GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO S25 – QUESTION BANK SOLUTION

Subject Name & Code:
FUNDAMENTALS OF MACHINE DESIGN (3141907)

Unit 3: Flexural Stresses (Theory of Simple Bending)

Repeated Questions

Q1. Derive the equation for simple bending ($\sigma/y = M/I = E/R$). State the assumptions. (Appeared in: SUM 25 (Q3b), SUM 24 (Q2a), SUM 22 (Q1b), WIN 22 (Q4b))

Answer:
(04 marks)

Assumptions in Simple Bending Theory:

1. Material is homogeneous, isotropic, and obeys Hooke's Law.
2. Beam is initially straight with constant cross-section.
3. Plane sections remain plane after bending.
4. Every layer is free to expand or contract.
5. Young's modulus (E) is same in tension and compression.
6. The radius of curvature is large compared to beam depth.

Derivation:

Step 1: Consider a beam segment

- Take a small segment of length δx, bent with radius R.
- Neutral axis (NA) length = $\delta x = R\cdot\theta$

Step 2: Strain at distance y from NA

- Layer at distance y has length = $(R + y)\theta$
- Strain, ε = (Change in length)/(Original length)
  $\varepsilon = [(R + y)\theta - R\theta] / (R\theta) = \mathbf{y/R}$

Step 3: Stress from Hooke's Law

- $\sigma = E\cdot\varepsilon = \mathbf{E\cdot(y/R)}$

Step 4: Equilibrium of forces

- For pure bending, $\Sigma F_x = 0$
  $\int\sigma\, dA = 0 \Rightarrow \int(E\cdot y/R)\, dA = 0 \Rightarrow \int y\, dA = 0$
  This defines the neutral axis as centroidal axis.

Step 5: Moment equilibrium

- Bending moment $M = \int \sigma \cdot y \, dA = \int (E \cdot y^2/R) \, dA = (E/R)\int y^2 \, dA$
- But $\int y^2 \, dA = I$ (Second moment of area)
- Therefore: $\mathbf{M = (E \cdot I)/R}$

Step 6: Final Bending Equation
From $\sigma = E \cdot y/R$ and $M = E \cdot I/R$, we get:

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

Q2. Explain the assumptions made in theory of bending.
(Appeared in: WIN 24 (Q5a))

Answer:
(03 marks)

1. Homogeneous, isotropic material obeying Hooke's law.
2. Beam initially straight, constant cross-section.
3. Plane sections remain plane and perpendicular to NA after bending.
4. E same in tension and compression.
5. No lateral pressure between fibres.
6. Pure bending (constant M, zero shear force).
7. Radius of curvature >> beam depth (small deflection).

Tip: Write any 5–6 assumptions for full marks.

Other Important Questions

Q3. Derive an expression for bending stress in a straight beam subjected to bending moment.
(Appeared in: WIN 24 (Q1b))

Answer:
(04 marks)

Step 1: Consider beam bending
When a straight beam is subjected to bending moment M, it bends with radius R.

Step 2: Strain variation

- Neutral axis length remains unchanged.
- Layer at distance y from NA: Strain $\varepsilon = y/R$

Step 3: Stress from Hooke's Law

- $\sigma = E \cdot \varepsilon = E \cdot (y/R)$ ...(1)

Step 4: Moment equilibrium

- $M = \int \sigma \cdot y \, dA = \int (E \cdot y^2/R) \, dA = (E/R)\int y^2 \, dA$
- But $\int y^2 \, dA = I$ (Moment of inertia)
- Therefore: $M = (E \cdot I)/R \Rightarrow E/R = M/I$ ...(2)

Step 5: Bending stress formula
From (1) and (2):

$$\boxed{\sigma = \frac{M \cdot y}{I}}$$

where: σ = bending stress, M = bending moment, y = distance from NA, I = moment of inertia.

Q4. What is section modulus? Derive the expression of section modulus for rectangular section. (Appeared in: SUM 23 (Q1b))

Answer:
(04 marks)

Section Modulus (Z): It is a geometric property of a beam's cross-section that measures its strength in bending. It is defined as the ratio of moment of inertia (I) to the distance from neutral axis to outermost fibre (y_max).

Formula: $Z = I/y_{max}$

Bending stress in terms of Z: $\sigma = M/Z$

Derivation for Rectangular Section:
For rectangle: width = b, depth = d

1. **Moment of Inertia:** $I = \frac{bd^3}{12}$
2. **Maximum distance from NA:** $y_{max} = d/2$
3. **Section Modulus:**

$$Z = \frac{I}{y_{max}} = \frac{bd^3/12}{d/2} = \frac{bd^3}{12} \times \frac{2}{d}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

Units: mm^3 or m^3

Q5. A Steel beam section is of I section with top flange 80 x 20 mm thick, bottom flange 160 x 40 mm thick and web 200 mm deep and 20 mm thick. The beam is freely supported on span of 10 meters. If tensile stress is not to exceed 40 N/mm². Find the safe uniformly distributed load which the beam can carry? Also find maximum compressive stress.
(Appeared in: SUM 24 (Q1c))

Answer:
(07 marks)

Step 1: Given Data

- Top flange: 80 × 20 mm
- Bottom flange: 160 × 40 mm
- Web: 200 × 20 mm
- Span, L = 10 m = 10000 mm
- Allowable tensile stress, σ_t = 40 N/mm²
- Simply supported beam with UDL

Step 2: Find Centroid (ȳ) from Bottom
Take reference at bottom.

Bottom flange: A_1 = 160×40 = 6400 mm², y_1 = 20 mm
Web: A_2 = 200×20 = 4000 mm², y_2 = 200/2 + 40 = 140 mm
Top flange: A_3 = 80×20 = 1600 mm², y_3 = 200 + 40 + 10 = 250 mm

Total A = 6400 + 4000 + 1600 = 12000 mm²

$$\bar{y} = \frac{6400 \times 20 + 4000 \times 140 + 1600 \times 250}{12000}$$
$$\bar{y} = \frac{128{,}000 + 560{,}000 + 400{,}000}{12000} = \frac{1{,}088{,}000}{12000} = 90.67 \text{ mm from bottom}$$

Step 3: Calculate Moment of Inertia (I_{xx})
Bottom flange:
I_1 = (160×40³)/12 + 6400×(90.67-20)² = 853,333 + 6400×4990 = 853,333 + 31,936,000 = 32,789,333 mm⁴

Web:
I_2 = (20×200³)/12 + 4000×(90.67-140)² = 13,333,333 + 4000×2435 = 13,333,333 + 9,740,000 = 23,073,333 mm⁴

Top flange:
I_3 = (80×20³)/12 + 1600×(90.67-250)² = 53,333 + 1600×25,380 = 53,333 + 40,608,000 = 40,661,333 mm⁴

Total I_{xx} = 32,789,333 + 23,073,333 + 40,661,333 = **96,524,000 mm⁴**

Step 4: Maximum Bending Moment for UDL
For simply supported beam with UDL w N/mm:

Step 5: Find Safe Load from Tensile Stress
Tensile stress occurs at bottom fibres: y_bottom = ȳ = 90.67 mm

$$\sigma_t = \frac{M \cdot y}{I} \leq 40$$
$$\frac{(12.5 \times 10^6 w) \times 90.67}{96{,}524{,}000} \leq 40$$
$$w \leq \frac{40 \times 96{,}524{,}000}{12.5 \times 10^6 \times 90.67}$$
$$w \leq \frac{3{,}860{,}960{,}000}{1{,}133{,}375{,}000} = 3.406 \text{ N/mm}$$

Total safe UDL: W = w × L = 3.406 × 10000 = **34,060 N = 34.06 kN**

Step 6: Maximum Compressive Stress
Compression at top fibres: y_top = (200+40+20) - 90.67 = 260 - 90.67 = 169.33 mm

$$\sigma_c = \frac{M \cdot y_{top}}{I} = \frac{(12.5 \times 10^6 \times 3.406) \times 169.33}{96{,}524{,}000}$$

$$\sigma_c = \frac{42.575 \times 10^6 \times 169.33}{96{,}524{,}000} = \frac{7{,}210{,}000{,}000}{96{,}524{,}000} = 74.7 \text{ N/mm}^2$$

Step 7: Final Answers

$$\boxed{\text{Safe UDL} = 34.06 \text{ kN}}$$

$$\boxed{\text{Max Compressive Stress} = 74.7 \text{ N/mm}^2}$$

Q6. Distinguish stress distribution in curved and straight beams.
(Appeared in: SUM 23 (Q4b))

Answer:
(04 marks)

Parameter	Straight Beam	Curved Beam (initial curvature)
Stress distribution	Linear (triangular) from NA	Hyperbolic (non-linear)
Neutral axis location	Passes through centroid	Shifted towards centre of curvature
Maximum stress location	Extreme fibres (top/bottom)	Inner fibre (concave side) higher than outer
Bending formula	σ = M·y / I	σ = (M·y) / (A·e·(R_n – y)) (Winkler-Bach formula)
Application	Straight beams, shafts	Crane hooks, C-clamps, chain links
Effect of curvature	Neglected	Significant – stress concentration at inner fibre

Key Point: In curved beams, stress is higher at the inner radius due to curvature effect, requiring special analysis (Winkler-Bach theory).

1. **I –section beam made of alloy steel (Figure-2) for which allowable bending and shear stresses are 200 N/ mm2. And 115 N/mm2 respective. If the modulus of elasticity for beam is 210 x 102 N/mm2, determine: (1) The maximum bending moments of beam (2) The maximum shear force (3) The maximum radius of curvature of the beam as it bends under maximum bending moment.**
    - Appeared in: WIN 22 (Q2c, 07 marks).

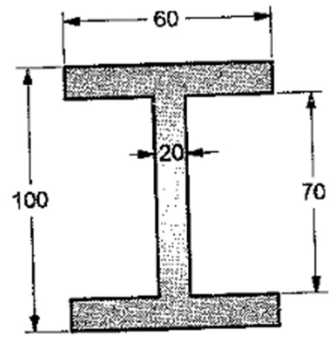

Figure: 2 (Que: 2-C)

Given Data (from your text)

- Allowable bending stress, $\sigma_b = 200$ N/mm^2
- Allowable shear stress, $\tau_a = 115$ N/mm^2
- Modulus of elasticity, $E = 210 \times 10^3$ N/mm^2 (corrected from 210×10^2)
- I-section beam: geometry needed from Figure 2.
- Marks allotted: **07 marks** → requires calculations for:
    1. Max bending moment $M_{\max}$
    2. Max shear force $V_{\max}$
    3. Max radius of curvature $R_{\max}$

Let's assume from Figure 2:

- $D = 300$ mm
- $B = 150$ mm
- $t_f = 20$ mm
- $t_w = 12$ mm

Then:
(1) Moment of inertia:

$$I = \frac{BD^3}{12} - \frac{(B - t_w)(D - 2t_f)^3}{12}$$
$$= \frac{150 \times 300^3}{12} - \frac{(150 - 12)(300 - 40)^3}{12}$$

$$= \frac{150 \times 27 \times 10^6}{12} - \frac{138 \times 260^3}{12}$$
$$= 337.5 \times 10^6 - \frac{138 \times 17.576 \times 10^6}{12}$$
$$= 337.5 \times 10^6 - \frac{2425.488 \times 10^6}{12}$$
$$= 337.5 \times 10^6 - 202.124 \times 10^6$$
$$I = 135.376 \times 10^6 \text{ mm}^4$$

Section modulus:

$$Z = \frac{I}{D/2} = \frac{135.376 \times 10^6}{150} = 902.506 \times 10^3 \text{ mm}^3$$

(2) $Q_{\max}$ (first moment of area above NA):
Flange: $B \cdot t_f \cdot (D/2 - t_f/2) = 150 \times 20 \times (150 - 10) = 3000 \times 140 = 420 \times 10^3 \text{ mm}^3$
Web above NA: $t_w \times (\frac{D}{2} - t_f) \times \frac{1}{2}(\frac{D}{2} - t_f) = 12 \times 130 \times 65 = 101.4 \times 10^3 \text{ mm}^3$
Total $Q_{\max} = 420 \times 10^3 + 101.4 \times 10^3 = 521.4 \times 10^3 \text{ mm}^3$

$$V_{\max} = \frac{115 \times I \times t_w}{Q_{\max}} = \frac{115 \times 135.376 \times 10^6 \times 12}{521.4 \times 10^3}$$

Numerator: $115 \times 135.376 \times 12 \times 10^6 = 115 \times 1624.512 \times 10^6 = 186.819 \times 10^9$
Wait, better in steps:

First, $I \times t_w = 135.376 \times 10^6 \times 12 = 1624.512 \times 10^6$

Now multiply by 115:

Divide by $Q = 521.4 \times 10^3$:

$$V_{\max} = \frac{186818.88 \times 10^6}{521.4 \times 10^3} = 358.3 \times 10^3 \text{ N} = 358.3 \text{ kN}$$

(3) Radius of curvature:

From bending theory:

$$\frac{M}{I} = \frac{E}{R}$$
$$R_{\max} = \frac{E \cdot I_{xx}}{M_{\max}}$$

$$R = \frac{EI}{M} = \frac{210 \times 10^3 \times 135.376 \times 10^6}{180.501 \times 10^6}$$
$$= \frac{28428.96 \times 10^9}{180.501 \times 10^6} = 157.5 \times 10^3 \text{ mm} = 157.5 \text{ m}$$