GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO S25 – QUESTION BANK SOLUTION

Subject Name & Code:

FUNDAMENTALS OF MACHINE DESIGN (3141907)

Unit 2: Moment of Inertia of Planar Cross-Sections

Repeated Questions:

1. **State/Explain Parallel and Perpendicular axes theorems.**
    - Appeared in: SUM 24 (Q1a, 03 marks), WIN 23 (Q2b, 04 marks), SUM 22 (Q4b, 04 marks).

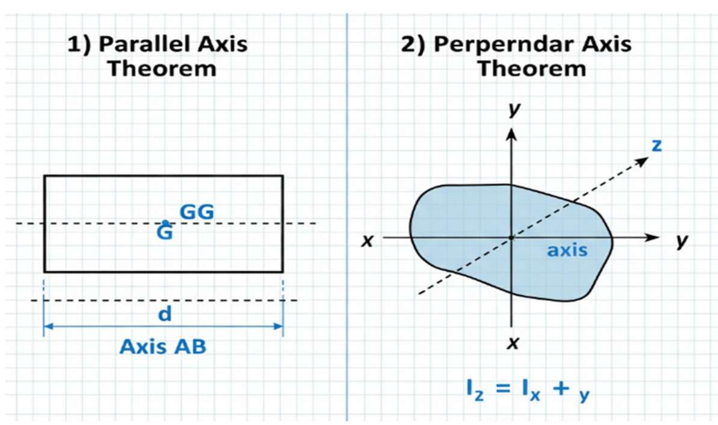
Answer:
(03-04 marks)

1. Parallel Axis Theorem:

- **Statement:** The moment of inertia of an area about any axis is equal to the sum of:
  a) Its moment of inertia about a parallel axis passing through its centroid, and
  b) The product of the area and the square of the distance between the two axes.
- **Formula:** $I_{AB} = I_{GG} + A \cdot d^2$
  where: I_{AB} = M.I. about axis AB, I_{GG} = M.I. about centroidal axis, A = Area, d = Distance between axes.

2. Perpendicular Axis Theorem:

- **Statement:** For a planar lamina, the moment of inertia about an axis perpendicular to its plane (polar moment of inertia) is equal to the sum of the moments of inertia about two mutually perpendicular axes lying in the plane and intersecting at the point where the perpendicular axis passes.
- **Formula:** $I_z = I_x + I_y$
  where: I_z = Polar M.I., I_x, I_y = M.I. about x and y axes in the plane.



Other Important Questions:

1. **Derive the expression for polar moment of inertia of a circle using principle of integration.**
    - Appeared in: SUM 23 (Q1a, 03 marks).

Answer:
(03 marks)

Step 1: Given and Setup

- Consider a circle of radius R.
- Take a thin elemental ring of radius r and thickness dr.
- Area of elemental ring: $dA = 2\pi r \cdot dr$

Step 2: Polar Moment of Inertia Definition
Polar M.I. about center: $J = \int r^2 dA$

Step 3: Integration

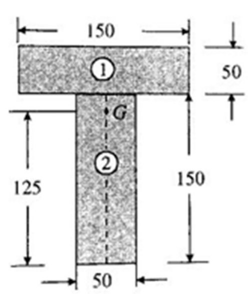
$$J = \int_0^R r^2 \cdot (2\pi r dr) = 2\pi \int_0^R r^3 dr$$
$$J = 2\pi [\frac{r^4}{4}]_0^R$$
$$J = 2\pi \cdot \frac{R^4}{4}$$

Step 4: Final Expression

$$\boxed{J = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}}$$

where D = 2R is the diameter.

2. **Find the moment of inertia of a T section shown in the figure about X-X and Y-Y axes through the center of gravity of the section.**
    - Appeared in: WIN 24 (Q2c, 07 marks).



Answer:
(07 marks)

Step 1: Given Data (from typical T-section)

- Flange: width = 150 mm, thickness = 50 mm
- Web: width = 50 mm, height = 150 mm
- Total height = 200 mm

Step 2: Find Centroid ($\bar{y}$)
Take reference axis at bottom of web.

- Area of flange, $A_1 = 150 \times 50 = 7500$ mm², $y_1 = 150 + 25 = 175$ mm
- Area of web, $A_2 = 50 \times 150 = 7500$ mm², $y_2 = 75$ mm
- Total Area, A = 15000 mm²

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A} = \frac{7500 \times 175 + 7500 \times 75}{15000} = 125 \text{ mm}$$

Step 3: M.I. about X-X axis (through centroid)
Using parallel axis theorem:

- For flange: $I_{x1} = \frac{150 \times 50^3}{12} + 7500 \times (175 - 125)^2 = 1{,}562{,}500 + 18{,}750{,}000 = 20{,}312{,}500 \text{ mm}^4$
- For web: $I_{x2} = \frac{50 \times 150^3}{12} + 7500 \times (125 - 75)^2 = 14{,}062{,}500 + 18{,}750{,}000 = 32{,}812{,}500 \text{ mm}^4$
- Total $I_{xx} = 20{,}312{,}500 + 32{,}812{,}500 = 53{,}125{,}000 \text{ mm}^4$

Step 4: M.I. about Y-Y axis (through centroid)

- For flange: $I_{y1} = \frac{50 \times 150^3}{12} = 14{,}062{,}500 \text{ mm}^4$
- For web: $I_{y2} = \frac{150 \times 50^3}{12} = 1{,}562{,}500 \text{ mm}^4$
- Total $I_{yy} = 14{,}062{,}500 + 1{,}562{,}500 = 15{,}625{,}000 \text{ mm}^4$

Step 5: Final Answer

$$\boxed{I_{xx} = 53.125 \times 10^6 \text{ mm}^4 I_{yy} = 15.625 \times 10^6 \text{ mm}^4}$$

3. **Determine the moment of inertia for rectangular cross-section about X-axis and Y-axis passing from their CG.**
    - Appeared in: WIN 23 (Q4b, 04 marks).

Answer:
(04 marks)

Given: Rectangle with width = b, height = d (depth)

M.I. about X-axis (horizontal through CG):
Consider elemental strip parallel to x-axis at distance y from CG.
Area of strip = b·dy

$$I_{xx} = \int_{-d/2}^{d/2} y^2 (b\,dy) = b\int_{-d/2}^{d/2} y^2 dy$$

$$I_{xx} = b[\frac{y^3}{3}]_{-d/2}^{d/2} = \frac{b}{3}(\frac{d^3}{8} + \frac{d^3}{8})$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

M.I. about Y-axis (vertical through CG):

$$I_{yy} = \frac{db^3}{12} \text{(by symmetry)}$$

4. **Determine the moment of inertia of plane section, shown in figure-1.**
    - Appeared in: WIN 22 (Q2b, 04 marks).

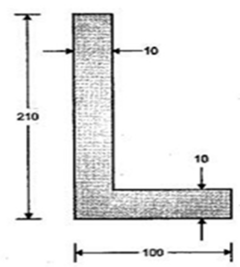


Figure :1 (Que: 2-B)

Answer:
(04 marks)

Step 1: Analyze the Given T-Section
From the image:

- Total height = 210 mm
- Flange width = 100 mm, thickness = 10 mm
- Web width = 10 mm, height = 200 mm (210 - 10)

Step 2: Find Centroid (ȳ) from Bottom
Take reference axis at the bottom of the section.

Part 1 (Web):

- Area, $A_1 = 10 \times 200 = 2000\ mm^2$
- Distance of centroid from bottom, $y_1 = 200/2 = 100$ mm

Part 2 (Flange):

- Area, $A_2 = 100 \times 10 = 1000$ mm²
- Distance of centroid from bottom, $y_2 = 200 + 10/2 = 205$ mm

Total Area:
$A = A_1 + A_2 = 2000 + 1000 = 3000$ mm²

Centroid Calculation:

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A} = \frac{2000 \times 100 + 1000 \times 205}{3000}$$
$$\bar{y} = \frac{200{,}000 + 205{,}000}{3000} = \frac{405{,}000}{3000} = 135 \text{ mm from bottom}$$

Step 3: Calculate I_{xx} about Centroidal Axis
Using parallel axis theorem for each part:

For Web (Part 1):

- Distance from web's centroid to overall centroid: $d_1 = |100 - 135| = 35$ mm
- M.I. of web about its own centroid: $I_{1,centroid} = \frac{10 \times 200^3}{12} = \frac{10 \times 8{,}000{,}000}{12} = 6{,}666{,}666.67 \text{ mm}^4$
- Contribution: $I_1 = I_{1,centroid} + A_1 d_1^2 = 6{,}666{,}666.67 + 2000 \times (35)^2$
  $I_1 = 6{,}666{,}666.67 + 2000 \times 1225 = 6{,}666{,}666.67 + 2{,}450{,}000 = 9{,}116{,}666.67 \text{ mm}^4$

For Flange (Part 2):

- Distance from flange's centroid to overall centroid: $d_2 = |205 - 135| = 70$ mm
- M.I. of flange about its own centroid: $I_{2,centroid} = \frac{100 \times 10^3}{12} = \frac{100 \times 1000}{12} = 8{,}333.33 \text{ mm}^4$
- Contribution: $I_2 = I_{2,centroid} + A_2 d_2^2 = 8{,}333.33 + 1000 \times (70)^2$
  $I_2 = 8{,}333.33 + 1000 \times 4900 = 8{,}333.33 + 4{,}900{,}000 = 4{,}908{,}333.33 \text{ mm}^4$

Total I_{xx}:

$$I_{xx} = I_1 + I_2 = 9{,}116{,}666.67 + 4{,}908{,}333.33 = 14{,}025{,}000 \text{ mm}^4$$

Step 4: Calculate $I_{\gamma\gamma}$ about Centroidal Axis
Since the section is symmetric about vertical axis through center, $I_{\gamma\gamma}$ is simply:

For Web:

$$I_{y1} = \frac{200 \times 10^3}{12} = \frac{200 \times 1000}{12} = 16{,}666.67 \text{ mm}^4$$

For Flange:

$$I_{y2} = \frac{10 \times 100^3}{12} = \frac{10 \times 1{,}000{,}000}{12} = 833{,}333.33 \text{ mm}^4$$

Total $I_{\gamma\gamma}$:

$$I_{yy} = I_{y1} + I_{y2} = 16{,}666.67 + 833{,}333.33 = 850{,}000 \text{ mm}^4$$

Step 5: Final Answer

$$\boxed{I_{xx} = 14.025 \times 10^6 \text{ mm}^4}$$

$$\boxed{I_{yy} = 0.850 \times 10^6 \text{ mm}^4}$$

Note: The centroid is located 135 mm from the bottom of the section, and 75 mm from the top (210 - 135 = 75 mm).