GUJARAT TECHNOLOGICAL UNIVERSITY

BE- 3 SEMESTER – OLD PAPER – S23 TO S25- QUESTION BANK & SOLUTION

Subject Name & Code:
Engineering Thermodynamics (3131905)

Unit 4: Entropy

Repeated Questions:

1. Prove that entropy is a property of the system. (04 Marks - W22, W23, W24)

Answer:

Definition of entropy: Entropy (S) is a thermodynamic property defined as $dS = \frac{\delta Q_{rev}}{T}$, where δQ_{rev} is the reversible heat transfer at absolute temperature T.

Proof that entropy is a property (path-independent):

Consider a system undergoing a reversible cycle (1–A–2–B–1). From **Clausius theorem** for a reversible cycle:

$$\oint \frac{\delta Q_{rev}}{T} = 0$$

Break the cycle into two reversible paths from state 1 to state 2: path A and path B.

$$\int_1^2 \left(\frac{\delta Q_{rev}}{T}\right)_A + \int_2^1 \left(\frac{\delta Q_{rev}}{T}\right)_B = 0$$

$$\int_1^2 \left(\frac{\delta Q_{rev}}{T}\right)_A = \int_1^2 \left(\frac{\delta Q_{rev}}{T}\right)_B$$

Since the integral $\int_1^2 \frac{\delta Q_{rev}}{T}$ has the same value for **any** reversible path between states 1 and 2, it depends only on the endpoints, not the path. Hence it represents a **point function** – a property. This property is called **entropy change**:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q_{rev}}{T}$$

➞ **Real-world application:** Entropy is used in T-s diagrams for power cycles (Rankine, Brayton) to calculate heat transfer as area under the curve.

2. Explain the principle of increase of entropy. (07 Marks - S25, W24)

Answer:

Statement: For any isolated system (or system + surroundings), the total entropy never decreases. It remains constant for reversible processes and increases for irreversible processes.

$$\Delta S_{universe} = \Delta S_{system} + \Delta S_{surroundings} \geq 0$$

Explanation:

From Clausius inequality for any cycle: $\oint \frac{\delta Q}{T} \leq 0$.
Consider a system changing from state 1 to 2 by an irreversible process. Devise a reversible process between the same states. Combining, we get:

$$\Delta S_{system} = S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

where equality holds for reversible process.
For an **adiabatic** system ($\delta Q = 0$): $\Delta S \geq 0$.
For an **isolated** system (no heat, work, or mass transfer): $\Delta S_{isolated} \geq 0$.
Physical meaning:

- Entropy is a measure of **disorder** or **energy dispersal**.
- Natural processes proceed toward increased entropy (e.g., heat flows from hot to cold, not reverse).
- The principle identifies the **direction** of time (arrow of time).

[DG PROMPT] – Entropy increase in isolated system
Title: Principle of Increase of Entropy
Description: Draw three boxes. Box 1: Two compartments separated by a partition – left side hot gas (red particles packed), right side cold gas (blue particles packed). Label "Initial state: Low entropy". An arrow pointing right labeled "Partition removed → Irreversible mixing". Box 2: Mixed gas with uniform color (purple), particles evenly spread. Label "Final state: High entropy". Below, write equation $\Delta S > 0$. Add a second example: Heat engine with hot reservoir at T_H and cold at T_L; show entropy increase of universe as $Q_L/T_L - Q_H/T_H > 0$ for irreversible engine.
Consequences:

- Perpetual motion of second kind (PMM-II) is impossible.
- Real processes are irreversible → entropy generation.
- Available energy (exergy) decreases as entropy increases.

➞ **Real-world application:** Design of heat exchangers, turbines, and refrigerators aims to minimize entropy generation (irreversibility) for better efficiency.

3. **Define and explain: Available energy, Unavailable energy, Exergy, Anergy, Irreversibility. (03 Marks - S24, W22, W23, S25)**

Answer:

1. **Available Energy (Exergy):** The **maximum useful work** obtainable from a system as it comes to equilibrium with its surroundings (dead state). It's the "quality" or "work potential" of energy.
    - *Example:* 100 kJ of heat at 500K has more available energy than 100 kJ at 300K.
2. **Unavailable Energy (Anergy):** The portion of energy that **cannot be converted** into useful work, even with ideal reversible processes. It's energy at dead state conditions.
    - *Formula:* For heat Q at temperature T: $Unavailable = Q \times \frac{T_0}{T}$ where T_0 = surroundings temperature.
3. **Exergy:** Same as available energy - the maximum theoretical useful work obtainable as system reaches equilibrium with environment.
4. **Anergy:** Same as unavailable energy - the portion of energy that cannot be converted to work.
5. **Irreversibility (I):** The **loss of available energy** or **increase in anergy** due to irreversible processes. It equals the entropy generation multiplied by surroundings temperature.

$$I = T_0 \times S_{gen}$$

    - Represents wasted work potential
    - Always positive for irreversible processes
    - Zero for reversible processes

Relationship: For energy transfer Q:

$$\text{Total Energy} = \text{Exergy (Available)} + \text{Anergy (Unavailable)}$$

Irreversibility measures how much exergy is destroyed (converted to anergy) in a process.

Other Important Questions:

1. **S23: Explain the Clausius inequality. (04 Marks)**

Answer:

Clausius Inequality: For any thermodynamic system undergoing a cyclic process, the cyclic integral of δQ/T is less than or equal to zero.

$$\oint \frac{\delta Q}{T} \leq 0$$

Interpretation:

- **Equality (= 0)** holds for **reversible cycles**
- **Inequality (< 0)** holds for **irreversible cycles**

Proof Basis: Consider a system undergoing any cycle. Replace actual heat interactions with equivalent reversible heat transfers using reversible Carnot devices. Analysis shows net entropy change of reservoirs ≥ 0, leading to the inequality.

Significance:

1. **Leads to Entropy Definition:** From $\oint \frac{\delta Q_{rev}}{T} = 0$, we deduce $\int_1^2 \frac{\delta Q_{rev}}{T}$ is path-independent, defining entropy S.
2. **Second Law Mathematical Statement:** $\oint \frac{\delta Q}{T} \leq 0$ is a general mathematical statement of Second Law.
3. **Direction of Processes:** For isolated systems, becomes $\Delta S \geq 0$ (entropy increase principle).
4. **Criterion for Reversibility:** Provides test whether a process is reversible (=0) or irreversible (<0).

Example Application: For heat engine between T_1 and T_2:

- Reversible: $\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0 \rightarrow$ Carnot efficiency
- Irreversible: $\frac{Q_1}{T_1} - \frac{Q_2}{T_2} < 0 \rightarrow$ lower efficiency

2. **W23: Derive an expression for exergy of a closed system. (07 Marks)**

Answer:

Given: A closed system at state (P, T, U, S, V) in an environment at (P_0, T_0).

To Find: Maximum useful work (exergy) obtainable as system comes to equilibrium with environment.

Assumptions:

1. Environment is infinite reservoir at constant P_0, T_0
2. Final state: System in equilibrium with environment (P_0, T_0, U_0, S_0, V_0)
3. Process is reversible

Derivation:

Step 1: Apply First Law to system:

$$Q - W = U_0 - U$$

where W = total work done by system

Step 2: Apply Second Law (entropy change of universe = 0 for reversible process):

$$\Delta S_{system} + \Delta S_{surroundings} = 0$$

$$(S_0 - S) + \frac{Q_{surr}}{T_0} = 0$$

Heat to surroundings: $Q_{surr} = -Q$ (heat from system to surroundings)

$$(S_0 - S) - \frac{Q}{T_0} = 0$$
$$Q = T_0(S_0 - S)$$

Step 3: Substitute Q into First Law:

$$T_0(S_0 - S) - W = U_0 - U$$
$$W = (U - U_0) - T_0(S - S_0)$$

Step 4: This total work W includes:

- Useful work (exergy, Φ)
- Work against atmosphere: $P_0(V_0 - V)$

So: $W = \Phi + P_0(V_0 - V)$

Step 5: Substitute and solve for exergy Φ:

$$\Phi = (U - U_0) - T_0(S - S_0) + P_0(V - V_0)$$

Final Expression for Closed System Exergy:

$$\Phi = (U - U_0) + P_0(V - V_0) - T_0(S - S_0)$$

Components:

1. $(U - U_0)$: Internal energy relative to environment
2. $P_0(V - V_0)$: Work against atmosphere
3. $T_0(S - S_0)$: Unavailable energy due to entropy

Special Cases:

1. **For steady flow:** $\psi = (h - h_0) - T_0(s - s_0) + \frac{C^2}{2} + gZ$
2. **For heat Q at temperature T:** $\Phi_{he} = Q(1 - \frac{T_0}{T})$

Exergy is **not conserved** - it's destroyed in irreversible processes by amount $I = T_0 S_{gen}$.

3. S25: Explain Guoy-Stodola theorem. (04 Marks)

Answer:

Guoy-Stodola theorem states that the **rate of exergy destruction (irreversibility)** is directly proportional to the **rate of entropy generation**:

$$\dot{I} = T_0 \cdot \dot{S}_{gen}$$

Where:

- $\dot{I}$ = Irreversibility (exergy destroyed, kW)
- T_0 = Environment temperature (K)
- $\dot{S}_{gen}$ = Rate of entropy generation (kW/K)

Explanation:

- Every irreversibility (friction, heat transfer across ΔT, mixing, throttling) generates entropy.
- The destroyed exergy represents lost work potential – extra fuel that must be burned to compensate.
- For a reversible process, $\dot{S}_{gen} = 0 \rightarrow \dot{I} = 0$.

Application:

- To improve system efficiency, identify components with high $\dot{S}_{gen}$ (e.g., combustor, heat exchanger) and redesign them to reduce temperature differences or pressure drops.

➞ **Real-world example:** In a heat exchanger with hot gas at 800 K and cold air at 300 K, the entropy generation rate can be calculated, multiplied by $T_0 = 298$ K to find kW of lost work.

4. W23: Explain the Third Law of Thermodynamics. (03 Marks)

Answer:

Third Law (Nernst heat theorem): The entropy of a perfectly crystalline substance approaches zero as the absolute temperature approaches zero Kelvin.

$$\lim_{T \to 0} S = 0$$

Key points:

- It provides an absolute reference for entropy values (unlike internal energy and enthalpy).
- Absolute entropies (Third Law entropies) are tabulated at 25°C and 1 bar.
- As $T \to 0$, all molecular motion ceases, and perfect order exists → zero entropy.

Consequences:

- It is impossible to reach absolute zero in a finite number of steps (unattainability principle).
- Heat capacities at low temperatures approach zero.

Example: At 0 K, a perfect crystal of copper has exactly zero entropy.

➞ Real-world application: Used in cryogenics and low-temperature physics to compute absolute entropies for chemical reaction equilibrium constants.