GUJARAT TECHNOLOGICAL UNIVERSITY

BE- 3 SEMESTER – OLD PAPER – S23 TO S25- QUESTION BANK & SOLUTION

Subject Name & Code:
Engineering Thermodynamics (3131905)

Unit 5: Energy (Exergy & Irreversibility)

Repeated Questions:

1. **Define: Exergy, Anergy, Dead state, Irreversibility.**
   ➤ Appeared in: W23 (Q3a, 03 marks), W24 (Q3a, 03 marks), S23 (Q5a, 03 marks), S25 (Q3a, 03 marks)

Answer:

1. **Exergy:** The **maximum theoretical useful work** obtainable from a system as it comes to equilibrium with its environment (dead state). It represents the **quality** or **work potential** of energy.
   - *Example:* 1 kg of steam at 200°C has more exergy than 1 kg of water at 80°C.
2. **Anergy:** The **unavailable portion** of energy that cannot be converted into useful work, even with ideal reversible processes. It's energy at dead state conditions.
   - *Formula:* For heat Q at temperature T: $Anergy = Q \times \frac{T_0}{T}$ where T_0 = surroundings temperature.
3. **Dead State:** The state of a system when it is in **complete equilibrium** with its environment. At dead state, exergy is zero.
4. **Irreversibility (I):** The **loss of exergy** or **increase in anergy** due to irreversible processes. It equals the entropy generation multiplied by the dead state temperature.

$$I = T_0 \times S_{gen} \geq 0$$

   - Always positive for real processes
   - Zero for ideal reversible processes
   - Represents wasted work potential

Relationship:

$$\text{Total Energy} = \text{Exergy (Available)} + \text{Anergy (Unavailable)}$$

Irreversibility measures the **destruction of exergy** (conversion to anergy) during a process.

2. **Explain types of irreversibility and their effects.**
   ➤ Appeared in: W23 (Q3b, 04 marks), W24 (Q3b, 04 marks)

Answer:

Types of irreversibilities (internal and external):

Type	Description	Effect

Type	Description	Effect
Friction	Mechanical friction in moving parts (piston, turbine blades, bearings)	Converts work into heat; increases entropy; reduces useful work output
Heat transfer across finite temperature difference	Heat flows from hot to cold across a temperature gap	Causes entropy generation; reduces exergy efficiency
Unrestrained expansion	Gas expanding suddenly into vacuum (free expansion)	No work done; entropy increases significantly
Mixing of different substances	Two different gases or fluids mixing spontaneously	Irreversible increase in entropy; exergy destroyed
Throttling (Joule-Thomson expansion)	Pressure drop across a valve without work extraction	Temperature change; entropy increases; no work recovered
Chemical reaction	Combustion or spontaneous reactions	Irreversible unless done in a reversible fuel cell

Overall effect of irreversibilities:

- **Exergy destruction** → Less work output from engines, more work input for refrigerators/pumps.
- **Entropy generation** → Universe entropy increases.
- **Efficiency reduction** → Real cycles have lower efficiency than Carnot.

➞ **Real-world application:** Engineers minimize irreversibilities by using lubrication, insulation, gradual expansion, and regenerators.

3. **Derive expression for exergy of a closed system / steady flow system.**
   ➤ **Appeared in: W23 (Q3c, 07 marks), S23 (Q5c, 07 marks)**

Answer:

(A) Exergy of a Closed System:

Given: System at state (P, T, U, S, V) in environment at (P_0, T_0, U_0, S_0, V_0)

Assumptions: Reversible process to dead state, environment is infinite reservoir

Derivation:

Step 1: First Law for process:

$$Q - W = U_0 - U$$

where W = total work done

Step 2: Second Law (reversible process):

$$\Delta S_{universe} = \Delta S_{system} + \Delta S_{surroundings} = 0$$

$$(S_0 - S) + \frac{Q_{surr}}{T_0} = 0$$

Heat to surroundings: $Q_{surr} = -Q$

$$(S_0 - S) - \frac{Q}{T_0} = 0$$

$$Q = T_0(S_0 - S)$$

Step 3: Substitute Q into First Law:

$$T_0(S_0 - S) - W = U_0 - U$$
$$W = (U - U_0) - T_0(S - S_0)$$

Step 4: Total work W includes:

- Useful work (exergy, Φ)
- Work against atmosphere: $P_0(V_0 - V)$

Thus: $W = \Phi + P_0(V_0 - V)$

Step 5: Solve for exergy Φ:

$$\Phi = (U - U_0) - T_0(S - S_0) - P_0(V_0 - V)$$
$$\Phi = (U - U_0) + P_0(V - V_0) - T_0(S - S_0)$$

Closed System Exergy:

$$\boxed{\Phi = (U - U_0) + P_0(V - V_0) - T_0(S - S_0)}$$

(B) Exergy of Steady Flow System (Flow Exergy, ψ):

For steady flow: Include kinetic and potential energy terms

Flow Exergy per unit mass:

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{C^2}{2} + gZ$$

Where:

- h, s = specific enthalpy and entropy at given state
- h_0, s_0 = values at dead state
- C = velocity relative to environment
- Z = elevation relative to reference

Total flow exergy rate: $\dot{\Psi} = \dot{m}\psi$

Special Cases:

1. **For heat Q at temperature T:** $\Phi_{heat} = Q(1 - \frac{T_0}{T})$
2. **For work W:** $\Phi_{work} = W$ (all work is exergy)
3. **At dead state:** $\Phi = 0, \psi = 0$

Exergy Balance: For any process:

$$\text{Exergy In} - \text{Exergy Out} - \text{Exergy Destruction} = \text{Exergy Accumulation}$$
$$\Phi_{in} - \Phi_{out} - I = \Delta\Phi_{system}$$

where $I = T_0 S_{gen} \geq 0$

Other Important Questions:

1. **Explain Guoy-Stodola theorem.**
   ➤ **Appeared in: S25 (Q3b, 04 marks)**

Answer:

Guoy-Stodola theorem states that the **rate of exergy destruction (irreversibility)** is directly proportional to the **rate of entropy generation**:

$$\dot{I} = T_0 \cdot \dot{S}_{gen}$$

Where:

- $\dot{I}$ = Irreversibility (exergy destroyed, kW)
- T_0 = Environment temperature (K)
- $\dot{S}_{gen}$ = Rate of entropy generation (kW/K)

Explanation:

- Every irreversibility (friction, heat transfer across ΔT, mixing, throttling) generates entropy.
- The destroyed exergy represents lost work potential – extra fuel that must be burned to compensate.
- For a reversible process, $\dot{S}_{gen} = 0 \rightarrow \dot{I} = 0$.

Application:

- To improve system efficiency, identify components with high $\dot{S}_{gen}$ (e.g., combustor, heat exchanger) and redesign them to reduce temperature differences or pressure drops.

➞ **Real-world example:** In a heat exchanger with hot gas at 800 K and cold air at 300 K, the entropy generation rate can be calculated, multiplied by $T_0 = 298$ K to find kW of lost work.

2. **A heat engine receives 1000 kW of heat from an infinite heat source at 559 K and rejects heat at 279 K to an infinite heat sink. Comment whether the cycle is possible or irreversible for following heat rejection amounts: (i) 400 kW, (ii) 490 kW, (iii) 850 kW.**
   ➤ **Appeared in: W23 (Q3c, 07 marks)**

Answer:

Given:

- Heat input: $Q_1 = 1000$ kW
- Source temperature: $T_1 = 559$ K
- Sink temperature: $T_2 = 279$ K

To Find: Nature of cycle for given Q_2 values

Formulas:

1. Carnot efficiency: $\eta_{max} = 1 - \frac{T_2}{T_1}$
2. Maximum work: $W_{max} = \eta_{max} \times Q_1$
3. Minimum Q_2: $Q_{2,min} = Q_1 - W_{max}$
4. Actual efficiency: $\eta = 1 - \frac{Q_2}{Q_1}$

Solution:

Step 1: Calculate Carnot efficiency

$$\eta_{max} = 1 - \frac{279}{559} = 1 - 0.4991 = 0.5009 \text{ or } 50.09\%$$

Step 2: Calculate maximum work and minimum Q_2

$$W_{max} = 0.5009 \times 1000 = 500.9 \text{ kW}$$
$$Q_{2,min} = 1000 - 500.9 = 499.1 \text{ kW}$$

Step 3: Analyze each case:

(i) $Q_2 = 400$ kW

$$W = Q_1 - Q_2 = 1000 - 400 = 600 \text{ kW}$$
$$\eta = \frac{600}{1000} = 0.60 \text{ or } 60\%$$
$$\eta = 60\% > \eta_{max} = 50.09\%$$

Conclusion: IMPOSSIBLE (violates Second Law)

(ii) $Q_2 = 490$ kW

$$W = 1000 - 490 = 510 \text{ kW}$$
$$\eta = \frac{510}{1000} = 0.51 \text{ or } 51\%$$
$$\eta = 51\% > \eta_{max} = 50.09\%$$

Conclusion: IMPOSSIBLE (slightly exceeds maximum)

(iii) $Q_2 = 850$ kW

$$W = 1000 - 850 = 150 \text{ kW}$$

$$\eta = \frac{150}{1000} = 0.15 \text{ or } 15\%$$
$$\eta = 15\% < \eta_{max} = 50.09\%$$

Conclusion: POSSIBLE and IRREVERSIBLE (actual efficiency less than Carnot)
Final Answer:

- (i) **Impossible** (η > η_max)
- (ii) **Impossible** (η > η_max)
- (iii) **Possible and Irreversible** (η < η_max)

3. **Explain available and unavailable energy.**
   ➤ **Appeared in: S24 (Q4a, 03 marks)**

Answer:

1. **Available Energy:** The **maximum portion** of energy that can be converted into **useful work** by ideal reversible processes as the system comes to equilibrium with its surroundings. It depends on:
   - Temperature of energy source relative to surroundings
   - State of the system relative to dead state
   - *Example:* High-temperature steam has more available energy than low-temperature hot water
2. **Unavailable Energy:** The **minimum portion** of energy that **must be rejected** to the surroundings according to the Second Law. It cannot be converted to work even by ideal processes.
   - For heat Q at temperature T: $Unavailable = Q \times \frac{T_0}{T}$
   - *Example:* In a heat engine, heat rejected to condenser is unavailable

Mathematical Relation: For heat transfer Q at temperature T:

$$\text{Available Energy} = Q(1 - \frac{T_0}{T})$$
$$\text{Unavailable Energy} = Q \times \frac{T_0}{T}$$

where T_0 = surroundings temperature
Characteristics:

- Available energy decreases as T approaches T_0
- At T = T_0, all energy is unavailable
- Higher T means higher fraction of available energy

Significance: Helps in designing efficient systems by maximizing available energy utilization.

4. **Derive relevant equation to get maximum work from a finite body and thermal energy reservoir.**
   ➤ **Appeared in: W23 (Q3c, 07 marks)**

Answer:

Given:

- Finite body: Mass m, specific heat C, initial temperature T, final temperature T_0
- Surroundings: Infinite reservoir at T_0
- Reversible heat engine operates between body and surroundings

To Find: Maximum work output
Assumptions:

1. Body's specific heat constant
2. Reversible processes

3. No other irreversibilities

Derivation:

Step 1: Consider differential process

As body temperature drops from T to T - dT, heat extracted:

$$\delta Q = -mCdT \text{(negative as body cools)}$$

Step 2: Maximum work from this heat using Carnot engine

Carnot efficiency at instantaneous temperature T:

$$\eta_{Carnot} = 1 - \frac{T_0}{T}$$

Work from δQ:

$$\delta W_{max} = \delta Q \times \eta_{Carnot} = -mCdT \times (1 - \frac{T_0}{T})$$

Step 3: Integrate from initial T to final T_0

$$W_{max} = \int_{T}^{T_0} -mC(1 - \frac{T_0}{T})dT$$
$$= mC \int_{T_0}^{T} (1 - \frac{T_0}{T})\, dT \text{(reversing limits)}$$

Step 4: Evaluate integral

$$W_{max} = mC[\int_{T_0}^{T} 1 \cdot dT - T_0 \int_{T_0}^{T} \frac{1}{T} dT]$$
$$= mC[(T - T_0) - T_0(\ln T - \ln T_0)]$$
$$= mC[(T - T_0) - T_0 \ln (\frac{T}{T_0})]$$

Maximum Work Equation:

$$\boxed{W_{max} = mC[(T - T_0) - T_0 \ln (\frac{T}{T_0})]}$$

Alternative Form: Using entropy change

Initial entropy of body: $S_i = mC\ln T$ (relative to reference)

Final entropy: $S_f = mC\ln T_0$

$$\Delta S = mC\ln (\frac{T_0}{T})$$

Heat transferred to surroundings: $Q_0 = T_0 \mid \Delta S \mid = mCT_0 \ln (\frac{T}{T_0})$

Total heat from body: $Q = mC(T - T_0)$

$$W_{max} = Q - Q_0 = mC(T - T_0) - mCT_0 \ln (\frac{T}{T_0})$$

Physical Interpretation:

- First term: Total heat available from cooling
- Second term: Minimum heat that must be rejected to surroundings
- Difference: Maximum useful work

Special Cases:

1. If T = T_0: $W_{max} = 0$ (no work possible)
2. As T → ∞: $W_{max} \to \infty$
3. This equals the **exergy** of the finite body at temperature T

6. A block of 100 kg of iron at temperature 100ºC is immersed in 50 kg of water at a temperature of 20ºC. Determine the entropy change for combined system of iron and water. Take Cp of iron and water as 0.42 kJ/kgK and 4.18 kJ/kgK respectively. (07 Marks)
Answer:
Given:

- Iron: $m_{Fe} = 100$ kg, $T_{Fe,i} = 100°C = 373$ K, $C_{p,Fe} = 0.42$ kJ/kgK
- Water: $m_w = 50$ kg, $T_{w,i} = 20°C = 293$ K, $C_{p,w} = 4.18$ kJ/kgK
- System is isolated (no heat loss to surroundings)

To Find: Total entropy change of iron + water system
Solution:
Step 1: Find final equilibrium temperature (T_f)
Heat lost by iron = Heat gained by water

$$m_{Fe}C_{p,Fe}(T_{Fe,i} - T_f) = m_w C_{p,w}(T_f - T_{w,i})$$
$$100 \times 0.42 \times (373 - T_f) = 50 \times 4.18 \times (T_f - 293)$$
$$42 \times (373 - T_f) = 209 \times (T_f - 293)$$
$$15666 - 42T_f = 209T_f - 61237$$
$$15666 + 61237 = 209T_f + 42T_f$$
$$76903 = 251T_f$$
$$T_f = \frac{76903}{251} = 306.39 \text{ K} = 33.39°C$$

Step 2: Calculate entropy change of iron

$$\Delta S_{Fe} = m_{Fe}C_{p,Fe}\ln\left(\frac{T_f}{T_{Fe,i}}\right)$$
$$\Delta S_{Fe} = 100 \times 0.42 \times \ln\left(\frac{306.39}{373}\right)$$
$$= 42 \times \ln(0.8214) = 42 \times (-0.1967) = -8.261 \text{ kJ/K}$$

Step 3: Calculate entropy change of water

$$\Delta S_w = m_w C_{p,w}\ln\left(\frac{T_f}{T_{w,i}}\right)$$
$$\Delta S_w = 50 \times 4.18 \times \ln\left(\frac{306.39}{293}\right)$$
$$= 209 \times \ln(1.0457) = 209 \times 0.0447 = 9.342 \text{ kJ/K}$$

Step 4: Calculate total entropy change

$$\Delta S_{total} = \Delta S_{Fe} + \Delta S_w = -8.261 + 9.342 = 1.081 \text{ kJ/K}$$

Verification: Total entropy change is **positive** (1.081 kJ/K > 0), confirming the process is **irreversible** (as expected for heat transfer through finite temperature difference).
Final Answer:

$$\boxed{\Delta S_{total} = 1.081 \text{ kJ/K}}$$

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7. 5 kg of water at 30° C is mixed with 1 kg of ice at 0° C. The process of mixing is adiabatic and the system is open to atmosphere. Make calculations for the temperature of mixture and the change of entropy for the spontaneous mixing process. Take specific heat of water = 4.187 kJ/kg K and latent heat of ice = 335 kJ/kg. (07 Marks)
Answer:
Given:

- Water: $m_w = 5$ kg, $T_w = 30°C = 303$ K
- Ice: $m_i = 1$ kg, $T_i = 0°C = 273$ K
- $C_p = 4.187$ kJ/kgK

- Latent heat of ice: $L = 335$ kJ/kg
- Adiabatic mixing at atmospheric pressure

To Find: (a) Final temperature (b) Total entropy change
Solution:
(a) Final Temperature:
Step 1: Check if all ice melts
Heat available from water cooling to 0°C:

$$Q_{available} = m_w C_p (T_w - 0) = 5 \times 4.187 \times 30 = 628.05 \text{ kJ}$$

Heat required to melt ice:

$$Q_{required} = m_i \times L = 1 \times 335 = 335 \text{ kJ}$$

Since $Q_{available} > Q_{required}$, all ice melts.
Step 2: Heat balance for final temperature T_f
Heat lost by water = Heat gained by ice (melting + heating melted water)

$$m_w C_p (T_w - T_f) = m_i L + m_i C_p (T_f - 0)$$
$$5 \times 4.187 \times (30 - T_f) = 1 \times 335 + 1 \times 4.187 \times T_f$$
$$20.935 \times (30 - T_f) = 335 + 4.187 T_f$$
$$628.05 - 20.935 T_f = 335 + 4.187 T_f$$
$$628.05 - 335 = 4.187 T_f + 20.935 T_f$$
$$293.05 = 25.122 T_f$$
$$T_f = \frac{293.05}{25.122} = 11.66°\text{C} = 284.66 \text{ K}$$

(b) Entropy Change:
Step 3: Entropy change of water (cools from 303 K to 284.66 K)

$$\Delta S_w = m_w C_p \ln\left(\frac{T_f}{T_w}\right) = 5 \times 4.187 \times \ln\left(\frac{284.66}{303}\right)$$
$$= 20.935 \times \ln(0.9395) = 20.935 \times (-0.0624) = -1.306 \text{ kJ/K}$$

Step 4: Entropy change of ice:
(i) Melting at 273 K:

$$\Delta S_{melt} = \frac{m_i L}{T_{melt}} = \frac{1 \times 335}{273} = 1.227 \text{ kJ/K}$$

(ii) Heating melted water from 273 K to 284.66 K:

$$\Delta S_{heat} = m_i C_p \ln\left(\frac{T_f}{T_{melt}}\right) = 1 \times 4.187 \times \ln\left(\frac{284.66}{273}\right)$$
$$= 4.187 \times \ln(1.0427) = 4.187 \times 0.0418 = 0.175 \text{ kJ/K}$$
$$\Delta S_{ice} = \Delta S_{melt} + \Delta S_{heat} = 1.227 + 0.175 = 1.402 \text{ kJ/K}$$

Step 5: Total entropy change

$$\Delta S_{total} = \Delta S_w + \Delta S_{ice} = -1.306 + 1.402 = 0.096 \text{ kJ/K}$$

Final Answer:

(a) Final temperature = 11.66°C	(b) $\Delta S_{total} = 0.096$ kJ/K