GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – Q&A BANK (Numerical)

Subject Name & Code:
Fluid Mechanics (3141906)

Assignment – 5 (Compressible Flow Fundamentals)

(Demonstrate fundamentals of compressible flow). Covering Unit 8.

Q-1: Mach Number & Speed of Sound: Exhaust gas leaves an internal combustion engine manifold at a temperature of 600 °C. Calculate the speed of sound in this gas and determine the velocity of the gas if the flow is at a Mach number of 0.85. (Assume the gas constant R = 287 J/kgK and the specific heat ratio γ = 1.33).

Answer:
Given:
Gas temp $T = 600°C = 873$ K
$R = 287$ J/kgK, $\gamma = 1.33$
Mach number $M = 0.85$
To Find: Speed of sound a, gas velocity V
Formula:

$$a = \sqrt{\gamma RT}$$
$$V = M \times a$$

Solution:

$$a = \sqrt{1.33 \times 287 \times 873} = \sqrt{1.33 \times 250{,}551} = \sqrt{333{,}233} = 577.3 \text{ m/s}$$
$$V = 0.85 \times 577.3 = 490.7 \text{ m/s}$$

Final Answer:

$$\boxed{a = 577 \text{ m/s}, V = 491 \text{ m/s}}$$

Q-2: Stagnation Properties: Air flows through a turbocharger intake duct at a velocity of 250 m/s a static pressure of 90 kPa and a static temperature of 15°C. Calculate the stagnation pressure and stagnation temperature. (Assume air properties: R = 287 J/kg.K, γ = 1.4, Cp = 1005 J/kgK.

Answer:
Given:

$$V = 250 \text{ m/s}$$

$P = 90$ kPa, $T = 15°C = 288$ K
$\gamma = 1.4$, $C_p = 1005$ J/kgK, $R = 287$
To Find: P_0, T_0
Solution:
Mach number: $a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 288} = \sqrt{115{,}718} = 340.2$ m/s

$$M = 250/340.2 = 0.735$$

$$T_0 = T\left(1+\frac{\gamma-1}{2}M^2\right) = 288(1+0.2\times(0.735)^2) = 288(1+0.108) = 288\times 1.108 = 319.1\ \text{K}$$

$$P_0 = P\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 90(1.108)^{3.5}$$

$$1.108^{3.5} = e^{3.5\ln\ 1.108} = e^{3.5\times 0.1026} = e^{0.3591} = 1.432$$

$$P_0 = 90\times 1.432 = 128.9\ \text{kPa}$$

Final Answer:

$$\boxed{T_0 = 319\ \text{K}, P_0 = 129\ \text{kPa}}$$

Q-3: Isentropic Flow in Nozzles: Air enters a converging-diverging nozzle from a reservoir where the pressure is 500 kPa and the temperature is 400 K. If the flow is isentropic, calculate the pressure, temperature, and velocity at the throat where the Mach number is 1.0.

Answer:
Given:
Reservoir: $P_0 = 500\ \text{kPa}$, $T_0 = 400\ \text{K}$
$\gamma = 1.4$ (air assumed)
To Find: At throat: P_*, T_*, V_*
Formula:

$$\frac{P_*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_*}{T_0} = \frac{2}{\gamma+1}$$

$$V_* = a_* = \sqrt{\gamma R T_*}$$

Solution:

$$\frac{P_*}{P_0} = \left(\frac{2}{2.4}\right)^{3.5} = (0.8333)^{3.5} = e^{3.5\ln\ 0.8333} = e^{3.5\times(-0.1823)} = e^{-0.638} = 0.528$$

$$P_* = 500\times 0.528 = 264\ \text{kPa}$$

$$\frac{T_*}{T_0} = \frac{2}{2.4} = 0.8333 \Rightarrow T_* = 400\times 0.8333 = 333.3\ \text{K}$$

$$a_* = \sqrt{1.4\times 287\times 333.3} = \sqrt{133{,}933} = 366\ \text{m/s}$$

Final Answer:

$$\boxed{P_* = 264\ \text{kPa}, T_* = 333\ \text{K}, V_* = 366\ \text{m/s}}$$

Q-4: Compressibility Regimes: Based on the compressibility of fluids, classify the flow regimes corresponding to Mach numbers M < 0.3, 0.8 < M < 1.2, and M > 1.2. Explain why flow behaves fundamentally differently in the supersonic regime compared to the subsonic regime.

Answer:

Mach Range	Regime	Behavior Difference
$M < 0.3$	Incompressible	Density changes negligible
$0.8 < M < 1.2$	Transonic	Mixed sub/supersonic, shock waves appear
$M > 1.2$	Supersonic	Shock waves, expansion fans, density changes dominate

Why different in supersonic?
Pressure disturbances cannot propagate upstream; flow behavior governed by oblique shocks and Prandtl-Meyer expansion instead of isentropic smooth changes.

Q-5: Mass Flow Rate Formulation: Derive the expression for the mass flow rate in a 1D isentropic flow in terms of the stagnation pressure (P0), stagnation temperature (T0), Mach number (M), and cross-sectional area (A).

Answer:
Given: 1D isentropic flow, P_0, T_0, M, A
Derivation:

$$\dot{m} = \rho A V$$

$\rho = \frac{P}{RT}$, $P = P_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}}$

$$T = T_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}$$

$$V = M\sqrt{\gamma RT} = M\sqrt{\gamma R T_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}}$$

Substitute and simplify:

$$\dot{m} = \frac{P_0 A M}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Final Answer:

$$\boxed{\dot{m} = \frac{P_0 A M}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

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