# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE-4 SEMESTER – OLD PAPER – S22 TO S25 – QUESTION BANK SOLUTION**

**Subject Name & Code:**
**FUNDAMENTALS OF MACHINE DESIGN (3141907)**

## Unit 4: Torsion

**Repeated Questions**

**Q1. Derive the torsion equation (T/J = τ/R = Gθ/L). State assumptions.**
**(Appeared in: SUM 24 (Q3c), SUM 22 (Q3b))**

**Answer:**
**(07 marks)**

**Assumptions in Torsion Theory:**

1. Material is homogeneous, isotropic, and obeys Hooke's Law.
2. Shaft is straight, circular, and of uniform cross-section.
3. Plane cross-sections remain plane and normal to the axis.
4. Radial lines remain radial during twisting.
5. Twisting moment is applied perpendicular to the axis.
6. Stresses are within the elastic limit.
7. Angle of twist is small.

**Derivation:**

**Step 1: Consider a circular shaft**

- Length L, radius R, fixed at one end
- Applied torque T at free end
- Angle of twist θ at free end

**Step 2: Shear strain in elemental ring**

- Consider small element at radius r, thickness dr
- Due to twist, point B moves to B'
- Arc length BB' = r·θ
- Shear strain, γ = BB'/L = (r·θ)/L

**Step 3: Shear stress from Hooke's Law**

- $\tau = G\cdot\gamma = G\cdot(r\cdot\theta/L)$ ...(1)
- This shows $\tau \propto r$ (linear variation)

**Step 4: Torque equilibrium**

- Torque dT on elemental ring = $\tau\cdot(2\pi r\cdot dr)\cdot r$
- $dT = \tau\cdot 2\pi r^2\cdot dr$
- Total torque $T = \int dT = \int_0^R \tau\cdot 2\pi r^2\cdot dr$

**Step 5: Substitute τ from (1)**

- $T = \int_0^R [G\cdot(r\cdot\theta/L)]\cdot 2\pi r^2\cdot dr$
- $T = (2\pi G\theta/L) \int_0^R r^3\cdot dr$
- $T = (2\pi G\theta/L)\cdot[r^4/4]_0^R$
- $T = (2\pi G\theta/L)\cdot(R^4/4)$

**Step 6: Polar moment of inertia**

- For solid circular shaft: $J = \pi R^4/2$
- Therefore: $T = (G\theta/L)\cdot J$

**Step 7: Final Torsion Equation**
From $\tau = G\cdot r\cdot\theta/L$ and $T = (G\theta/L)\cdot J$:

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

---

**Other Important Questions**

**Q2. Define torsional rigidity. Derive an expression for angular deflection of a shaft subjected to twisting moment.**
**(Appeared in: WIN 24 (Q5b))**

**Answer:**
**(04 marks)**

**Torsional Rigidity:** It is the torque required to produce a unit angle of twist in a shaft of unit length. It represents the resistance of a shaft to torsional deformation.

**Formula:** Torsional Rigidity = $G\cdot J$
where G = Shear modulus, J = Polar moment of inertia.

**Derivation of Angular Deflection:**

From torsion equation:

$$\frac{T}{J} = \frac{G\theta}{L}$$

Rearranging for angular deflection θ:

$$\theta = \frac{T \cdot L}{G \cdot J}$$

**Where:**

- $\theta$ = Angle of twist in radians
- T = Applied torque (N·mm)
- L = Length of shaft (mm)
- G = Shear modulus ($N/mm^2$)
- J = Polar moment of inertia ($mm^4$)

**For solid circular shaft:** $J = \pi d^4/32$
**For hollow shaft:** $J = \pi(d_o^4 - d_i^4)/32$

**Units:** $\theta$ in radians (1 rad = $180/\pi$ degrees)

---

**Q3. Derive and explain the expression of polar modulus and polar moment of inertia of a solid shaft.**
**(Appeared in: SUM 23 (Q2b))**

**Answer:**
**(04 marks)**

**1. Polar Moment of Inertia (J):**

- It is the resistance of a shaft to torsional deformation.
- For solid circular shaft of diameter d:

$$J = \frac{\pi d^4}{32}$$

- **Derivation:** Consider elemental area dA at radius r.
  $dA = 2\pi r \cdot dr$
  $J = \int r^2 \cdot dA = \int_0^{d/2} r^2 \cdot (2\pi r \cdot dr) = 2\pi \int_0^{d/2} r^3 \cdot dr = 2\pi \cdot [r^4/4]_0^{d/2} = \pi d^4/32$

**2. Polar Modulus ($Z_p$):**

- It is the ratio of polar moment of inertia to the outer radius.
- Also called torsional section modulus.

$$Z_p = \frac{J}{R} = \frac{J}{d/2} = \frac{\pi d^4/32}{d/2} = \frac{\pi d^3}{16}$$

- Used in torsion formula: $\tau\_max = T/Z_p$

**Significance:**

- J measures resistance to angle of twist
- $Z_p$ measures resistance to shear stress
- Both increase with diameter ($\propto d^4$ and $d^3$ respectively)

---

**Q4. Discuss torsion in solid shaft and hollow shaft.**
**(Appeared in: SUM 25 (Q4a))**

**Answer:**
**(03 marks)**

| Aspect | Solid Shaft | Hollow Shaft |
|---|---|---|
| **Cross-section** | Complete circular area | Annular area |
| **Material Usage** | More material | Less material |
| **Weight** | Heavier | Lighter |
| **Strength/Weight Ratio** | Lower | Higher |
| **Polar Moment (J)** | $J = \pi d^4/32$ | $J = \pi(d_o^4 - d_i^4)/32$ |
| **Stress Distribution** | Zero at center, max at surface | More uniform |
| **Applications** | General purpose, low cost | Where weight saving is critical (aircraft, automobiles) |
| **Manufacturing Cost** | Lower | Higher |

**Key Point:** For the same outer diameter and material, a hollow shaft has higher torsional strength and stiffness per unit weight compared to a solid shaft.

---

**Q5. A steel wire of diameter d = 4 mm is bent around a cylindrical drum of radius $R_0$ = 0.5 m, E = 200 GPa, σ_p = 1200 MPa. Determine M and σ_max.**
**(Appeared in: SUM 23 (Q3c))**

**Answer:**
**(07 marks)**

**Step 1: Given Data**

- Wire diameter, d = 4 mm = 0.004 m
- Drum radius, $R_0$ = 0.5 m
- Young's modulus, E = 200 GPa = $200 \times 10^9$ Pa
- Proportional limit, σ_p = 1200 MPa = $1200 \times 10^6$ Pa

**Step 2: Radius of curvature after bending**
When wire is bent around drum, neutral axis is at radius:

$$R = R_0 + d/2 = 0.5 + 0.002 = 0.502 \text{ m}$$

**Step 3: Maximum bending stress formula**

$$\sigma_{max} = \frac{E \cdot y}{R}$$

where y = distance from NA to outermost fibre = d/2 = 0.002 m

$$\sigma_{max} = \frac{200 \times 10^9 \times 0.002}{0.502}$$
$$\sigma_{max} = \frac{400 \times 10^6}{0.502} = 796.8 \times 10^6 \text{ Pa} = 796.8 \text{ MPa}$$

**Step 4: Check against proportional limit**
σ_max = 796.8 MPa < σ_p = 1200 MPa
∴ Wire remains elastic.

**Step 5: Calculate bending moment (M)**
For circular cross-section:

- Moment of inertia, $I = \pi d^4/64 = \pi(0.004)^4/64 = \pi(256\times10^{-12})/64 = 12.57\times10^{-12}$ $m^4$
- Section modulus, $Z = \pi d^3/32 = \pi(0.004)^3/32 = \pi(64\times10^{-9})/32 = 6.28\times10^{-9}$ $m^3$

Using σ = M/Z:

$$M = \sigma_{max} \times Z = 796.8 \times 10^6 \times 6.28 \times 10^{-9}$$

**Step 6: Final Answers**

$$\boxed{\sigma_{max} = 796.8\ \text{MPa}}$$