009375, h_f = 18$$

$$Q^2 = 8.789 \times 10^{-5}$$

$$fLQ^2 = 0.007 \times 4000 \times 8.789 \times 10^{-5} = 28 \times 8.789 \times 10^{-5} = 2.461 \times 10^{-3}$$

$$D^5 = \frac{2.461 \times 10^{-3}}{12.1 \times 18} = \frac{2.461 \times 10^{-3}}{217.8} = 1.130 \times 10^{-5}$$

$$D = (1.130 \times 10^{-5})^{1/5}$$

$$0.1^5 = 1 \times 10^{-5}, 0.11^5 = 1.61 \times 10^{-5} \rightarrow \text{interpolate: } D \approx 0.102 \text{ m} = 102 \text{ mm}$$

**Final Answer:**

$$D \approx 100 \text{ mm (say 100 mm pipe)}$$

2. Calculate the discharge through a pipe of diameter 250 mm when the difference of pressure head between two ends of a pipe 500 mm apart is 3.5 m of water. Take friction factor = 0.04.
- **Appeared in:** 3141906(5) (OR Q4b, 07 marks)
  - **Frequency Tag:** Most Repeated
  - **Input Variables:** D = 250 mm, length L = 500 mm (likely a typo – probably 500 m),  $h_f = 3.5$  m water,  $f = 0.04$

**Answer:****Given:**

$$\text{Diameter } D = 250 \text{ mm} = 0.25 \text{ m}$$

$$\text{Length } L = 500 \text{ mm?} \rightarrow \text{Likely typo, should be 500 m. Assume } L = 500 \text{ m}$$

$$\text{Head loss } h_f = 3.5 \text{ m of water}$$

$$\text{Friction factor } f = 0.04$$

**To find:** Discharge  $Q$ **Solution:**

Darcy-Weisbach:

$$h_f = \frac{fLQ^2}{12.1D^5}$$

$$Q^2 = \frac{h_f \times 12.1D^5}{fL}$$

$$D^5 = (0.25)^5 = 0.25^2 = 0.0625, \times 0.0625 = 0.00390625, \times 0.25 = 0.00097656$$

$$12.1D^5 = 12.1 \times 0.00097656 = 0.011816$$

$$h_f \times 12.1D^5 = 3.5 \times 0.011816 = 0.041356$$

$$Q^2 = \frac{0.041356}{0.04 \times 500} = \frac{0.041356}{20} = 0.0020678$$

$$Q = \sqrt{0.0020678} = 0.04547 \text{ m}^3/\text{s} = 45.47 \text{ L/s}$$

**Final Answer:**

$$Q = 45.5 \text{ L/s}$$

3. Determine the optimum diameter of a pipe to carry 100 L/s of crude oil (density = 950 kg/m<sup>3</sup>, dynamic viscosity = 0.08 kg/(m · s)) and still maintain laminar flow. Also determine the power required for transport over one kilometre distance.
- **Appeared in:** WIN24 (OR Q4c, 07 marks)
  - **Frequency Tag:** Most Repeated
  - **Input Variables:** Q = 100 L/s = 0.1 m<sup>3</sup>/s, ρ = 950 kg/m<sup>3</sup>, μ = 0.08 Pa·s, L = 1 km, flow regime = laminar

**Answer:**

**Given:**

$$Q = 100 \text{ L/s} = 0.1 \text{ m}^3/\text{s}$$

$$\rho = 950 \text{ kg/m}^3, \mu = 0.08 \text{ Pa} \cdot \text{s}$$

$$L = 1 \text{ km} = 1000 \text{ m}$$

Flow regime: laminar

**To find:** Optimum diameter (maximum possible for laminar?) Actually “optimum diameter” to maintain laminar flow – means find diameter such that  $Re \leq 2000$  (critical). Then power required.

**Solution:**

**Step 1 – Reynolds number for laminar limit**

$$Re = \frac{\rho V D}{\mu} \leq 2000$$

$$V = \frac{4Q}{\pi D^2} \rightarrow Re = \frac{4\rho Q}{\pi \mu D}$$

Set  $Re = 2000$ :

$$2000 = \frac{4 \times 950 \times 0.1}{\pi \times 0.08 \times D} = \frac{380}{0.2513D} = \frac{1512.5}{D}$$

$$D = \frac{1512.5}{2000} = 0.756 \text{ m}$$

Thus for laminar flow, D must be  $\geq 0.756$  m (larger D gives lower Re). Optimum = minimum D that keeps laminar? Usually largest possible for laminar is 0.756 m.

**Step 2 – Power required**

For laminar flow, Hagen-Poiseuille:

$$h_f = \frac{128\mu L Q}{\pi \rho g D^4}$$

$$\text{But power } P = \rho g Q h_f = \frac{128\mu Q^2}{\pi D^4}$$

With  $D = 0.756$  m:

$$D^4 = (0.756)^4 = 0.756^2 \times 0.756^2 = 0.5715 \times 0.5715 = 0.3266$$

$$P = \frac{128 \times 0.08 \times 1000 \times (0.1)^2}{\pi \times 0.3266} = \frac{128 \times 0.08 \times 1000 \times 0.01}{1.026}$$

$$= \frac{102.4}{1.026} = 99.8 \text{ W}$$

**Final Answer:**

$$\boxed{D_{\text{opt}} = 0.756 \text{ m}, P = 100 \text{ W}}$$

**Topic/Formula Group: Flow Between Parallel Plates****Marks Group: 7****Numerical Question:**

- Glycerine of specific gravity 1.28 and viscosity 8.07 poise flows between two large parallel flat plates 1.5 cm apart. The rate of flow is 4.4 m<sup>2</sup>/hour per metre width of the plates. Determine: (i) maximum velocity, (ii) maximum shear stress, (iii) pressure gradient, (iv) Reynolds number.
  - Appeared in:** 3141906 (OR Q3c, 07 marks)
  - Frequency Tag:** Single/Unique
  - Input Variables:** SG = 1.28,  $\mu = 8.07$  poise = 0.807 Pa·s, gap  $h = 1.5$  cm = 0.015 m, flow rate per unit width = 4.4 m<sup>2</sup>/hr = 4.4/3600 m<sup>2</sup>/s

**Answer:****Given:**

$$SG = 1.28 \rightarrow \rho = 1280 \text{ kg/m}^3$$

$$\text{Viscosity } \mu = 8.07 \text{ poise} = 0.807 \text{ Pa} \cdot \text{s}$$

$$\text{Gap } h = 1.5 \text{ cm} = 0.015 \text{ m}$$

$$\text{Flow rate per unit width } q = 4.4 \text{ m}^2/\text{hour} = 4.4/3600 = 0.001222 \text{ m}^2/\text{s}$$

**To find:** (i) maximum velocity, (ii) maximum shear stress, (iii) pressure gradient, (iv) Reynolds number

**Solution for laminar flow between parallel plates (both stationary):****Step 1 – Pressure gradient**

$$\text{For plane Poiseuille flow: } q = \frac{h^3}{12\mu} \left( -\frac{dp}{dx} \right)$$

$$-\frac{dp}{dx} = \frac{12\mu q}{h^3} = \frac{12 \times 0.807 \times 0.001222}{(0.015)^3}$$

$$(0.015)^3 = 3.375 \times 10^{-6}$$

$$12 \times 0.807 \times 0.001222 = 12 \times 0.000986 = 0.01183$$

$$-\frac{dp}{dx} = \frac{0.01183}{3.375 \times 10^{-6}} = 3505 \text{ Pa/m}$$

**Step 2 – Maximum velocity**

$$u_{\max} = \frac{h^2}{8\mu} \left( -\frac{dp}{dx} \right) = \frac{(0.015)^2}{8 \times 0.807} \times 3505 = \frac{0.000225}{6.456} \times 3505 = 3.485 \times 10^{-5} \times 3505$$

$$= 0.1222 \text{ m/s}$$

**Step 3 – Maximum shear stress**

$$\text{At wall: } \tau_{\max} = \frac{h}{2} \left( -\frac{dp}{dx} \right) = \frac{0.015}{2} \times 3505 = 0.0075 \times 3505 = 26.29 \text{ Pa}$$

**Step 4 – Reynolds number**

$$\text{Hydraulic diameter } D_h = 2h = 0.03 \text{ m}$$

$$\text{Average velocity } V_{\text{avg}} = q/h = 0.001222/0.015 = 0.08147 \text{ m/s}$$

$$Re = \frac{\rho V D_h}{\mu} = \frac{1280 \times 0.08147 \times 0.03}{0.807} = \frac{3.128}{0.807} = 3.876 \text{ (laminar)}$$

**Final Answer:**

$$u_{\max} = 0.122 \text{ m/s}, \tau_{\max} = 26.3 \text{ Pa}, -\frac{dp}{dx} = 3505 \text{ Pa/m}, Re = 3.88$$

**Topic/Formula Group: Minor Losses (Contraction)****Marks Group: 7****Numerical Question:**

1. A pipe of 150 mm diameter is attached to a 100 mm diameter pipe by means of a flange in the same horizontal axis. Rate of flow is  $2 \text{ m}^3/\text{min}$  and a manometer shows a pressure difference reading of 80 mm. Find: (i) loss of head due to contraction, (ii) coefficient of contraction.
  - **Appeared in:** 3141906 (Q3c, 07 marks)
  - **Frequency Tag:** Single/Unique
  - **Input Variables:**  $D_1 = 150 \text{ mm}$ ,  $D_2 = 100 \text{ mm}$ ,  $Q = 2 \text{ m}^3/\text{min} = 1/30 \text{ m}^3/\text{s}$ , manometer reading = 80 mm (fluid unspecified)

**Answer:****Given:**

$$D_1 = 150 \text{ mm} = 0.15 \text{ m}$$

$$D_2 = 100 \text{ mm} = 0.10 \text{ m}$$

$$Q = 2 \text{ m}^3/\text{min} = 2/60 = 0.03333 \text{ m}^3/\text{s}$$

Manometer reading (pressure difference) = 80 mm of (fluid unspecified – assume mercury or water? Usually water in manometer, but let's assume water for simplicity; if mercury, convert). Assume manometer fluid = water.

**To find:** (i) Loss of head due to contraction, (ii) Coefficient of contraction  $C_c$

**Solution:****Step 1 – Velocities**

$$A_1 = \frac{\pi}{4} (0.15)^2 = 0.01767 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (0.1)^2 = 0.007854 \text{ m}^2$$

$$V_1 = Q/A_1 = 0.03333/0.01767 = 1.886 \text{ m/s}$$

$$V_2 = Q/A_2 = 0.03333/0.007854 = 4.245 \text{ m/s}$$

**Step 2 – Pressure drop from manometer**

Manometer reading 80 mm water  $\rightarrow \Delta P = \rho g h = 1000 \times 9.81 \times 0.08 = 784.8 \text{ Pa}$

Head difference = 0.08 m of water.

**Step 3 – Loss of head**

Apply Bernoulli between section 1 (upstream) and section 2 (downstream in smaller pipe) including loss:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_L$$

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = 0.08 \text{ m (from manometer)}$$

$$\frac{V_1^2 - V_2^2}{2g} = \frac{(1.886)^2 - (4.245)^2}{2 \times 9.81} = \frac{3.556 - 18.02}{19.62} = \frac{-14.464}{19.62} = -0.737 \text{ m}$$

$h_L = 0.08 + (-0.737) = -0.657 \text{ m}$  – negative? That implies pressure drop is not enough to account for velocity increase. Actually for contraction,  $P_1 - P_2$  is positive and large. The manometer reading should be in mercury to get a larger head. Let's assume the 80 mm is mercury.

Recalc with mercury:  $\frac{P_1 - P_2}{\rho_w g} = x \left( \frac{\rho_m}{\rho_w} - 1 \right) = 0.08 \times (13.6 - 1) = 0.08 \times 12.6 =$

1.008 m of water

Then  $h_L = 1.008 - 0.737 = 0.271$  m

#### Step 4 – Coefficient of contraction

Loss head for sudden contraction:  $h_L = \frac{V_2^2}{2g} \left( \frac{1}{C_c} - 1 \right)^2$

$$0.271 = \frac{(4.245)^2}{2 \times 9.81} \left( \frac{1}{C_c} - 1 \right)^2 = \frac{18.02}{19.62} \left( \frac{1}{C_c} - 1 \right)^2 = 0.9185 \left( \frac{1}{C_c} - 1 \right)^2$$

$$\left( \frac{1}{C_c} - 1 \right)^2 = \frac{0.271}{0.9185} = 0.295$$

$$\frac{1}{C_c} - 1 = \sqrt{0.295} = 0.543$$

$$\frac{1}{C_c} = 1.543 \Rightarrow C_c = 0.648$$

**Final Answer:**

$$h_L = 0.271 \text{ m water, } C_c = 0.648$$

### Assignment – 3 ( Question Related To unit-6&7)

**Q-1: Reynolds Number & Flow Regime: Jatropa oil (Specific Gravity = 0.92, dynamic viscosity = 0.035 Pa.s) flows through a 50 mm diameter pipe. Calculate the maximum volumetric flow rate (Q) for which the flow will remain strictly laminar (Assume critical Reynolds number  $Re = 2000$ ).**

**Answer:**

**Given:**

$$SG = 0.92 \rightarrow \rho = 920 \text{ kg/m}^3$$

$$\mu = 0.035 \text{ Pa} \cdot \text{s}$$

$$D = 50 \text{ mm} = 0.05 \text{ m}$$

$$Re_{crit} = 2000$$

**To Find:** Max volumetric flow rate  $Q$  for laminar flow

**Formula:**

$$Re = \frac{\rho V D}{\mu} = 2000 \rightarrow V = \frac{2000}{\rho D}$$

$$Q = V \times \frac{\pi D^2}{4}$$

**Solution:**

$$V = \frac{2000 \times 0.035}{920 \times 0.05} = \frac{70}{46} = 1.5217 \text{ m/s}$$

$$Q = 1.5217 \times \frac{\pi(0.05)^2}{4} = 1.5217 \times 0.0019635 = 0.002988 \text{ m}^3/\text{s}$$

$$Q \approx 2.99 \text{ lps}$$

**Final Answer:**

$$Q_{max} = 0.00299 \text{ m}^3/\text{s} \approx 3 \text{ lps}$$

**Q-2: Hagen-Poiseuille Flow: For the laminar flow of the Jatropha oil calculated in Question 1, determine the pressure drop over a 15 m length of the horizontal pipe. Also, calculate the maximum velocity and the shear stress at the pipe wall.**

**Answer:**

**Given:**

$$\text{From Q1: } Q = 0.002988 \text{ m}^3/\text{s}$$

$$D = 0.05 \text{ m}$$

$$L = 15 \text{ mm} = 0.015 \text{ m}$$

$$\mu = 0.035 \text{ Pa} \cdot \text{s}$$

$$\rho = 920 \text{ kg/m}^3$$

**To Find:**  $\Delta P$ , max velocity  $u_{max}$ , wall shear stress  $\tau_w$

**Solution:**

$$V = \frac{Q}{A} = \frac{0.002988}{\frac{\pi \times 0.05^2}{4}} = 1.5217 \text{ m/s (same as above)}$$

$$\Delta P = \frac{128\mu L Q}{\pi D^4} = \frac{128 \times 0.035 \times 0.015 \times 0.002988}{\pi \times (0.05)^4}$$

$$\text{Num} = 128 \times 0.035 \times 0.015 \times 0.002988 = 128 \times 1.5687 \times 10^{-6} = 0.0002008$$

$$\text{Denom} = \pi \times 6.25 \times 10^{-6} = 1.9635 \times 10^{-5}$$

$$\Delta P = \frac{0.0002008}{1.9635 \times 10^{-5}} = 10.23 \text{ Pa}$$

$$u_{max} = 2V = 3.0434 \text{ m/s}$$

$$\tau_w = \frac{\Delta P \cdot D}{4L} = \frac{10.23 \times 0.05}{4 \times 0.015} = \frac{0.5115}{0.06} = 8.525 \text{ Pa}$$

**Final Answer:**

$$\Delta P = 10.23 \text{ Pa}, u_{max} = 3.04 \text{ m/s}, \tau_w = 8.53 \text{ Pa}$$

**Q-3: Darcy-Weisbach & Friction: Water flows through a rough pipe of diameter 400 mm and length 2500 m at a rate of 0.4 m<sup>3</sup>/s. If the kinematic viscosity of water is 1 x 10<sup>-6</sup> m<sup>2</sup>/s and the absolute roughness is 0.05 mm, use the Moody diagram concept (or Colebrook equation) to find the friction factor and subsequently calculate the major head loss.**

**Answer:**

**Given:**

$$D = 0.4 \text{ m}, L = 2500 \text{ m}$$

$$Q = 0.4 \text{ m}^3/\text{s}$$

$$\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Absolute roughness } \varepsilon = 0.05 \text{ mm} = 5 \times 10^{-5} \text{ m}$$

**To Find:** Friction factor  $f$ , major head loss  $h_f$

**Solution:**

$$V = \frac{Q}{A} = \frac{0.4}{\pi/4 \times (0.4)^2} = \frac{0.4}{0.12566} = 3.183 \text{ m/s}$$

$$Re = \frac{VD}{\nu} = \frac{3.183 \times 0.4}{1 \times 10^{-6}} = 1.273 \times 10^6$$

$$\text{Relative roughness } \varepsilon/D = \frac{5 \times 10^{-5}}{0.4} = 1.25 \times 10^{-4}$$

From Moody chart for  $Re = 1.27 \times 10^6$ ,  $\varepsilon/D = 0.000125$ :  
 $f \approx 0.014$  (smooth pipe zone?) Check Colebrook: For such low roughness,  $f$  close to smooth pipe:  $f = 0.012$  to  $0.014$ . Let's take  $f = 0.013$ .

$$\begin{aligned} h_f &= f \frac{L V^2}{D 2g} = 0.013 \times \frac{2500}{0.4} \times \frac{(3.183)^2}{2 \times 9.81} \\ &= 0.013 \times 6250 \times \frac{10.13}{19.62} = 0.013 \times 6250 \times 0.5165 \\ &= 0.013 \times 3228 = 41.96 \text{ m of water} \end{aligned}$$

**Final Answer:**

$$f \approx 0.013, h_f \approx 42.0 \text{ m}$$

**Q-4: Minor Losses:** An engine cooling system features a sudden expansion in a pipe from 50 mm to 100 mm diameter. If the flow rate of the coolant is 12 lps, calculate the head loss due to the sudden expansion.

**Answer:**

**Given:**

$$\begin{aligned} D_1 &= 50 \text{ mm} = 0.05 \text{ m} \\ D_2 &= 100 \text{ mm} = 0.1 \text{ m} \\ Q &= 12 \text{ lps} = 0.012 \text{ m}^3/\text{s} \end{aligned}$$

Water:  $\rho = 1000$

**To Find:** Head loss  $h_L$

**Formula:**

$$h_L = \frac{(V_1 - V_2)^2}{2g} \text{ or } h_L = K \frac{V_1^2}{2g} \text{ with } K = \left(1 - \frac{A_1}{A_2}\right)^2$$

**Solution:**

$$\begin{aligned} A_1 &= 0.0019635 \text{ m}^2, A_2 = 0.007854 \text{ m}^2 \\ V_1 &= 0.012/0.0019635 = 6.112 \text{ m/s} \\ V_2 &= 0.012/0.007854 = 1.528 \text{ m/s} \\ h_L &= \frac{(6.112 - 1.528)^2}{2 \times 9.81} = \frac{(4.584)^2}{19.62} = \frac{21.01}{19.62} = 1.071 \text{ m} \end{aligned}$$

**Final Answer:**

$$h_L = 1.07 \text{ m}$$

**Q-5: Pipes in Series & Parallel:** Two pipes of lengths 1000 m and 800 m, and diameters 300 mm and 200 mm respectively, are connected in series. Find the diameter of an equivalent pipe of length 1800 m that will carry the same discharge for the same total head loss.

**Answer:**

**Given:**

Pipe 1:  $L_1 = 1000 \text{ m}$ ,  $D_1 = 0.3 \text{ m}$

Pipe 2:  $L_2 = 800 \text{ m}$ ,  $D_2 = 0.2 \text{ m}$

Series connection.

Equivalent pipe:  $L_e = 1800 \text{ m}$ , find  $D_e$

**Formula (Darcy-Weisbach, same  $f$  for all pipes):**

$$h_f = h_{f1} + h_{f2} \rightarrow \frac{fL_e Q^2}{12.1D_e^5} = \frac{fL_1 Q^2}{12.1D_1^5} + \frac{fL_2 Q^2}{12.1D_2^5}$$

Cancel common terms:

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5}$$

**Solution:**

$$\frac{1800}{D_e^5} = \frac{1000}{(0.3)^5} + \frac{800}{(0.2)^5}$$

$$(0.3)^5 = 0.00243, (0.2)^5 = 0.00032$$

$$\frac{1000}{0.00243} = 411522, \frac{800}{0.00032} = 2.5 \times 10^6$$

$$\text{Sum} = 2,911,522$$

$$D_e^5 = \frac{1800}{2911522} = 0.0006183$$

$$D_e = (0.0006183)^{1/5} = (6.183 \times 10^{-4})^{0.2}$$

$$\text{Let's calculate: } \log D_e = \frac{1}{5}(\log 6.183 \times 10^{-4}) = \frac{1}{5}(0.7912 - 4) = \frac{1}{5}(-3.2088) = -0.64176$$

$$D_e = 10^{-0.64176} = 0.228 \text{ m} \approx 228 \text{ mm}$$

**Final Answer:**

$$\boxed{D_e = 228 \text{ mm}}$$