GUJARAT TECHNOLOGICAL UNIVERSITY

BE- 3 SEMESTER – OLD PAPER – S23 TO S25- QUESTION BANK & SOLUTION

Subject Name & Code:
Engineering Thermodynamics (3131905)

Unit 7: Gas Power Cycles (Otto, Diesel, Dual)

Repeated Questions:

1. **Derive the air standard efficiency equation for the Otto cycle. (07 Marks - S24, W24)**

Answer:
Given: Otto cycle with air as ideal gas, constant specific heats
Processes:
1. **1→2:** Isentropic compression
2. **2→3:** Constant volume heat addition
3. **3→4:** Isentropic expansion
4. **4→1:** Constant volume heat rejection

Parameters:
- Compression ratio: $r = \frac{V_1}{V_2}$
- Cut-off ratio: Not applicable (constant volume heat addition)
- Pressure ratio: $r_p = \frac{P_3}{P_2}$
- $\gamma = \frac{C_p}{C_v}$ (ratio of specific heats)

Derivation:
Step 1: Heat addition (Q_in)
At constant volume (2→3):

$$Q_{in} = mC_v(T_3 - T_2)$$

Step 2: Heat rejection (Q_out)
At constant volume (4→1):

$$Q_{out} = mC_v(T_4 - T_1)$$

Step 3: Thermal efficiency

$$\eta_{Otto} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Step 4: Relate temperatures using isentropic relations
For isentropic process (1→2):

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} = r^{\gamma-1}$$
$$T_2 = T_1 r^{\gamma-1}$$

For constant volume process (2→3):

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} = r_p$$
$$T_3 = T_2 r_p = T_1 r^{\gamma-1} r_p$$

For isentropic process (3→4):

$$\frac{T_3}{T_4} = (\frac{V_4}{V_3})^{\gamma-1} = (\frac{V_1}{V_2})^{\gamma-1} = r^{\gamma-1}$$
$$T_4 = \frac{T_3}{r^{\gamma-1}} = \frac{T_1 r^{\gamma-1} r_p}{r^{\gamma-1}} = T_1 r_p$$

Step 5: Substitute in efficiency formula

$$\eta_{Otto} = 1 - \frac{T_1 r_p - T_1}{T_1 r^{\gamma-1} r_p - T_1 r^{\gamma-1}}$$
$$= 1 - \frac{T_1(r_p - 1)}{T_1 r^{\gamma-1}(r_p - 1)}$$
$$= 1 - \frac{1}{r^{\gamma-1}}$$

Final Otto Cycle Efficiency:

$$\boxed{\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}}$$

Key Observations:

1. Depends only on **compression ratio r** and **γ**
2. Independent of heat added
3. Higher r gives higher efficiency
4. Practical limit: r ≈ 8-12 (knocking limitation)

2. **Derive the air standard efficiency equation for the Diesel cycle. (07 Marks - S23, W22)**

Answer:

Diesel Cycle Processes:

1. **1→2:** Isentropic compression
2. **2→3:** Constant pressure heat addition
3. **3→4:** Isentropic expansion
4. **4→1:** Constant volume heat rejection

Parameters:

- Compression ratio: $r = \frac{V_1}{V_2}$
- Cut-off ratio: $\rho = \frac{V_3}{V_2}$
- $\gamma = \frac{C_p}{C_v}$

Derivation:

Step 1: Heat addition (Q_in)
At constant pressure (2→3):

$$Q_{in} = mC_p(T_3 - T_2)$$

Step 2: Heat rejection (Q_out)
At constant volume (4→1):

$$Q_{out} = mC_v(T_4 - T_1)$$

Step 3: Thermal efficiency

$$\eta_{Diesel} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2}$$

Step 4: Relate temperatures
For isentropic compression (1→2):

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} = r^{\gamma-1}$$
$$T_2 = T_1 r^{\gamma-1}$$

For constant pressure (2→3):

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} = \rho$$
$$T_3 = T_2\rho = T_1 r^{\gamma-1}\rho$$

For isentropic expansion (3→4):

$$\frac{T_3}{T_4} = (\frac{V_4}{V_3})^{\gamma-1} = (\frac{V_1}{V_3})^{\gamma-1}$$

Since $\frac{V_1}{V_3} = \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} = \frac{r}{\rho}$

$$\frac{T_3}{T_4} = (\frac{r}{\rho})^{\gamma-1}$$
$$T_4 = T_3(\frac{\rho}{r})^{\gamma-1} = T_1 r^{\gamma-1}\rho(\frac{\rho}{r})^{\gamma-1} = T_1\rho^{\gamma}$$

Step 5: Substitute in efficiency formula

$$\eta = 1 - \frac{1}{\gamma} \cdot \frac{T_1\rho^{\gamma} - T_1}{T_1 r^{\gamma-1}\rho - T_1 r^{\gamma-1}}$$
$$= 1 - \frac{1}{\gamma} \cdot \frac{T_1(\rho^{\gamma} - 1)}{T_1 r^{\gamma-1}(\rho - 1)}$$
$$= 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{1}{\gamma} \cdot \frac{\rho^{\gamma} - 1}{\rho - 1}$$

Final Diesel Cycle Efficiency:

$$\boxed{\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{1}{\gamma} \cdot \frac{\rho^{\gamma} - 1}{\rho - 1}}$$

Key Observations:

1. Depends on r, ρ, and γ
2. For same r, Diesel η < Otto η
3. Higher r increases efficiency
4. Higher ρ decreases efficiency (heat addition at lower average temperature)

3. **Compare Otto, Diesel, and Dual cycles for the same compression ratio and heat supplied. (04 Marks - S23, S25, W24)**

Answer:
Comparison Basis: Same compression ratio (r), same heat input (Q_in), same working fluid (air with constant γ)
Efficiency Order: $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$
Reason:

1. **Otto Cycle:** All heat added at **constant volume** (highest temperature)
    - Maximum pressure and temperature highest
    - Heat addition at minimum volume → highest average temperature
2. **Dual Cycle:** Heat added partly at constant volume, partly at constant pressure
    - Moderate maximum pressure and temperature
    - Average temperature between Otto and Diesel
3. **Diesel Cycle:** All heat added at **constant pressure**
    - Lowest maximum pressure and temperature
    - Heat addition during expansion → lowest average temperature

Mathematical Proof:
For same r and Q_in:
- Otto: All Q_in at T_2 (immediately after compression)
- Dual: Some Q_in at T_2, some during expansion
- Diesel: All Q_in during expansion (at rising volume)

Since $\eta = 1 - \frac{Q_{out}}{Q_{in}}$ and Q_in same:
- Higher average T during heat addition → lower Q_out → higher η
- Otto has highest average T → highest η

Pressure-Temperature Comparison:
- P_{max}: Otto > Dual > Diesel
- T_{max}: Otto > Dual > Diesel
- Expansion ratio: Diesel > Dual > Otto

Practical Significance:
- Otto engines need higher octane fuel (anti-knock)
- Diesel engines can use higher compression ratios
- Dual cycle represents actual combustion (CI engines with pre-combustion)

4. **State assumptions made for analysis of air standard cycles.**
   ➤ Appeared in: W23 (Q4a, 03 marks), W24 (Q4a, 03 marks), S25 (Q4a, 03 marks)

Answer:
The following assumptions are made in **air standard cycle analysis**:
1. **Working fluid is air** – considered as an ideal gas with constant specific heats (C_p, C_v).
2. **Closed cycle** – same air is recirculated (no mass exchange).
3. **Heat addition and rejection** are external (from a hot reservoir and to a cold reservoir).
4. **All processes are internally reversible** – no friction, no pressure drops, no heat losses.
5. **Combustion is replaced by heat addition** from an external source – chemical reaction effects ignored.
6. **Exhaust and intake strokes are replaced by heat rejection** to the surroundings.

Why these assumptions?
They simplify analysis to compare ideal performance of different cycles without complexities of real engines.
➞ **Real-world application:** Air standard efficiency overpredicts actual engine efficiency but serves as a benchmark for improvement.

Other Important Questions:
1. **S24: An oil engine works on the dual combustion cycle. Find the ideal efficiency. (04 Marks)**

Answer:
For the **dual combustion cycle** (also called the **mixed cycle** or **limited pressure cycle**), the **air-standard thermal efficiency** is given by:
Processes:

1. **1→2:** Isentropic compression
2. **2→3:** Constant volume heat addition
3. **3→4:** Constant pressure heat addition
4. **4→5:** Isentropic expansion
5. **5→1:** Constant volume heat rejection

Parameters:

- Compression ratio: $r = \frac{V_1}{V_2}$
- Cut-off ratio: $\rho = \frac{V_4}{V_3}$
- Explosion ratio (pressure ratio): $r_p = \frac{P_3}{P_2}$
- $\gamma = \frac{C_p}{C_v}$ (ratio of specific heats, for air $\gamma = 1.4$)

Derivation of Efficiency Formula:

Step 1: Heat addition

- Constant volume portion (2→3): $Q_{23} = mC_v(T_3 - T_2)$
- Constant pressure portion (3→4): $Q_{34} = mC_p(T_4 - T_3)$
- Total heat input: $Q_{in} = m[C_v(T_3 - T_2) + C_p(T_4 - T_3)]$

Step 2: Heat rejection

- Constant volume (5→1): $Q_{out} = mC_v(T_5 - T_1)$

Step 3: Thermal efficiency

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Step 4: Relate temperatures using isentropic and process relations

- For isentropic compression (1→2): $T_2 = T_1 r^{\gamma-1}$
- For constant volume (2→3): $T_3 = T_2 r_p = T_1 r^{\gamma-1} r_p$
- For constant pressure (3→4): $T_4 = T_3 \rho = T_1 r^{\gamma-1} r_p \rho$
- For isentropic expansion (4→5): $T_5 = T_4 (\frac{V_4}{V_5})^{\gamma-1}$

Since $V_5 = V_1$ and $V_4 = V_3 \rho$, and $V_3 = V_2$:

$$\frac{V_4}{V_5} = \frac{V_2 \rho}{V_1} = \frac{\rho}{r}$$

$$T_5 = T_4 (\frac{\rho}{r})^{\gamma-1} = T_1 r^{\gamma-1} r_p \rho (\frac{\rho}{r})^{\gamma-1} = T_1 r_p \rho^\gamma$$

Step 5: Substitute into efficiency formula

$$\eta = 1 - \frac{T_1 r_p \rho^\gamma - T_1}{(T_1 r^{\gamma-1} r_p - T_1 r^{\gamma-1}) + \gamma(T_1 r^{\gamma-1} r_p \rho - T_1 r^{\gamma-1} r_p)}$$
$$= 1 - \frac{T_1(r_p \rho^\gamma - 1)}{T_1 r^{\gamma-1}(r_p - 1) + \gamma T_1 r^{\gamma-1} r_p (\rho - 1)}$$
$$= 1 - \frac{r_p \rho^\gamma - 1}{r^{\gamma-1}[(r_p - 1) + \gamma r_p (\rho - 1)]}$$

Final Dual Cycle Efficiency Formula:

$$\boxed{\eta = 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{r_p \rho^\gamma - 1}{(r_p - 1) + \gamma r_p (\rho - 1)}}$$

For air ($\gamma = 1.4$):

$$\eta = 1 - \frac{1}{r^{0.4}} \cdot \frac{r_p \rho^{1.4} - 1}{(r_p - 1) + 1.4 r_p (\rho - 1)}$$

Special Cases:

1. **When** $\rho = 1$ (no constant pressure heat addition):
$$\eta = 1 - \frac{1}{r^{\gamma-1}} \text{(Otto cycle efficiency)}$$

2. **When** $r_p = 1$ (no constant volume heat addition):
$$\eta = 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)} \text{(Diesel cycle efficiency)}$$

To find the ideal efficiency for given values of r, r_p, and ρ, substitute into the above formula with $\gamma = 1.4$ for air.

2. **W22: Compare Otto, Diesel, and Dual cycles for the same maximum pressure and temperature. (04 Marks)**

Answer:
Comparison Basis: Same maximum pressure (P_{max}), same maximum temperature (T_{max}), same working fluid
Efficiency Order: $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$
Reasoning:

1. **Compression Ratio:**
   - Diesel: **Highest** compression ratio (r)
   - Dual: Moderate r
   - Otto: **Lowest** r
2. **Heat Addition:**
   - Otto: All heat at constant volume (smallest volume)
   - Dual: Part constant volume, part constant pressure
   - Diesel: All heat at constant pressure (largest volume)
3. **Expansion Ratio:**
   - Diesel: **Largest** expansion (from largest volume to V_1)
   - Dual: Moderate expansion
   - Otto: **Smallest** expansion
4. **Heat Rejection:**
   For same P_max, T_max:
   - Diesel rejects **least** heat (Q_out minimum)
   - Otto rejects **most** heat
   - Dual is intermediate

Mathematical Explanation:
Since $\eta = 1 - \frac{Q_{out}}{Q_{in}}$:

- For same P_max, T_max, all cycles have same T_3, P_3
- But T_1 (after heat rejection) varies: Otto has highest T_1, Diesel lowest
- Thus Q_out = mC_v(T_4 - T_1) is smallest for Diesel

Pressure-Volume Diagram Comparison:

- All cycles end at same P_max, T_max point
- Otto: Smallest area enclosed (lowest work)
- Diesel: Largest area enclosed (highest work)
- Dual: Intermediate area

Practical Implication:

- Diesel engines can be more efficient than Otto for same peak conditions
- But Diesel needs higher compression ratios (stronger construction)
- Dual cycle represents compromise

3. W24: Derive an expression for the thermal efficiency of an Otto cycle. (04 Marks)

Answer:

Given: Otto cycle with air as ideal gas, constant specific heats

Processes:

1. **1→2:** Isentropic compression
2. **2→3:** Constant volume heat addition
3. **3→4:** Isentropic expansion
4. **4→1:** Constant volume heat rejection

Parameters:

- Compression ratio: $r = \frac{V_1}{V_2}$
- $\gamma = \frac{C_p}{C_v}$ (ratio of specific heats)

Derivation:

Step 1: Heat addition (Q_in)

At constant volume (2→3):

$$Q_{in} = mC_v(T_3 - T_2)$$

Step 2: Heat rejection (Q_out)

At constant volume (4→1):

$$Q_{out} = mC_v(T_4 - T_1)$$

Step 3: Thermal efficiency

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Step 4: Relate temperatures using isentropic relations

For isentropic compression (1→2):

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} = r^{\gamma-1}$$
$$T_2 = T_1 r^{\gamma-1}$$

For constant volume heat addition (2→3):

$$\frac{P_3}{P_2} = \frac{T_3}{T_2}$$
$$T_3 = T_2 \times \frac{P_3}{P_2}$$

For isentropic expansion (3→4):

$$\frac{T_3}{T_4} = (\frac{V_4}{V_3})^{\gamma-1} = (\frac{V_1}{V_2})^{\gamma-1} = r^{\gamma-1}$$
$$T_4 = \frac{T_3}{r^{\gamma-1}}$$

Step 5: Substitute and simplify

From Step 4: $T_3 = T_2 \times \frac{P_3}{P_2} = T_1 r^{\gamma-1} \times \frac{P_3}{P_2}$

And $T_4 = \frac{T_3}{r^{\gamma-1}} = T_1 \times \frac{P_3}{P_2}$

Substitute into efficiency formula:

$$\eta = 1 - \frac{T_1 \frac{P_3}{P_2} - T_1}{T_1 r^{\gamma-1} \frac{P_3}{P_2} - T_1 r^{\gamma-1}}$$

$$= 1 - \frac{T_1(\frac{P_3}{P_2} - 1)}{T_1 r^{\gamma-1}(\frac{P_3}{P_2} - 1)}$$
$$= 1 - \frac{1}{r^{\gamma-1}}$$

Final Expression:

$$\boxed{\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}}$$

Key Points:

1. Efficiency depends only on **compression ratio (r)** and **ratio of specific heats (γ)**
2. Higher compression ratio gives higher efficiency
3. Independent of heat added or maximum temperature
4. Practical limit: r ≈ 8-12 due to knocking in SI engines
5. For air (γ = 1.4): η increases from 47% at r=6 to 56% at r=8 to 61% at r=10

4. **In an engine working on Otto cycle, air has a pressure of 1 bar and temperature 30°C at the entry. Air is compressed with a compression ratio of 6. The heat is added at constant volume until the temperature rises to 1500°C. Determine (i) air standard efficiency, (ii) pressure and temperature at the end of compression, (iii) heat supplied, (iv) mean effective pressure. Take Cv = 0.718 kJ/kg K, R = 0.287 kJ/kg K.**
   ➤ Appeared in: W23 (Q4c, 07 marks)

Answer:

Given:

- $P_1 = 1 \text{ bar} = 100 \text{ kPa}, T_1 = 30°\text{C} = 303 \text{ K}$
- Compression ratio: $r = 6$
- $T_3 = 1500°\text{C} = 1773 \text{ K}$
-
- $\gamma = \frac{C_p}{C_v} = \frac{R+C_v}{C_v} = \frac{0.287+0.718}{0.718} = \frac{1.005}{0.718} = 1.4$

Solution:

(i) Air standard efficiency

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{6^{0.4}} = 1 - \frac{1}{6^{0.4}}$$
$$6^{0.4} = e^{0.4 \ln 6} = e^{0.4 \times 1.7918} = e^{0.7167} = 2.048$$
$$\eta = 1 - \frac{1}{2.048} = 1 - 0.4883 = 0.5117 \text{ or } 51.17\%$$

(ii) Pressure and temperature at end of compression (Point 2)

$$T_2 = T_1 r^{\gamma-1} = 303 \times 6^{0.4} = 303 \times 2.048 = 620.5 \text{ K}$$
$$P_2 = P_1 r^{\gamma} = 100 \times 6^{1.4} = 100 \times 12.29 = 1229 \text{ kPa} = 12.29 \text{ bar}$$

(iii) Heat supplied per kg

$$q_{in} = C_v(T_3 - T_2) = 0.718 \times (1773 - 620.5)$$
$$= 0.718 \times 1152.5 = 827.5 \text{ kJ/kg}$$

(iv) Mean effective pressure (MEP)

First find net work: $w_{net} = \eta \times q_{in} = 0.5117 \times 827.5 = 423.4 \text{ kJ/kg}$
Specific volume at point 1:

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 303}{100} = \frac{86.96}{100} = 0.8696 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{0.8696}{6} = 0.1449 \text{ m}^3/\text{kg}$$

MEP = $\frac{w_{net}}{v_1 - v_2} = \frac{423.4}{0.869 \quad .1449} = \frac{423.4}{0.7247} = 584.4$ kPa = 5.844 bar

Final Answers:

(i) $\eta = 51.17\%$

(ii) $T_2 = 620.5$ K, $P_2 = 12.29$ bar

(iii) $q_{in} = 827.5$ kJ/kg

(iv) MEP = 5.844 bar

5. **An engine working on the Otto cycle is supplied with air at 0.1 MPa, 35°C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency and mean effective pressure (for air, Cp = 1.005, Cv = 0.718 and R = 0.287 kJ/kg K).**

   ➤ **Appeared in: S23 (Q4c, 07 marks)**

Answer:

Given:

- $P_1 = 0.1$ MPa $= 100$ kPa, $T_1 = 35$°C $= 308$ K
- $r = 8$
- $q_{in} = 2100$ kJ/kg
-
- $\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.718} = 1.4$

Solution:

(i) Cycle efficiency

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{0.4}}$$
$$8^{0.4} = e^{0.4 \ln 8} = e^{0.4 \times 2.0794} = e^{0.8318} = 2.297$$
$$\eta = 1 - \frac{1}{2.297} = 1 - 0.4353 = 0.5647 \text{ or } 56.47\%$$

(ii) Temperature and pressure at end of compression (Point 2)

$$T_2 = T_1 r^{\gamma-1} = 308 \times 8^{0.4} = 308 \times 2.297 = 707.5 \text{ K}$$
$$P_2 = P_1 r^{\gamma} = 100 \times 8^{1.4} = 100 \times 18.38 = 1838 \text{ kPa} = 1.838 \text{ MPa}$$

(iii) Maximum temperature (Point 3)

From heat addition: $q_{in} = C_v(T_3 - T_2)$

$$2100 = 0.718(T_3 - 707.5)$$
$$T_3 - 707.5 = \frac{2100}{0.718} = 2924.8$$
$$T_3 = 2924.8 + 707.5 = 3632.3 \text{ K}$$

(iv) Maximum pressure (Point 3)

For constant volume process (2→3): $\frac{P_3}{P_2} = \frac{T_3}{T_2}$

$$P_3 = P_2 \times \frac{T_3}{T_2} = 1838 \times \frac{3632.3}{707.5} = 1838 \times 5.134 = 9436 \text{ kPa} = 9.436 \text{ MPa}$$

(v) Mean effective pressure

Net work: $w_{net} = \eta \times q_{in} = 0.5647 \times 2100 = 1185.9$ kJ/kg
Specific volume at point 1:

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 308}{100} = \frac{88.40}{100} = 0.8840 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{0.8840}{8} = 0.1105 \text{ m}^3/\text{kg}$$

MEP = $\frac{w_{net}}{v_1 - v_2} = \frac{1185.9}{0.8840 - .1105} = \frac{1185.9}{0.7735} = 1533.3$ kPa $= 1.533$ MPa

Final Answers:

- Maximum temperature: 3632.3 K
- Maximum pressure: 9.436 MPa
- Cycle efficiency: 56.47%
- Mean effective pressure: 1.533 MPa

6. **In an engine operated on Otto cycle, the compression ratio is 6 and the compression starts at 30°C and 100 kPa. The maximum temperature of the cycle is 1250°C. Determine: (i) heat supplied per kg of air, (ii) air standard efficiency of the cycle, (iii) Mean effective pressure of the cycle. Take Cv = 0.716 and γ = 1.4.**
   ➤ Appeared in: W23 (Q4c, 07 marks)

Answer:

Given:

- $r = 6, T_1 = 30°\text{C} = 303 \text{ K}, P_1 = 100$ kPa
- $T_3 = 1250°\text{C} = 1523$ K
-
-

Solution:

(i) Air standard efficiency

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{6^{0.4}}$$
$$6^{0.4} = 2.048 (\text{from earlier})$$
$$\eta = 1 - \frac{1}{2.048} = 1 - 0.4883 = 0.5117 \text{ or } 51.17\%$$

(ii) Heat supplied per kg

First find T_2:

$$T_2 = T_1 r^{\gamma-1} = 303 \times 6^{0.4} = 303 \times 2.048 = 620.5 \text{ K}$$
$$q_{in} = C_v(T_3 - T_2) = 0.716 \times (1523 - 620.5) = 0.716 \times 902.5 = 646.2 \text{ kJ/kg}$$

(iii) Mean effective pressure

Net work: $w_{net} = \eta \times q_{in} = 0.5117 \times 646.2 = 330.6$ kJ/kg
Specific volume at point 1:

$$v_1 = \frac{RT_1}{P_1} = \frac{0.2864 \times 303}{100} = \frac{86.78}{100} = 0.8678 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{0.8678}{6} = 0.1446 \text{ m}^3/\text{kg}$$

MEP = $\frac{w_{net}}{v_1 - v_2} = \frac{330.6}{0.8678 - .1446} = \frac{330.6}{0.7232} = 457.0$ kPa $= 4.57$ bar

Final Answers:

(i) $q_{in} = 646.2$ kJ/kg
(ii) $\eta = 51.17\%$

(iii) MEP = 4.57 bar

7. A diesel engine has compression ratio of 20. The compression begins at 0.1 MPa and 35°C. The heat added is 1700 kJ/kg. Calculate maximum pressure and temperature of cycle, work done per kg of air and cycle efficiency.
➤ Appeared in: S25 (Q4c, 07 marks)

Answer:
Given:
- $r = 20, P_1 = 0.1 \text{ MPa} = 100 \text{ kPa}, T_1 = 35°\text{C} = 308 \text{ K}$
- $q_{in} = 1700 \text{ kJ/kg}$
- For air:

Solution:
Step 1: Temperature and pressure after compression (Point 2)

$$T_2 = T_1 r^{\gamma-1} = 308 \times 20^{0.4}$$
$$20^{0.4} = e^{0.4\ln 20} = e^{0.4\times 2.9957} = e^{1.1983} = 3.314$$
$$T_2 = 308 \times 3.314 = 1020.7 \text{ K}$$
$$P_2 = P_1 r^{\gamma} = 100 \times 20^{1.4} = 100 \times 66.29 = 6629 \text{ kPa} = 6.629 \text{ MPa}$$

Step 2: Cut-off ratio (ρ) from heat addition
For constant pressure heat addition: $q_{in} = C_p(T_3 - T_2)$

$$1700 = 1.005(T_3 - 1020.7)$$
$$T_3 - 1020.7 = \frac{1700}{1.005} = 1691.5$$
$$T_3 = 1020.7 + 1691.5 = 2712.2 \text{ K}$$
$$\rho = \frac{T_3}{T_2} = \frac{2712.2}{1020.7} = 2.657$$

Step 3: Maximum pressure (Point 3)
Since 2→3 is constant pressure:

$$P_3 = P_2 = 6.629 \text{ MPa}$$

Step 4: Cycle efficiency

$$\eta = 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{1}{\gamma} \cdot \frac{\rho^{\gamma} - 1}{\rho - 1}$$
$$= 1 - \frac{1}{3.314} \times \frac{1}{1.4} \times \frac{2.657^{1.4} - 1}{2.657 - 1}$$

First calculate $2.657^{1.4} = e^{1.4\ln 2.657} = e^{1.4\times 0.9773} = e^{1.3682} = 3.928$

$$\frac{\rho^{\gamma} - 1}{\rho - 1} = \frac{3.928 - 1}{2.657 - 1} = \frac{2.928}{1.657} = 1.767$$
$$\eta = 1 - \frac{1}{3.314} \times \frac{1}{1.4} \times 1.767 = 1 - 0.3018 \times 0.7143 \times 1.767$$
$$= 1 - 0.3018 \times 1.262 = 1 - 0.3808 = 0.6192 \text{ or } 61.92\%$$

Step 5: Work done per kg

$$w_{net} = \eta \times q_{in} = 0.6192 \times 1700 = 1052.6 \text{ kJ/kg}$$

Step 6: Temperature at end of expansion (Point 4)

$$T_4 = T_1 \rho^{\gamma} = 308 \times 2.657^{1.4} = 308 \times 3.928 = 1209.8 \text{ K}$$

Final Answers:

- Maximum pressure: 6.629 MPa
- Maximum temperature: 2712.2 K
- Cycle efficiency: 61.92%
- Work done: 1052.6 kJ/kg

8. **An oil engine working on the dual combustion cycle has a compression ratio of 14 and the explosion ratio obtained from an indicator card is 1.4. If the cut-off occurs at 6 per cent of the stroke, find the ideal efficiency. Take γ for air = 1.4.**
   ➤ Appeared in: S24 (Q3b, 04 marks)

Answer:

Given:

- Dual combustion cycle
- Compression ratio: $r = 14$
- Explosion ratio (pressure ratio): $r_p = 1.4$
- Cut-off occurs at 6% of stroke
- $\gamma = 1.4$

To Find: Ideal efficiency

Solution:

Step 1: Find cut-off ratio (ρ)

Let V_1 = volume at beginning of compression

V_2 = volume at end of compression

V_3 = volume at end of constant volume heat addition

V_4 = volume at end of constant pressure heat addition

Given: Cut-off occurs at 6% of stroke

Stroke volume = $V_1 - V_2$

Cut-off volume = 0.06 × stroke volume = $0.06(V_1 - V_2)$

But $V_4 = V_3 + 0.06(V_1 - V_2)$

And $V_3 = V_2$ (constant volume process 2→3)

So: $V_4 = V_2 + 0.06(V_1 - V_2)$

Cut-off ratio: $\rho = \frac{V_4}{V_3} = \frac{V_2 + 0.06(V_1 - V_2)}{V_2}$

$$\rho = 1 + 0.06(\frac{V_1}{V_2} - 1) = 1 + 0.06(r - 1)$$

$$\rho = 1 + 0.06(14 - 1) = 1 + 0.06 \times 13 = 1 + 0.78 = 1.78$$

Step 2: Dual cycle efficiency formula

$$\eta = 1 - \frac{1}{r^{\gamma-1}} \cdot [\frac{r_p \rho^\gamma - 1}{(\gamma r_p(\rho - 1) + (r_p - 1))}]$$

Step 3: Calculate required terms

$$r^{\gamma-1} = 14^{0.4} = e^{0.4l\quad 14} = e^{0.4\times 2.6391} = e^{1.0556} = 2.874$$

$$\rho^\gamma = 1.78^{1.4} = e^{1.4\ln\ 1.78} = e^{1.4\times 0.5766} = e^{0.8072} = 2.242$$

$$r_p \rho^\gamma = 1.4 \times 2.242 = 3.139$$

Step 4: Substitute in formula

Numerator: $r_p \rho^\gamma - 1 = 3.139 - 1 = 2.139$

Denominator: $\gamma r_p(\rho - 1) + (r_p - 1) = 1.4 \times 1.4 \times (1.78 - 1) + (1.4 - 1)$

$$= 1.96 \times 0.78 + 0.4 = 1.5288 + 0.4 = 1.9288$$

$$\frac{r_p\rho^\gamma - 1}{\text{Denominator}} = \frac{2.139}{1.9288} = 1.109$$

Step 5: Calculate efficiency

$$\eta = 1 - \frac{1}{2.874} \times 1.109 = 1 - 0.348 \times 1.109 = 1 - 0.386 = 0.614 \text{ or } 61.4\%$$

Final Answer:

$$\boxed{\eta = 61.4\%}$$

9. **The air standard efficiency of an Otto cycle is 60% and γ = 1.4. Calculate the compression ratio.**
   ➤ **Appeared in: S24 (Q3b, 04 marks)**

Answer:

Given:

- Otto cycle efficiency: $\eta = 60\% = 0.60$
- $\gamma = 1.4$

To Find: Compression ratio (r)

Solution:

Step 1: Otto cycle efficiency formula

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

Step 2: Rearrange for r

$$\frac{1}{r^{\gamma-1}} = 1 - \eta = 1 - 0.60 = 0.40$$
$$r^{\gamma-1} = \frac{1}{0.40} = 2.5$$

Step 3: Solve for r

$$r^{0.4} = 2.5$$

Take natural log:

$$0.4\ln r = \ln 2.5 = 0.9163$$
$$\ln r = \frac{0.9163}{0.4} = 2.2908$$
$$r = e^{2.2908} = 9.88$$

Step 4: Verify

$$\eta = 1 - \frac{1}{9.88^{0.4}} = 1 - \frac{1}{e^{0.4\times\ln 9.88}} = 1 - \frac{1}{e^{0.4\times 2.2908}} = 1 - \frac{1}{e^{0.9163}} = 1 - \frac{1}{2.5} = 1 - 0.4 = 0.60$$

Final Answer:

$$\boxed{r = 9.88}$$