GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – MID TERM – S26 – ANSWER BANK

Subject Name & Code:

FLUID MECHANICS - BE04000161

Important Question For Mid Sem & GTU Exam - Answer Bank

3 Marks Questions

Q1. State Pascal's law and briefly explain its real-world application in automotive hydraulic brakes.

- **Pascal's law** states that pressure applied to an enclosed fluid at rest is transmitted equally and undiminished in all directions throughout the fluid.
- **Real-world application (automotive hydraulic brakes):**
    - Driver applies force on brake pedal → master cylinder creates pressure.
    - Pressure transmits equally through brake fluid to all wheel cylinders (slave cylinders).
    - Same pressure acts on larger-area pistons → force multiplication → braking force at wheels.

Q2. Explain the concept of the fluid continuum. Why is this assumption necessary in macroscopic fluid mechanics?
Ans:

- **Fluid continuum** assumes that fluid properties (density, velocity, pressure, etc.) vary continuously in space and time, ignoring molecular structure.
- Fluid is treated as a continuous, infinitely divisible substance with no gaps.
- **Necessity in macroscopic fluid mechanics:**
    - Allows use of differential calculus (partial differential equations like Navier-Stokes).
    - Properties at a point are defined as statistical averages over a small volume containing many molecules.
    - Fails only in rarefied gases (very low density) or nanoscale flows.

Q3. Define compressibility and bulk modulus. How are these two fluid properties related to each other?

- **Compressibility (β)** = fractional change in volume per unit change in pressure:
$$\beta = -\frac{1}{V}\frac{dV}{dP}$$
- **Bulk modulus (K)** = resistance to compression = reciprocal of compressibility:
$$K = \frac{1}{\beta} = -V\frac{dP}{dV}$$
- **Relationship:** $K = \frac{1}{\beta}$ → higher bulk modulus means lower compressibility (fluid is stiffer).
- **Example:** Water has high K (~2.2 GPa), hence nearly incompressible; air has low K,

highly compressible.

Q4. What is an inclined manometer? State its primary advantage over a standard vertical U-tube manometer for pressure measurement.

- **Inclined manometer** – a U-tube manometer with one limb inclined at an angle θ to the horizontal.
- Pressure difference causes a small vertical rise h, but the liquid moves a longer distance L along the inclined tube: $h = L\sin\,\theta$.
- **Primary advantage:**
  - **Increased sensitivity** – for the same pressure difference, the inclined tube gives a longer liquid column movement (L = h / sinθ).
  - Enables measurement of very small pressure differences (draft pressures, low velocities).

Q5. A sample of a Jatropha oil biodiesel blend has a mass of 850 kg and occupies a volume of 0.95 m³. Calculate its density, specific weight, and specific gravity.

Step 1: Given Data

- Mass, $m = 850\,\mathrm{kg}$
- Volume, $V = 0.95\,\mathrm{m}^3$
- Acceleration due to gravity, $g = 9.81\,\mathrm{m/s}^2$ (Standard value)
- Density of water, $\rho_w = 1000\,\mathrm{kg/m}^3$ (Standard value)

Step 2: Calculate Density (ρ)

$$\rho = \frac{m}{V} = \frac{850}{0.95}$$
$$\rho = 894.74\,\mathrm{kg/m}^3$$

Step 3: Calculate Specific Weight (γ)

$$\gamma = \rho \cdot g = 894.74 \times 9.81$$
$$\gamma = 8777.4\,\mathrm{N/m}^3 \mathrm{or} 8.777\,\mathrm{kN/m}^3$$

Step 4: Calculate Specific Gravity (s)

$$s = \frac{\rho}{\rho_w} = \frac{894.74}{1000}$$
$$s = 0.895$$

Final Answer:

$$\boxed{\rho = 894.74\,\mathrm{kg/m}^3,\ \gamma = 8777.4\,\mathrm{N/m}^3,\ s = 0.895}$$

Q6. Calculate the capillary rise of a fuel in a glass tube of a fuel injector with a 2.5 mm diameter. The surface tension of the fuel is 0.025 N/m, its specific gravity is 0.82, and the contact angle is 0°.

Step 1: Given Data

- Diameter, $d = 2.5\,\mathrm{mm} = 0.0025\,\mathrm{m}$
- Radius, $r = \frac{d}{2} = 0.00125\,\mathrm{m}$
- Surface tension, $\sigma = 0.025\,\mathrm{N/m}$
- Specific Gravity, $s = 0.82$
- Contact angle, $\theta = 0^\circ \Rightarrow \cos\,\theta = 1$
- Density of water, $\rho_w = 1000\,\mathrm{kg/m}^3$

- $g = 9.81\ \text{m/s}^2$

Step 2: Calculate Density of Fuel

$$\rho = s \cdot \rho_w = 0.82 \times 1000 = 820\ \text{kg/m}^3$$

Step 3: Apply Capillary Rise Formula

$$h = \frac{2\sigma\cos\ \theta}{\rho g r}$$

Substitute the values:

$$h = \frac{2 \times 0.025 \times 1}{820 \times 9.81 \times 0.00125}$$

Step 4: Simplify the Expression

$$h = \frac{0.05}{820 \times 9.81 \times 0.00125}$$

First, calculate the denominator:

$$820 \times 9.81 = 8044.2$$
$$8044.2 \times 0.00125 = 10.05525$$

Thus:

$$h = \frac{0.05}{10.05525} = 0.004972\ \text{m}$$

Step 5: Convert to Millimeters

$$h = 0.004972 \times 1000 = 4.97\ \text{mm}$$

Final Answer:

$$\boxed{h = 4.97\ \text{mm}}$$

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4 Marks Questions

Q1. Differentiate between dynamic viscosity and kinematic viscosity. Discuss their specific significance when selecting lubricants for machinery.

- **Differentiation:**

Feature	Dynamic Viscosity (μ)	Kinematic Viscosity (ν)
Definition	Shear stress per velocity gradient ($\mu = \tau / (du/dy)$)	Ratio of dynamic viscosity to density ($\nu = \mu/\rho$)
Symbol	μ (mu)	ν (nu)
Formula	$\mu = \tau / (du/dy)$	$\nu = \mu / \rho$
Units (SI)	Pascal-second (Pa·s) or $N\cdot s/m^2$	m^2/s (Stoke, cSt)
Nature	Represents absolute viscosity, independent of fluid density.	Represents viscous effect coupled with inertial effect (density).
Physical meaning	Internal friction resistance to flow	Diffusivity of momentum
Influenced by	Temperature & pressure	Temperature, pressure & density

Significance for lubricant selection:

- **Dynamic viscosity** – determines oil film thickness and load-carrying capacity in bearings, gears (high μ = better film at low speeds).
- **Kinematic viscosity** – used in viscosity grades (ISO VG, SAE) for pumps, hydraulic systems; affects flow rate, energy loss, and startup behavior (low ν at cold start may cause insufficient lubrication).
- **Real-world:** Engine oils must have high ν at high temperature to maintain film, but low ν at low temperature for easy starting.

Q2. Describe the concept of vapor pressure. Explain the mechanism by which it leads to cavitation in pumps and cooling systems.

Ans:

- **Vapor pressure** – the pressure at which a liquid boils (changes to vapor) at a given temperature.
- At a given temperature, if absolute pressure falls below vapor pressure, liquid vaporizes spontaneously.

Cavitation mechanism:

1. In pumps or cooling systems, local pressure drops (e.g., at pump impeller inlet, sharp edges, or narrow passages).
2. When pressure ≤ vapor pressure, tiny vapor bubbles form (cavities).
3. Bubbles travel to higher-pressure regions and **collapse violently**, producing shock waves.
4. Repeated collapse erodes metal surfaces (pitting), causes noise, vibration, and loss of

efficiency.

Consequences:

- Pump damage, reduced flow, overheating in cooling systems.
- **Prevention:** Maintain NPSH (Net Positive Suction Head) above vapor pressure.

Q3. Discuss the phenomenon of surface tension and capillarity. How do these properties influence the performance of fuel injectors and brake fluids?

Ans:

- **Surface tension (σ)** – cohesive force between liquid molecules at the free surface, acting like a stretched membrane.
- **Capillarity** – rise or fall of liquid in a small tube due to combination of surface tension and adhesive forces.

Influence on fuel injectors:

- Capillary rise in injector nozzle holes can affect fuel atomization.
- Surface tension resists droplet breakup; lower σ (e.g., with additives) produces finer spray → better combustion.
- Capillary effects can cause fuel retention or clogging in micro-orifices.

Influence on brake fluids:

- Brake fluid must have **low surface tension** to wet rubber seals and metal surfaces, preventing leaks.
- Capillarity helps fluid penetrate narrow clearances (e.g., between piston and cylinder) for proper sealing.
- High surface tension could cause vapor lock or poor bleeding.

Q4. Differentiate between absolute pressure, gauge pressure, and vacuum pressure. Provide the mathematical relationship between them.

Ans:

Type	Definition	Reference	Example
Absolute pressure (Pabs)	Pressure measured relative to perfect vacuum (zero pressure)	Absolute zero	Atmospheric pressure = 101.3 kPa abs
Gauge pressure (Pgauge)	Pressure measured relative to local atmospheric pressure	Atmospheric	Tire pressure = 220 kPa gauge
Vacuum pressure (Pvac)	Pressure below atmospheric; also called negative gauge pressure	Atmospheric	Suction pressure = -30 kPa gauge = 30 kPa vacuum

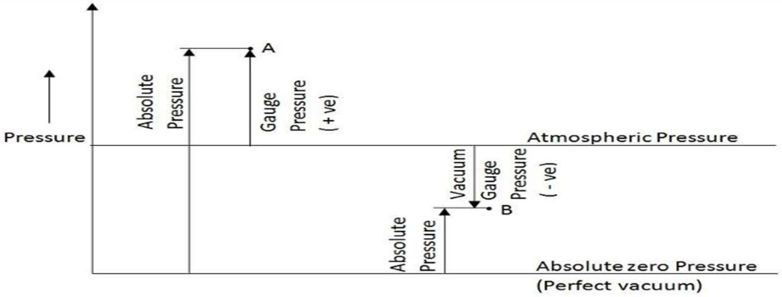


Fig.1.14 – Relationship between pressures

- **Mathematical Relationships:**

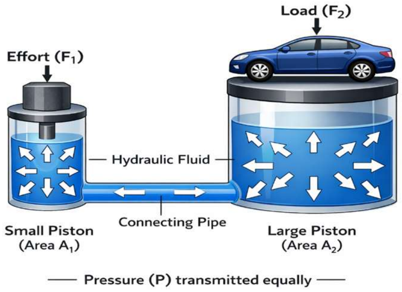
Absolute pressure = Atmospheric pressure + Gauge pressure

$$p_{abs} = p_{atm} + p_{gauge}$$

Vacuum pressure = Atmospheric pressure - Absolute pressure.

$$p_{vac} = p_{atm} - p_{abs}$$

Q5. With the help of a neat sketch, explain the working principle of a hydraulic lift based on Pascal's law.
Diagram Generation Prompt:



- **Working Principle (Pascal's Law):**
  Working:
- Two connected cylinders: small (master) and large (slave), filled with oil.
- Force F_1 applied on small piston area A_1 → pressure $p = F_1/A_1$.
- Same pressure acts on large piston area A_2 → force $F_2 = p \times A_2 = F_1 \times (A_2/A_1)$.
- Since $A_2 >> A_1$, F_2 is many times larger than F_1.
  Advantages: Heavy loads lifted with small effort. **Applications:** Car lifts, hydraulic jacks, presses.

Q6. A flat plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a shear stress of 2.5 N/m² to maintain this velocity. Determine the dynamic and kinematic viscosity of the lubricant between the plates if its specific gravity is 0.85.

Step 1: Given Data

- Distance between plates, $dy = 0.05\,\text{mm} = 0.05 \times 10^{-3} = 5 \times 10^{-5}\,\text{m}$
- Velocity of moving plate, $u = 1.2\,\text{m/s}$
- Since the fixed plate is stationary, velocity gradient, $du = u - 0 = 1.2\,\text{m/s}$
- Shear stress, $\tau = 2.5\,\text{N/m}^2$
- Specific Gravity, $s = 0.85$
- Density of water, $\rho_w = 1000\,\text{kg/m}^3$

Step 2: Calculate Velocity Gradient

$$\frac{du}{dy} = \frac{1.2}{5 \times 10^{-5}} = 24000\,\text{s}^{-1}$$

Step 3: Apply Newton's Law of Viscosity to Find Dynamic Viscosity (μ)

Newton's law states:

$$\tau = \mu \frac{du}{dy}$$

Rearranging for μ:

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{2.5}{24000}$$

$$\mu = 1.04167 \times 10^{-4}\ \mathrm{Pa \cdot s}$$

Step 4: Calculate Density of Lubricant (ρ)

$$\rho = s \cdot \rho_w = 0.85 \times 1000 = 850\ \mathrm{kg/m^3}$$

Step 5: Calculate Kinematic Viscosity (ν)

The relationship is:

$$\nu = \frac{\mu}{\rho} = \frac{1.04167 \times 10^{-4}}{850}$$

$$\nu = 1.2255 \times 10^{-7}\ \mathrm{m^2/s}$$

Final Answer:

$$\boxed{\mu = 1.042 \times 10^{-4}\ \mathrm{Pa \cdot s},\ \nu = 1.226 \times 10^{-7}\ \mathrm{m^2/s}}$$

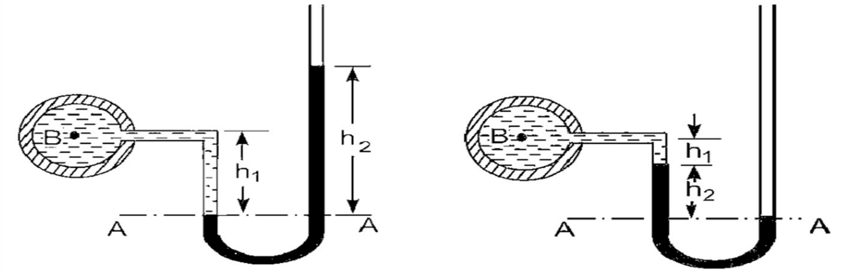
7 Marks Questions

Q1. Explain the working principle of a differential U-tube manometer. Derive the hydrostatic equation used to determine the pressure difference between two points in a pipe containing a fluid under pressure.

- **Working Principle:**
  A differential U-tube manometer measures the pressure difference between two points (A and B) in a pipeline or between two vessels.
- It consists of a U-shaped tube containing a heavy manometric fluid (e.g., mercury).
- The two ends are connected to points A and B. The difference in manometric fluid levels gives the pressure difference.

- **Derivation of Hydrostatic Equation:**

Diagram:



(a) For gauge pressure (b) For vacuum pressure

Fig.1.16 – U-tube Manometer

Consider two points A and B at different elevations, with a manometer fluid of density ρ_m. The fluid in the pipes has density ρ.

Let:

- Pressure at A = P_A, at B = P_B.
- Elevation difference between A and B = Δz (positive if A is higher).
- Manometer reading = h (difference in manometer fluid levels).

Starting from A, move through the manometer:

$P_A + \rho g(z_A - z_1)$ (down to interface) then through manometer fluid: $-\rho_m gh$ then through pipe fluid from interface to B:

$$+\rho g(z_2 - z_B)$$

Set equal to P_B:

$$P_A + \rho g(z_A - z_1) - \rho_m gh + \rho g(z_2 - z_B) = P_B$$

Note that (z_A - z_1) + (z_2 - z_B) = (z_A - z_B) - (z_1 - z_2) = Δz - h (if manometer fluid occupies height h).

Thus:

$P_A - P_B = \rho_m gh - \rho g(\Delta z - h) - \rho gh$? Wait, careful.

Simplified common form (for horizontal pipes, Δz=0):

$$P_A - P_B = (\rho_m - \rho)gh$$

General case (including elevation):

$$P_A + \rho g Z_A = P_B + \rho g Z_B + (\rho_m - \rho)gh$$

or

$$P_A - P_B = \rho g(Z_B - Z_A) + (\rho_m - \rho)gh$$

Where Z_A, Z_B are vertical coordinates of A and B from a datum.

Final equation (most common form):

$$\boxed{P_A - P_B = (\rho_m - \rho)gh + \rho g(Z_B - Z_A)}$$

Q2. Discuss the conditions of equilibrium (stable, unstable, and neutral) for floating and submerged bodies. Explain how buoyancy and stability principles dictate the design of automotive fuel tanks and float gauges.

Ans:

Equilibrium conditions for submerged bodies (completely underwater):

- **Stable:** Center of gravity (G) below center of buoyancy (B) → righting moment when tilted.
- **Unstable:** G above B → overturning moment.
- **Neutral:** G coincides with B → no moment, remains at any tilt.

Equilibrium conditions for floating bodies (partially submerged):

Stability depends on **metacentric height (GM)**.

- **Stable:** Metacenter (M) above G → GM > 0.
- **Unstable:** M below G → GM < 0.
- **Neutral:** M coincides with G → GM = 0.

Buoyancy principle: Upward force = weight of displaced fluid.

Application to automotive fuel tanks:

- Fuel tanks are floating bodies (partially filled with fuel + air).
- Tank shape is wide and flat → low center of gravity → high GM → stable against sloshing.
- Baffles inside reduce fuel movement, maintaining B position.
- Low fuel level: G may shift; designers ensure GM remains positive.

Application to float gauges (fuel level sensor):

- Float (hollow) rides on fuel surface – a floating body.
- Float must be stable (wide, low G) so it doesn't tilt and give false readings.
- Buoyancy force changes with fuel height, moving a potentiometer or magnetic sensor.
- Metacentric height design ensures float remains upright even during vehicle motion.

Q3. Derive an expression for the total hydrostatic force and the position of the center of pressure for a vertically submerged plane surface. Explain why the center of pressure always lies below the center of gravity.

1. Total Hydrostatic Force (F)

Consider a vertical plane surface of total area A, submerged in a static liquid of density ρ.

- Free surface of liquid at top.
- Take an elemental horizontal strip of area dA = b × dh at a depth h below the free surface.
- Pressure on the strip: $p = \rho g h$
- Force on the strip: $dF = p \times dA = \rho g h \times dA$

Integrate over entire area:

$$F = \int dF = \int_A \rho g h \, dA = \rho g \int_A h \, dA$$

The integral ∫ h dA is the **first moment of area** about the free surface.

Let $\bar{h}$ = depth of centroid (center of gravity of the area) from free surface.

Then:

$$\int_A h \, dA = A \bar{h}$$

Therefore:

$$\boxed{F = \rho g \bar{h} A}$$

In words: Total hydrostatic force = pressure at centroid × total area.

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2. Center of Pressure (h*)

Center of pressure is the point where the resultant force F acts. Its depth h* from free surface is found by taking moments of the distributed force about the free surface.
Moment of resultant force about free surface: $F \times h^*$
Moment of distributed forces about free surface:

$$\int h\, dF = \int h(\rho g h\, dA) = \rho g \int h^2 dA$$

Set equal:

$$F \cdot h^* = \rho g \int h^2 dA$$

$$h^* = \frac{\rho g \int h^2 dA}{F}$$

But $F = \rho g \bar{h} A$, so:

$$h^* = \frac{\int h^2 dA}{\bar{h} A}$$

The integral ∫ h² dA is the **second moment of area** (moment of inertia) about the free surface axis. Denote it as I_o.
Thus:

$$h^* = \frac{I_o}{\bar{h} A}$$

Using **parallel axis theorem**:

$$I_o = I_G + A\bar{h}^2$$

where I_G = moment of inertia of the area about its own centroidal axis (parallel to free surface).
Substitute:

$$h^* = \frac{I_G + A\bar{h}^2}{\bar{h} A} = \frac{I_G}{\bar{h} A} + \bar{h}$$

Final expression:

$$\boxed{h^* = \bar{h} + \frac{I_G}{A\bar{h}}}$$

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3. Why Center of Pressure lies below Center of Gravity (Centroid)

- The term $\frac{I_G}{A\bar{h}}$ is always positive (since $I_G > 0$, $A > 0$, $\bar{h} > 0$ for submerged surface).
- Hence $h^* > \bar{h}$.
- **Physical reason:** Pressure increases linearly with depth. The lower parts of the surface experience higher pressure, so the resultant force acts at a point lower than the centroid.

Example: For a vertical rectangular plate of height H, top edge at free surface: $\bar{h} = H/2$, $I_G = bH^3/12$, then $h^* = 2H/3$ – indeed below centroid.

Q4. Derive an expression for the total hydrostatic force and the center of pressure for an inclined plane surface submerged in a static fluid.

Ans:
Consider a plane surface of area A, inclined at an angle θ to the horizontal (or angle to vertical). The free surface of the liquid is horizontal.
Let the line of intersection of the plane surface with the free surface be the axis (or its extension). Measure distance **s** along the plane from this intersection.

- Depth of any point at distance s: $h = s\sin\,\theta$
- Centroid of the area lies at distance $\bar{s}$ from intersection → depth of centroid: $\bar{h} = \bar{s}\sin\,\theta$

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1. Total Hydrostatic Force (F)
Take an elemental area dA at distance s.
Pressure: $p = \rho g h = \rho g (s \sin\theta)$
Force on element: $dF = p\, dA = \rho g \sin\theta \cdot s\, dA$
Integrate over area A:

$$F = \int dF = \rho g \sin\,\theta \int_A s\, dA$$

The integral ∫ s dA is the first moment of area about the intersection line = $A\bar{s}$.
Thus:

$$F = \rho g \sin\,\theta \cdot A\bar{s} = \rho g(\bar{s}\sin\,\theta)A = \rho g\bar{h}A$$

Same result as vertical case:

$$\boxed{F = \rho g\bar{h}A}$$

Note: Force depends only on depth of centroid, not on angle of inclination.

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2. Center of Pressure (along the plane: s*)
Take moments about the intersection line:
Moment of resultant force: $F \times s^*$
Moment of distributed forces: $\int s\, dF = \int s(\rho g \sin\,\theta\, s\, dA) = \rho g \sin\,\theta \int s^2 dA$
Set equal:

$$F \cdot s^* = \rho g \sin\,\theta \int s^2 dA$$
$$s^* = \frac{\rho g \sin\,\theta \int s^2 dA}{F}$$

Substitute $F = \rho g\bar{h}A = \rho g(\bar{s}\sin\,\theta)A$:

$$s^* = \frac{\rho g \sin\,\theta \int s^2 dA}{\rho g \bar{s} \sin\,\theta A} = \frac{\int s^2 dA}{\bar{s}A}$$

∫ s^2 dA = second moment of area about the intersection line = I_o.

$$s^* = \frac{I_o}{\bar{s}A}$$

Parallel axis theorem: $I_o = I_G + A\bar{s}^2$, where I_G = moment of inertia about centroidal axis parallel to intersection line.

Thus:

$$s^* = \frac{I_G + A\bar{s}^2}{\bar{s}A} = \bar{s} + \frac{I_G}{A\bar{s}}$$

Center of pressure depth (h*):

$$h^* = s^*\sin\,\theta = \left(\bar{s} + \frac{I_G}{A\bar{s}}\right)\sin\,\theta = \bar{h} + \frac{I_G\sin\,\theta}{A\bar{s}}$$

Final expressions:

$$\boxed{s^* = \bar{s} + \frac{I_G}{A\bar{s}}}$$

$$\boxed{h^* = \bar{h} + \frac{I_G\sin\,\theta}{A\bar{s}}}$$

Q5. Define the terms 'Center of Buoyancy', 'Meta-center', and 'Meta-centric height'. Explain with diagrams how the relative positions of the center of gravity and the meta-center determine the stability of a floating body.

- **Definitions:**

  Center of Buoyancy (B): Centroid of the displaced volume of fluid; the point where the buoyant force acts vertically upward.

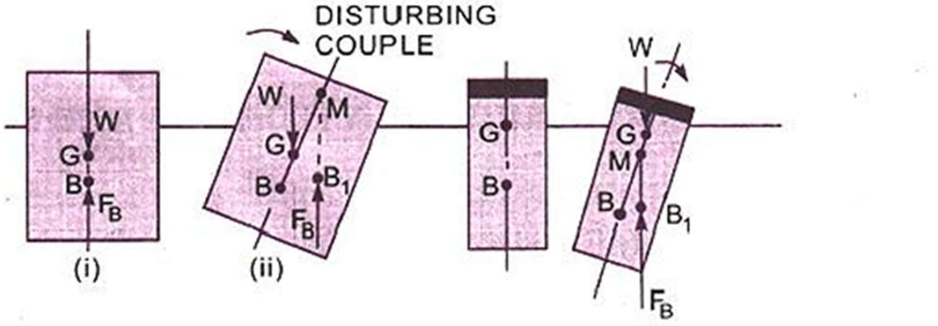
  Meta-center (M): Intersection of the line of action of buoyant force (through B) with the centerline of the floating body when the body is tilted by a small angle.

  Meta-centric height (GM): Distance between the center of gravity (G) and the metacenter (M). GM = positive if M above G.

- **Stability Determination (using diagrams):**

  The stability of a floating body is determined by the relative positions of G and M.

Diagram Generation Prompts:



(a) Stable equilibrium *M* is above *G*

(b) Unstable equilibrium *M* is below *G*.

- **Explanation:**

Condition	Relative position	GM	Behaviour
Stable	M above G	GM > 0	Righting moment → returns to upright
Unstable	M below G	GM < 0	Overturning moment → capsizes

Condition	Relative position	GM	Behaviour
Neutral	M coincides with G	GM = 0	Remains at any tilt

Q6. A differential U-tube manometer containing mercury is used to measure the pressure difference between two pipes, A and B, containing liquids of specific gravities 0.8 and 1.0 respectively. The center of pipe A is 0.5 m above the center of pipe B. The mercury level in the limb connected to pipe A is 0.4 m below the center of pipe B, and the difference in mercury levels is 0.3 m. Calculate the pressure difference between pipes A and B.

Step 1: Given Data

- Specific gravity of fluid in A, $s_1 = 0.8$
- Specific gravity of fluid in B, $s_2 = 1.0$
- Specific gravity of mercury, $s_m = 13.6$ (Standard value)
- Height of pipe A above pipe B, $z_A - z_B = +0.5$ m (So $z_B - z_A = -0.5$ m)
- Specific weight of water, $\gamma_w = 9810$ N/m^3

Step 2: Determine the Geometric Parameters (with reference to Pipe B's center as elevation 0 m)

- **Elevation of Pipe A:** $+0.5$ m
- **Elevation of left mercury meniscus (connected to A):** It is 0.4 m *below* pipe B.

$$\text{Elevation}_{\text{left}} = 0 - 0.4 = -0.4 \text{ m}$$

- **Height of fluid column from Pipe A to left meniscus (h_1):**

$$h_1 = \text{Elevation}_A - \text{Elevation}_{\text{left}} = 0.5 - (-0.4) = 0.9 \text{ m}$$

- **Difference in mercury levels (h):** 0.3 m. Since the left meniscus is lower, the right meniscus is higher.

$$\text{Elevation}_{\text{right}} = \text{Elevation}_{\text{left}} + h = -0.4 + 0.3 = -0.1 \text{ m}$$

- **Height of fluid column from right meniscus to Pipe B (h_2):** (Moving *up* from right meniscus to B)

$$h_2 = \text{Elevation}_B - \text{Elevation}_{\text{right}} = 0 - (-0.1) = 0.1 \text{ m}$$

Step 3: Calculate Specific Weights

$$\gamma_1 = s_1 \cdot \gamma_w = 0.8 \times 9810 = 7848 \text{ N/m}^3$$
$$\gamma_2 = s_2 \cdot \gamma_w = 1.0 \times 9810 = 9810 \text{ N/m}^3$$
$$\gamma_m = s_m \cdot \gamma_w = 13.6 \times 9810 = 133416 \text{ N/m}^3$$

Step 4: Apply the Hydrostatic Equation (Equating pressures at the datum - the lower left meniscus)

From the left limb side (A to Datum):

$$P_{\text{datum}} = P_A + \gamma_1 h_1$$
$$P_{\text{datum}} = P_A + (7848 \times 0.9)$$

From the right limb side (B to Datum):
First, pressure at the right meniscus:

$$P_{\text{right}} = P_B + \gamma_2 h_2 = P_B + (9810 \times 0.1)$$

Now, go down through the mercury from the right meniscus to the datum:

$$P_{\text{datum}} = P_{\text{right}} + \gamma_m h$$

$$P_{\text{datum}} = [P_B + (9810 \times 0.1)] + (133416 \times 0.3)$$

Step 5: Equate the Two Expressions and Solve for $P_A - P_B$

$$P_A + (7848 \times 0.9) = P_B + (9810 \times 0.1) + (133416 \times 0.3)$$

Rearrange to isolate $P_A - P_B$:

$$P_A - P_B = (9810 \times 0.1) + (133416 \times 0.3) - (7848 \times 0.9)$$

Calculate each term:

- $9810 \times 0.1 = 981$
- $133416 \times 0.3 = 40024.8$
- $7848 \times 0.9 = 7063.2$

$$P_A - P_B = 981 + 40024.8 - 7063.2$$
$$P_A - P_B = 41005.8 - 7063.2$$
$$P_A - P_B = 33942.6 \text{ N/m}^2$$

Final Answer:

$$\boxed{P_A - P_B = 33942.6 \text{ Pa (or 33.94 kPa)}}$$

Unit 5: Dimensional Analysis & Similitude

3 & 4-Marks Questions

Q.1 What do you mean by fundamental units and derived units?

Step 1: Definition of Fundamental Units

- **Fundamental Units** (also called Base Units) are the basic units of measurement that are **independently defined** and cannot be expressed in terms of other units.
- They form the foundation upon which all other units are built.
- In the **MLT system** (Mass, Length, Time) commonly used in fluid mechanics, the fundamental units are:
    - **Mass (M)** - measured in kilograms (kg)
    - **Length (L)** - measured in meters (m)
    - **Time (T)** - measured in seconds (s)
- In the **FLT system** (Force, Length, Time), the fundamental units are Force (F), Length (L), and Time (T).

Step 2: Definition of Derived Units

- **Derived Units** are units that can be expressed in terms of fundamental units using mathematical relationships.
- They are obtained by combining fundamental units according to the physical laws defining the quantity.

Step 3: Examples

Quantity	Relationship	Dimension (MLT)	Derived Unit
Velocity	Displacement / Time	LT^{-1}	m/s
Acceleration	Velocity / Time	LT^{-2}	m/s²
Force	Mass × Acceleration	MLT^{-2}	Newton (N) or kg·m/s²
Pressure	Force / Area	$ML^{-1}T^{-2}$	Pascal (Pa) or N/m²
Energy	Force × Distance	$ML^{2}T^{-2}$	Joule (J) or N·m

Q.2 Determine the dimension of the following quantities: (i) Angular Acceleration, (ii) Discharge (iii) Force (iv) Specific Weight

Step 1: General Approach

We use the **MLT system** (Mass, Length, Time) to determine dimensions. The dimension of a quantity is expressed as $M^{a}L^{b}T^{c}$.

Step 2: (i) Angular Acceleration (α)

- Angular velocity (ω) = Angle / Time = $\frac{1}{T} = T^{-1}$ (Angle is dimensionless)
- Angular acceleration (α) = Rate of change of angular velocity = $\frac{\omega}{T} = \frac{T^{-1}}{T}$

$$\boxed{\alpha = T^{-2}}$$

- **Dimension:** $M^{0}L^{0}T^{-2}$

Step 3: (ii) Discharge (Q)

- Discharge (also called flow rate or volumetric flow rate) = Volume / Time
- Volume = L^3
- Time = T

$$Q = \frac{L^3}{T} = L^3T^{-1}$$

$$\boxed{Q = L^3T^{-1}}$$

- **Dimension:** $M^0L^3T^{-1}$

Step 4: (iii) Force (F)

- From Newton's Second Law: Force = Mass × Acceleration
- Mass = M
- Acceleration = Velocity / Time = $\frac{LT^{-1}}{T} = LT^{-2}$

$$F = M \times LT^{-2} = MLT^{-2}$$

$$\boxed{F = MLT^{-2}}$$

- **Dimension:** $M^1L^1T^{-2}$

Step 5: (iv) Specific Weight (γ)

- Specific weight = Weight / Volume
- Weight = Mass × Acceleration due to gravity = $M \times LT^{-2} = MLT^{-2}$
- Volume = L^3

$$\gamma = \frac{MLT^{-2}}{L^3} = ML^{-2}T^{-2}$$

$$\boxed{\gamma = ML^{-2}T^{-2}}$$

- **Dimension:** $M^1L^{-2}T^{-2}$

Final Answer Table:

Quantity	Symbol	Dimension (MLT)
Angular Acceleration	α	$M^0L^0T^{-2}$
Discharge	Q	$M^0L^3T^{-1}$
Force	F	$M^1L^1T^{-2}$
Specific Weight	γ	$M^1L^{-2}T^{-2}$

Q.3 What is meant by dimensional analysis? What are its limitations and advantages?

Step 1: Definition of Dimensional Analysis

- **Dimensional analysis** is a mathematical technique used in engineering and physics to study the relationships between physical quantities by examining their **dimensions** (M, L, T, etc.).
- It is based on the fundamental principle of **dimensional homogeneity**, which states that every term in a valid physical equation must have the same dimensions.
- It helps in:
  - Deriving relationships between variables without complete theory

  - Converting units from one system to another
  - Planning experiments and reducing the number of variables
  - Checking the dimensional correctness of equations

Step 2: Advantages of Dimensional Analysis

Advantage	Explanation
1. Simplification	It reduces the number of variables by grouping them into dimensionless numbers (e.g., from 7 variables to 3 dimensionless groups).
2. Experimental Design	It guides experimenters by indicating which variables are important and how to present results using dimensionless parameters.
3. Scale Modeling	It enables the design of scaled models (prototype testing) using similitude principles.
4. Equation Verification	It helps check the dimensional correctness of derived or empirical equations.
5. Missing Variable Detection	It can sometimes reveal if a relevant variable has been omitted from analysis.

Step 3: Limitations of Dimensional Analysis

Limitation	Explanation
1. No Constant Information	It does not provide the value of dimensionless constants (e.g., the factor $\pi/4$ in discharge formula). These must be found experimentally.
2. Requires Complete Variable List	The analysis is only as good as the list of variables considered. If a significant variable is omitted, the result will be incorrect.
3. Cannot Differentiate	It cannot distinguish between quantities with the same dimensions (e.g., energy and torque both have dimension ML^2T^{-2}).
4. No Directional Information	It provides only the form of the relationship, not the physical mechanism or directional sense.
5. Selection of Repeating Variables	The choice of repeating variables is arbitrary and requires physical insight.

Q.4 Explain the dimensionless numbers: (i) Reynold's Number (ii) Mach Number

Step 1: Definition of Dimensionless Numbers

- **Dimensionless numbers** are pure numbers that have no units. They represent the ratio of two forces or effects and are used to characterize the flow regime in fluid mechanics.

Step 2: (i) Reynold's Number (Re)

Definition:

Reynold's number is the ratio of **inertia force** to **viscous force** in a flowing fluid.
Mathematical Expression:

$$Re = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

Where:

- ρ = Density of fluid (ML^{-3})
- V = Characteristic velocity (LT^{-1})
- L = Characteristic length (L) (e.g., pipe diameter)
- μ = Dynamic viscosity ($ML^{-1}T^{-1}$)
- ν = Kinematic viscosity (L^2T^{-1})

Physical Significance:

- **Low Re (<2000):** Viscous forces dominate. Flow is **laminar** (smooth, orderly).
- **High Re (> 4000):** Inertia forces dominate. Flow is **turbulent** (chaotic, with eddies).
- **Transitional Re ($2000 < Re < 4000$):** Flow is unstable and switches between laminar and turbulent.

Applications:

- Determining flow regime in pipes
- Scaling models in ship and aircraft design
- Analyzing boundary layer behavior

Step 3: (ii) Mach Number (M)
Definition:
Mach number is the ratio of **inertia force** to **elastic force** in a fluid flow. More commonly, it is defined as the ratio of the **velocity of flow** to the **velocity of sound** in the same fluid medium.
Mathematical Expression:

$$M = \frac{\text{Inertia Force}}{\text{Elastic Force}} = \frac{V}{C}$$

Where:

- V = Velocity of flow (LT^{-1})
- C = Velocity of sound in the fluid (LT^{-1}) = $\sqrt{\frac{K}{\rho}}$ (where K is bulk modulus)

Physical Significance:
Mach number classifies flow regimes based on compressibility effects:

Mach Number Range	Regime	Characteristics
$M < 0.8$	Subsonic	Compressibility effects negligible; density constant
$0.8 < M < 1.2$	Transonic	Mixed flow; shock waves begin to form
$1.2 < M < 5.0$	Supersonic	Compressibility effects dominant; shock waves present
$M > 5.0$	Hypersonic	Very high speeds; extreme temperatures

Applications:

- Aircraft and missile design
- Wind tunnel testing at high speeds
- Compressible flow analysis in nozzles and diffusers

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7 Marks Questions

Ex. 1. Fluid of density ρ and viscosity μ flows at an average velocity ν through a circular pipe of diameter D. Show by dimensional analysis, that the shear stress of the pipe wall is $\tau_0 = \rho\nu^2\Phi\left(\frac{\rho\nu D}{\mu}\right)$

Step 1: Identify the Variables and Their Dimensions

The wall shear stress τ_0 depends on:

- Diameter of pipe, D [L]
- Average velocity, V [LT^{-1}] *(Note: The question uses ν for velocity, but we'll use V to avoid confusion with kinematic viscosity)*
- Fluid density, ρ [ML^{-3}]
- Fluid viscosity, μ [$ML^{-1}T^{-1}$]
- (Possibly) Roughness height, but for a smooth pipe it's not considered.

Let the functional relationship be:

$$\tau_0 = f(D, V, \rho, \mu)$$

Step 2: Write Dimensions of Each Variable

Variable	Symbol	Dimension
Wall shear stress	τ_0	$ML^{-1}T^{-2}$ (Force/Area)
Diameter	D	L
Velocity	V	LT^{-1}
Density	ρ	ML^{-3}
Dynamic viscosity	μ	$ML^{-1}T^{-1}$

Number of variables, $n = 5$

Number of fundamental dimensions, $m = 3$ (M, L, T)

According to Buckingham's π-theorem, number of dimensionless π-terms = $n - m = 5 - 3 = 2$.

Step 3: Select Repeating Variables

Choose $m = 3$ repeating variables that together contain all fundamental dimensions (M, L, T). Common choices are:

- ρ (contains M)
- V (contains L and T)
- D (contains L)

These three variables are independent and cannot themselves form a dimensionless group.

Step 4: Form the First π-Term (π_1)

Let:

$$\pi_1 = \rho^a V^b D^c \cdot \tau_0$$

Write the dimensional equation:

$$[M^0L^0T^0] = [ML^{-3}]^a[LT^{-1}]^b[L]^c \cdot [ML^{-1}T^{-2}]$$

Equate exponents for M, L, and T:

For M: $a + 1 = 0 \Rightarrow a = -1$

For T: $-b - 2 = 0 \Rightarrow -b = 2 \Rightarrow b = -2$
For L: $-3a + b + c - 1 = 0$
Substitute $a = -1$ and $b = -2$:

$$-3(-1) + (-2) + c - 1 = 0$$
$$3 - 2 + c - 1 = 0$$
$$c + 0 = 0 \Rightarrow c = 0$$

Therefore:

$$\pi_1 = \rho^{-1}V^{-2}D^0 \cdot \tau_0 = \frac{\tau_0}{\rho V^2}$$

Step 5: Form the Second π-Term (π_2)
Let:

$$\pi_2 = \rho^a V^b D^c \cdot \mu$$

Write the dimensional equation:

$$[M^0L^0T^0] = [ML^{-3}]^a[LT^{-1}]^b[L]^c \cdot [ML^{-1}T^{-1}]$$

Equate exponents:
For M: $a + 1 = 0 \Rightarrow a = -1$
For T: $-b - 1 = 0 \Rightarrow -b = 1 \Rightarrow b = -1$
For L: $-3a + b + c - 1 = 0$
Substitute $a = -1$ and $b = -1$:

$$-3(-1) + (-1) + c - 1 = 0$$
$$3 - 1 + c - 1 = 0$$
$$1 + c = 0 \Rightarrow c = -1$$

Therefore:

$$\pi_2 = \rho^{-1}V^{-1}D^{-1} \cdot \mu = \frac{\mu}{\rho V D}$$

Step 6: Form the Functional Relationship
According to Buckingham's theorem:

$$\pi_1 = \Phi(\pi_2)$$

Substitute the expressions:

$$\frac{\tau_0}{\rho V^2} = \Phi\left(\frac{\mu}{\rho V D}\right)$$

Step 7: Rearrange to Obtain the Desired Form
The second π-term is the reciprocal of the Reynolds number:

$$\frac{\mu}{\rho V D} = \frac{1}{Re} \text{ where} Re = \frac{\rho V D}{\mu}$$

Therefore:

$$\frac{\tau_0}{\rho V^2} = \Phi\left(\frac{1}{Re}\right)$$

Or more commonly written as:

$$\frac{\tau_0}{\rho V^2} = \Phi(Re)$$

Thus:

$$\boxed{\tau_0 = \rho V^2\,\Phi\left(\frac{\rho V D}{\mu}\right)}$$

This shows that the wall shear stress is proportional to ρV^2 times some function of the Reynolds number.

Ex. 2. Find the expression for drag force on a smooth sphere of diameter D, moving with a uniform velocity V in a fluid of density ρ and dynamic viscosity μ.

Step 1: Identify the Variables and Their Dimensions

The drag force F_D depends on:

- Diameter of sphere, D [L]
- Velocity of sphere relative to fluid, V [LT^{-1}]
- Fluid density, ρ [ML^{-3}]
- Fluid viscosity, μ [$ML^{-1}T^{-1}$]

Let the functional relationship be:

$$F_D = f(D, V, \rho, \mu)$$

Step 2: Write Dimensions of Each Variable

Variable	Symbol	Dimension
Drag Force	F_D	MLT^{-2}
Diameter	D	L
Velocity	V	LT^{-1}
Density	ρ	ML^{-3}
Dynamic viscosity	μ	$ML^{-1}T^{-1}$

Number of variables, $n = 5$

Number of fundamental dimensions, $m = 3$ (M, L, T)

According to Buckingham's π-theorem, number of dimensionless π-terms = $n - m = 5 - 3 = 2$.

Step 3: Select Repeating Variables

Choose $m = 3$ repeating variables:

- ρ (contains M)
- V (contains L and T)
- D (contains L)

These variables are independent and together contain all fundamental dimensions.

Step 4: Form the First π-Term (π_1)

Let:

$$\pi_1 = \rho^a V^b D^c \cdot F_D$$

Write the dimensional equation:

$$[M^0 L^0 T^0] = [ML^{-3}]^a [LT^{-1}]^b [L]^c \cdot [MLT^{-2}]$$

Equate exponents for M, L, and T:

For M: $a + 1 = 0 \Rightarrow a = -1$

For T: $-b - 2 = 0 \Rightarrow -b = 2 \Rightarrow b = -2$

For L: $-3a + b + c + 1 = 0$

Substitute $a = -1$ and $b = -2$:

$$-3(-1) + (-2) + c + 1 = 0$$
$$3 - 2 + c + 1 = 0$$

$$2 + c = 0 \Rightarrow c = -2$$

Therefore:

$$\pi_1 = \rho^{-1}V^{-2}D^{-2} \cdot F_D = \frac{F_D}{\rho V^2 D^2}$$

Step 5: Form the Second π-Term (π_2)
Let:

$$\pi_2 = \rho^a V^b D^c \cdot \mu$$

Write the dimensional equation:

$$[M^0 L^0 T^0] = [ML^{-3}]^a [LT^{-1}]^b [L]^c \cdot [ML^{-1}T^{-1}]$$

Equate exponents:
For M: $a + 1 = 0 \Rightarrow a = -1$
For T: $-b - 1 = 0 \Rightarrow -b = 1 \Rightarrow b = -1$
For L: $-3a + b + c - 1 = 0$
Substitute $a = -1$ and $b = -1$:

$$-3(-1) + (-1) + c - 1 = 0$$
$$3 - 1 + c - 1 = 0$$
$$1 + c = 0 \Rightarrow c = -1$$

Therefore:

$$\pi_2 = \rho^{-1}V^{-1}D^{-1} \cdot \mu = \frac{\mu}{\rho V D}$$

Step 6: Identify the Dimensionless Numbers

- $\pi_1 = \frac{F_D}{\rho V^2 D^2}$ is a form of the **Drag Coefficient** (C_D) multiplied by a constant. In standard form, the drag coefficient is defined as:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{F_D}{\frac{1}{2}\rho V^2 \left(\frac{\pi D^2}{4}\right)} = \frac{8F_D}{\pi \rho V^2 D^2}$$

So π_1 is proportional to C_D.

- $\pi_2 = \frac{\mu}{\rho V D} = \frac{1}{Re}$, where $Re = \frac{\rho V D}{\mu}$ is the **Reynolds Number**.

Step 7: Form the Functional Relationship
According to Buckingham's theorem:

$$\pi_1 = \Phi(\pi_2)$$

Substitute the expressions:

$$\frac{F_D}{\rho V^2 D^2} = \Phi\left(\frac{\mu}{\rho V D}\right)$$

Or in terms of Reynolds number:

$$\frac{F_D}{\rho V^2 D^2} = \Phi\left(\frac{1}{Re}\right) = \Psi(Re)$$

Step 8: Rearrange to Obtain the Expression for Drag Force

$$F_D = \rho V^2 D^2 \cdot \Psi(Re)$$

In standard fluid mechanics, this is written as:

$$\boxed{F_D = C_D \cdot \frac{1}{2} \rho V^2 A}$$

where $A = \frac{\pi D^2}{4}$ is the projected area of the sphere, and $C_D = f(Re)$ is the drag coefficient obtained from experiments.

The final expression shows that the drag force depends on:

- Dynamic pressure $\frac{1}{2}\rho V^2$
- Projected area $\frac{\pi D^2}{4}$
- Drag coefficient C_D, which is a function of the Reynolds number

UNIT 7: COMPRESSIBLE FLUID FLOW

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1. Define compressible fluid flow.

- **Definition:** Compressible fluid flow is the branch of fluid mechanics that deals with fluids where the **density changes (ρ ≠ constant)** within the flow field are significant enough to influence the solution.
- **Key Criteria:** Density changes are primarily driven by high-pressure variations caused by high-velocity flows, typically when the flow velocity approaches or exceeds the speed of sound.
- **Governing Factors:** Temperature and pressure variations lead to considerable density variations, requiring the use of thermodynamic principles (equations of state) alongside fluid mechanics equations.
- **Examples:** Flow through nozzles, turbines, compressors, and high-speed aircraft.

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2. What is Mach number?

- **Definition:** The Mach number (M) is a dimensionless quantity in fluid dynamics representing the ratio of the **velocity of an object or flow (V)** relative to a fluid to the **local speed of sound (a)** in that fluid.
- **Formula:** It is mathematically expressed as:

$$M = \frac{V}{a}$$

- **Significance:** It is the single most important parameter in compressible flow, used to:
  - Classify flow regimes (subsonic, supersonic, etc.)
  - Quantify the importance of compressibility effects
  - Determine whether density changes are negligible
- **Physical Meaning:** It represents the ratio of **inertia forces to elastic forces** in the fluid.

---

3. What is stagnation pressure?

- **Definition:** Stagnation pressure (also known as **total pressure** or **pitot pressure**) is the pressure a fluid attains when it is brought to rest **isentropically** (i.e., without friction or heat transfer) from its actual flow condition.
- **Physical Meaning:** It represents the sum of the static pressure and the dynamic pressure due to the fluid's kinetic energy.
- **For Incompressible Flow:** The relationship is:

$$P_0 = P + \frac{1}{2}\rho V^2$$

where:

- P_0 = Stagnation pressure
- P = Static pressure
- ρ = Density
- V = Flow velocity
- **For Compressible Flow:** The relationship is more complex and depends on Mach number:

$$P_0 = P\left[1+\frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

where k is the specific heat ratio.

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4. What is stagnation temperature?

- **Definition:** Stagnation temperature (or **total temperature**) is the temperature a fluid attains when it is brought to rest **adiabatically** (i.e., without any heat transfer) from its actual flow condition.
- **Physical Meaning:** It represents the **total energy content** of the fluid per unit mass. It is the sum of the static enthalpy $\left(c_p T\right)$ and the kinetic energy $\left(\frac{V^2}{2}\right)$.
- **Mathematical Expression:**

$$T_0 = T + \frac{V^2}{2c_p}$$

or in dimensionless form:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

where:

- T_0 = Stagnation temperature
- T = Static temperature
- c_p = Specific heat at constant pressure
- k = Specific heat ratio
- M = Mach number
- **Key Property:** Stagnation temperature remains constant in an adiabatic flow, even if the velocity changes.

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5. Define subsonic, sonic and supersonic flow.

Subsonic Flow:

- **Definition:** Flow where the fluid velocity is **less than** the local speed of sound.
- **Mach Number Range:** $M < 1$
- **Characteristics:**
  - Density changes are negligible at low subsonic speeds ($M < 0.3$)
  - Density changes become significant as M approaches 1
  - Pressure disturbances can propagate upstream
  - Streamlines are smooth and continuous

Sonic Flow:

- **Definition:** Flow where the fluid velocity is **exactly equal** to the local speed of sound.
- **Mach Number Range:** $M = 1$
- **Characteristics:**
  - This is a critical condition in gas dynamics
  - Occurs at the throat (minimum area) of a nozzle
  - Mass flow rate is maximum at this condition

Supersonic Flow:

- **Definition:** Flow where the fluid velocity is **greater than** the local speed of sound.
- **Mach Number Range:** $M > 1$
- **Characteristics:**
  - Flow behavior is fundamentally different from subsonic flow
  - Area increase causes velocity to increase (opposite of subsonic flow)
  - Shock waves and expansion fans form
  - Pressure disturbances cannot propagate upstream

6. Define Mach cone.

- **Definition:** A Mach cone is a pressure shock wave pattern formed in the air when an object (like an aircraft) travels at **supersonic speeds** ($M > 1$).
- **Formation:** As the object moves faster than the sound waves it generates, the waves cannot propagate forward and instead coalesce into a cone-shaped shock front.
- **Diagram Prompt:** Create a diagram showing a supersonic aircraft moving from left to right. Draw concentric circles representing sound waves emitted from the aircraft at different times. Show how these circles overlap to form a V-shaped cone behind the aircraft. Label the cone boundary, the aircraft, and indicate the direction of motion.
- **Key Elements:**
    - **Mach Angle (μ):** The half-angle of the cone, measured from the direction of motion. It is given by:

      $$\sin \mu = \frac{a}{V} = \frac{1}{M}$$

    - **Zone of Action:** The region **inside** the cone, where the disturbances (sound) are present and can be heard.
    - **Zone of Silence:** The region **outside** the cone, which remains undisturbed by the object's passage. An observer in this zone will not hear the object until after it has passed.
- **Sonic Boom:** The intersection of the Mach cone with the ground creates a sonic boom.

7. Derive the expression for velocity of sound in compressible fluid.

- **Given:** Consider a plane pressure wave (sound wave) propagating through a stationary compressible fluid in a constant area duct.
- **To Find:** Expression for the velocity of sound, a.
- **Assumptions:**
    1. Frictionless flow
    2. No heat transfer (adiabatic)
    3. The disturbance is small ($dP, d\rho$ are infinitesimal)
    4. The process is **reversible and adiabatic**, hence **isentropic**
- **Solution:**

Step 1: Setup Control Volume

Imagine the wave is stationary and the fluid is flowing from right to left into the wave at velocity a. The fluid ahead of the wave is undisturbed at (P, ρ). The fluid behind the wave is at $(P + dP, \rho + d\rho)$ and has a velocity dV relative to the wave.

Step 2: Apply Continuity Equation

Mass flow into the control volume = Mass flow out

$$\rho A a = (\rho + d\rho) A (a - dV)$$

Simplifying:

$$\rho a = (\rho + d\rho)(a - dV)$$
$$\rho a = \rho a - \rho\, dV + a\, d\rho - d\rho\, dV$$

Neglecting the product of small terms $d\rho\, dV$:

$$0 = -\rho\, dV + a\, d\rho$$

$$\rho\, dV = a\, d\rho \text{...(Equation 1)}$$

Step 3: Apply Momentum Equation

Net force in the direction of flow = Rate of change of momentum

$$PA - (P + dP)A = \dot{m}(V_{out} - V_{in})$$
$$-dP \cdot A = (\rho a A)[(a - dV) - a]$$
$$-dP = \rho a(-dV)$$
$$dP = \rho a\, dV \text{...(Equation 2)}$$

Step 4: Combine Equations

From Equation 1: $dV = \frac{a\, d\rho}{\rho}$

Substitute into Equation 2:

$$dP = \rho a \left(\frac{a\, d\rho}{\rho}\right)$$
$$dP = a^2\, d\rho$$

Step 5: Final Expression

Therefore, the velocity of sound is:

$$a^2 = \frac{dP}{d\rho}$$
$$a = \sqrt{\frac{dP}{d\rho}}$$

Step 6: Apply Isentropic Relation

For an isentropic process, $\frac{P}{\rho^k} = \text{constant}$

Taking natural log and differentiating:

$$\ln\, P - k\ln\, \rho = \text{constant}$$
$$\frac{dP}{P} - k\frac{d\rho}{\rho} = 0$$
$$\frac{dP}{d\rho} = \frac{kP}{\rho}$$

Using the ideal gas law, $\frac{P}{\rho} = RT$:

$$\frac{dP}{d\rho} = kRT$$

Step 7: Final Answer

The velocity of sound in a compressible fluid is:

$$\boxed{a = \sqrt{\frac{dP}{d\rho}}}$$

For a perfect gas under isentropic conditions, this becomes:

$$\boxed{a = \sqrt{kRT}}$$

where:

- k = Specific heat ratio $\left(\frac{c_p}{c_v}\right)$
- R = Specific gas constant
- T = Absolute temperature

---

8. Explain Mach number and classify flows based on Mach number.

Explanation of Mach Number:

The Mach number (M) is a dimensionless parameter defined as the ratio of the **local flow velocity (V)** to the **local speed of sound (a)** in the fluid.

$$M = \frac{V}{a}$$

It is the fundamental governing parameter in compressible flow because it quantifies the significance of compressibility effects. It represents the ratio of **inertia forces to elastic forces** in the fluid.

Physical Significance:

- Low Mach number ($M < 0.3$): Fluid's kinetic energy is small compared to its internal energy, allowing density to be considered constant.
- High Mach number ($M > 0.3$): Density variations become significant, fundamentally altering flow behavior.

Classification of Flows Based on Mach Number:

Flow Regime	Mach Number Range	Characteristics
1. Incompressible Flow	$M < 0.3$	• Density changes are negligible (<5%) • Fluid is treated as having constant density • Example: Flow of water in pipes, low-speed air flow
2. Subsonic Flow	$0.3 \leq M < 0.8$	• Density changes are significant and must be considered • Flow velocity is less than the speed of sound • Streamlines are smooth • Pressure changes can propagate upstream • Example: Flow over commercial aircraft during takeoff and landing
3. Transonic Flow	$0.8 \leq M \leq 1.2$	• Mixed flow regime with both subsonic and supersonic regions • Shock waves begin to form on the body surface • Sharp increase in drag • Example: Flow over modern airliners at cruising speed
4. Supersonic Flow	$1.2 < M < 5.0$	• Flow velocity is greater than the speed of sound • Velocity increases with increasing area (opposite to subsonic flow) • Shock waves and expansion fans are prominent • Example: Flow over a supersonic fighter jet

Flow Regime	Mach Number Range	Characteristics
5. Hypersonic Flow	$M \geq 5.0$	• Extremely high-speed flow • High kinetic energy causes significant aerodynamic heating • Gas dissociates and ionizes • Example: Re-entry of space vehicle, ICBMs

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9. Derive stagnation temperature and pressure relation.

- **Given:** A flowing fluid with static properties (T, P) and velocity V. The fluid is brought to rest isentropically.
- **To Find:** The relationship between stagnation properties (T_0, P_0) and static properties (T, P, M).
- **Assumptions:**
  1. Steady, one-dimensional flow
  2. Perfect gas with constant specific heats
  3. Adiabatic and reversible (isentropic) deceleration process

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Part A: Derivation of Stagnation Temperature (T_0)

Step 1: Apply Steady Flow Energy Equation (SFEE)

For an adiabatic process (no heat transfer) with no work done, the SFEE states that stagnation enthalpy (h_0) is constant.

$$h_0 = h + \frac{V^2}{2}$$

For a perfect gas, $h = c_p T$, so:

$$c_p T_0 = c_p T + \frac{V^2}{2}$$

where T_0 is the stagnation temperature.

Step 2: Rearrange the equation

$$c_p T_0 - c_p T = \frac{V^2}{2}$$

$$c_p (T_0 - T) = \frac{V^2}{2}$$

$$T_0 - T = \frac{V^2}{2 c_p}$$

$$\frac{T_0}{T} = 1 + \frac{V^2}{2 c_p T}$$

Step 3: Express in terms of Mach Number

We know:

- Speed of sound: $a = \sqrt{kRT}$
- Mach number: $M = \frac{V}{a}$

- For a perfect gas: $c_p = \frac{kR}{k-1}$

Substitute c_p and a:

$$\frac{V^2}{2c_pT} = \frac{V^2}{2\left(\frac{kR}{k-1}\right)T}$$

$$\frac{V^2}{2c_pT} = \frac{V^2(k-1)}{2kRT}$$

Since $a^2 = kRT$, then $\frac{V^2}{kRT} = \left(\frac{V}{a}\right)^2 = M^2$:

$$\frac{V^2}{2c_pT} = \frac{k-1}{2}M^2$$

Step 4: Final Relation for Stagnation Temperature

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\boxed{T_0 = T\left[1+\frac{k-1}{2}M^2\right]}$$

Part B: Derivation of Stagnation Pressure (P_0)

Step 1: Apply Isentropic Relation

For an isentropic process, the relationship between pressure and temperature is:

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$$

Step 2: Substitute Temperature Ratio

Substitute the temperature ratio derived above:

$$\frac{P_0}{P} = \left[1+\frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

Step 3: Final Relation for Stagnation Pressure

$$\boxed{P_0 = P\left[1+\frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}}$$

Summary of Final Results:

Property	**Relation**
Stagnation Temperature	$T_0 = T\left[1+\frac{k-1}{2}M^2\right]$

Property	**Relation**
Stagnation Pressure	$P_0 = P\left[1+\frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$

where:

- T_0, P_0 = Stagnation (total) properties
- T, P = Static properties
- M = Mach number
- k = Specific heat ratio $\left(\frac{c_p}{c_v}\right)$

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