

GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO S25 – QUESTION BANK SOLUTION

Subject Name & Code:

FUNDAMENTALS OF MACHINE DESIGN (3141907)

Unit 7: Shafts and Keys

Repeated Questions

Q1. Design a shaft subjected to torque/bending/combined loading.

(Appeared in: SUM 24 (Q2c), WIN 23 (Q3c), SUM 22 (Q3c), WIN 24 (Q2c-OR))

Answer:

(07 marks - General Procedure)

Step-by-Step Shaft Design Procedure:

1. Determine Forces and Moments:

- Calculate all forces from pulleys, gears, etc.
- Find bearing reactions using equilibrium equations.
- Draw shear force and bending moment diagrams.

2. Calculate Torque:

- For power transmission: $T = \frac{60 \times P}{2\pi N}$ N·mm
where P in watts, N in rpm

3. Select Material and Allowable Stress:

- Choose material (usually mild steel, alloy steel)
- Determine allowable shear stress (τ_{max}) based on yield strength and FOS
- ASME code: $\tau_{max} = 0.3 \times S_{yt}$ or $0.18 \times S_{ut}$, whichever is smaller

4. Consider Shock and Fatigue Factors:

- ASME code for combined loading:

$$d^3 = \frac{16}{\pi \tau_{max}} \sqrt{(K_b M)^2 + (K_t T)^2}$$

where K_b = combined shock & fatigue factor for bending

K_t = combined shock & fatigue factor for torsion

5. Calculate Shaft Diameter:

- **For pure torsion:** $d = \sqrt[3]{\frac{16}{\pi\tau}}$
- **For pure bending:** $d = \sqrt[3]{\frac{32}{\pi\sigma}}$
- **For combined loading (ASME):** Use formula in step 4

6. Check for Deflection and Rigidity:

- Bending deflection: $\delta = \frac{PL^3}{48EI}$ for simply supported
- Torsional deflection: $\theta = \frac{TL}{GJ}$
- Limits: Bending deflection $\leq 0.001 \times \text{span}$, Torsional deflection $\leq 0.25^\circ/\text{m}$

7. Standardize Diameter:

- Round up to nearest standard size (R10/R20 series)

Example from SUM 24 (Q2c):

*A shaft transmitting 100 KW at 300 rpm, length 3 m, two pulleys each 1500 N at 1 m from ends, $\tau = 0.5S_{yt}$, FOS=2, $S_{yt}=240 \text{ N/mm}^2$, check rigidity $0.25^\circ/\text{m}$, $G=84 \times 10^3 \text{ N/mm}^2$, $K_b=1.5$, $K_t=1.0$ *

Solution:

1. **Torque:**
2. **Allowable τ :** $\tau = \frac{0.5 \times 240}{2} = 60 \text{ N/mm}^2$
3. **Bending Moment:** Symmetrical loading $\rightarrow$ max BM at center $= 1500 \times 0.5 = 750 \text{ N}\cdot\text{m} = 0.75 \times 10^6 \text{ N}\cdot\text{mm}$
4. **ASME formula:**

$$d^3 = \frac{16}{\pi \times 60} \sqrt{(1.5 \times 0.75 \times 10^6)^2 + (1.0 \times 3.183 \times 10^6)^2}$$

$$d^3 = 0.0849 \times \sqrt{(1.125 \times 10^6)^2 + (3.183 \times 10^6)^2}$$

$$d^3 = 0.0849 \times \sqrt{1.265 \times 10^{12} + 1.013 \times 10^{13}}$$

$$d^3 = 0.0849 \times \sqrt{1.1395 \times 10^{13}} = 0.0849 \times 3.375 \times 10^6$$

$$d^3 = 286,500 \Rightarrow d = 65.9 \text{ mm}$$

Take **d = 66 mm**

5. **Check torsional rigidity:**

$$J = \frac{\pi d^4}{32} = \frac{\pi (66)^4}{32} = 1.86 \times 10^6 \text{ mm}^4$$

$$\theta = \frac{TL}{GJ} = \frac{3.183 \times 10^6 \times 3000}{84 \times 10^3 \times 1.86 \times 10^6}$$

$$\theta = 0.061 \text{ radians} = 3.5^\circ \text{ total}$$

Per meter: $3.5/3 = 1.17^\circ/\text{m} > 0.25^\circ/\text{m} \rightarrow$ **Not rigid enough**

6. Increase diameter for rigidity:

Need $\theta \leq 0.25^\circ/\text{m} = 0.00436 \text{ rad/m}$

$$J = \frac{TL}{G\theta} = \frac{3.183 \times 10^6 \times 1000}{84 \times 10^3 \times 0.00436} = 8.69 \times 10^6 \text{ mm}^4$$

$$d = \sqrt[4]{\frac{32J}{\pi}} = \sqrt[4]{\frac{32 \times 8.69 \times 10^6}{\pi}} = 97.2 \text{ mm}$$

Take **d = 100 mm** for rigidity.

Final: Strength requires 66 mm, rigidity requires 100 mm $\rightarrow$ **Use d = 100 mm**

Q2. Compare/Discuss Hollow shaft and Solid shaft. / How the hollow shafts are beneficial over the solid shaft?

(Appeared in: SUM 25 (Q5a), WIN 22 (Q3a))

Answer:

(03 marks)

Aspect	Solid Shaft	Hollow Shaft
Material Usage	More material	Less material
Weight	Heavier	Lighter
Strength-to-Weight Ratio	Lower	Higher
Cost (Material)	Lower	Higher (but less material)
Manufacturing Complexity	Simpler	More complex
Polar Moment (J)	$J = \frac{\pi d^4}{32}$	$J = \frac{\pi(d_o^4 - d_i^4)}{32}$

Aspect	Solid Shaft	Hollow Shaft
Torsional Strength	Lower for same weight	Higher for same weight
Stiffness (GJ)	Lower for same weight	Higher for same weight
Natural Frequency	Lower	Higher
Applications	General purpose, low speed	Aerospace, automotive, high speed

Benefits of Hollow Shafts:

1. **Higher strength-to-weight ratio**
2. **Higher stiffness-to-weight ratio**
3. **Better for high-speed applications** (higher natural frequency)
4. **Can pass utilities through center** (wires, fluids)
5. **Less material cost** for same strength

Other Important Questions

Q3. Derive the design equation for shaft subjected to twisting moment only.
(Appeared in: WIN 22 (Q3b))

Answer:
(04 marks)

Step 1: Torsion Formula

From torsion equation:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

where T = torque, J = polar moment of inertia, τ = shear stress, r = radius, G = shear modulus, θ = angle of twist, L = length.

Step 2: Maximum Shear Stress

At outer surface, $r = d/2$:

$$\tau_{max} = \frac{T \times (d/2)}{J}$$

Step 3: Polar Moment for Solid Circular Shaft

$$J = \frac{\pi d^4}{32}$$

Step 4: Substitute and Rearrange

$$\tau_{max} = \frac{T \times (d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3}$$

Step 5: Design Equation

For design, $\tau_{max} \leq$ allowable shear stress (τ_{allow}):

$$\tau_{allow} = \frac{16T}{\pi d^3}$$

$$d^3 = \frac{16T}{\pi \tau_{allow}}$$

$$d = \sqrt[3]{\frac{16T}{\pi \tau_{allow}}}$$

For hollow shaft with outer diameter d_o and inner diameter d_i :

$$d_o^3(1 - k^4) = \frac{16T}{\pi \tau_{allow}}, \text{ where } k = \frac{d_i}{d_o}$$

Q4. Derive the expression of design of shaft subjected to combined twisting moment and bending moment with usual notations.

(Appeared in: SUM 23 (Q3c))

Answer:

(07 marks)

Step 1: Combined Loading

A shaft subjected to bending moment M and twisting moment T experiences:

- **Bending stress:** $\sigma_b = \frac{32M}{\pi d^3}$
- **Shear stress:** $\tau = \frac{16T}{\pi d^3}$

Step 2: Principal Stresses

For biaxial stress state ($\sigma_x = \sigma_b$, $\tau_{xy} = \tau$):

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

Maximum shear stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

Step 3: Apply Failure Theories

A. Maximum Shear Stress Theory (Tresca):

$$\tau_{max} = \frac{S_{yt}}{2 \times FOS} = \tau_{allow}$$

$$\sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = \tau_{allow}$$

Substitute σ_b and τ :

$$\sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2} = \tau_{allow}$$

$$\frac{16}{\pi d^3} \sqrt{M^2 + T^2} = \frac{S_{yt}}{2 \times FOS}$$

$$d^3 = \frac{32 \times FOS}{\pi S_{yt}} \sqrt{M^2 + T^2}$$

B. Maximum Distortion Energy Theory (von Mises):

$$\sigma_{eq} = \sqrt{\sigma_b^2 + 3\tau^2} = \frac{S_{yt}}{FOS}$$

Substitute:

$$\sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + 3\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \sqrt{M^2 + 3T^2} = \frac{S_{yt}}{FOS}$$

$$d^3 = \frac{16 \times FOS}{\pi S_{yt}} \sqrt{M^2 + 3T^2}$$

Step 4: ASME Code Formula (with shock factors)

$$d^3 = \frac{16}{\pi \tau_{allow}} \sqrt{(K_b M)^2 + (K_t T)^2}$$

where K_b = combined shock & fatigue factor for bending (1.5-2.0)

K_t = combined shock & fatigue factor for torsion (1.0-1.5)

Step 5: Most Common Design Equation

$$d^3 = \frac{16}{\pi \tau_{allow}} \sqrt{M^2 + T^2} \text{ (simplified)}$$

or

$$d^3 = \frac{16}{\pi \tau_{allow}} \sqrt{(K_b M)^2 + (K_t T)^2} \text{ (ASME with factors)}$$

Q5. Determine the diameter of a solid shaft used for transmitting 30 kW power at 230 r.p.m. Determine the diameter of hollow shaft, if the solid shaft is to be replaced by the hollow shaft. Take the ratio of inside to outside diameter is 6:8 and allowable shear stress of shaft as 50 MPa.

(Appeared in: WIN 22 (Q3c))

Answer:

(07 marks)

Part A: Solid Shaft

Step 1: Given Data

- Power, $P = 30 \text{ kW} = 30,000 \text{ W}$
- Speed, $N = 230 \text{ rpm}$
- Allowable shear stress, $\tau = 50 \text{ MPa} = 50 \text{ N/mm}^2$

Step 2: Calculate Torque

Step 3: Diameter of Solid Shaft

$$\begin{aligned} d^3 &= \frac{16T}{\pi \tau} = \frac{16 \times 1.2456 \times 10^6}{\pi \times 50} \\ d^3 &= \frac{19.93 \times 10^6}{157.08} = 126,900 \\ d &= \sqrt[3]{126,900} = 50.25 \text{ mm} \end{aligned}$$

Take $d_{\text{solid}} = 52 \text{ mm}$

Part B: Hollow Shaft

Step 4: Given Ratio

$$\frac{d_i}{d_o} = \frac{6}{8} = 0.75 = k$$

Step 5: Torque Capacity Formula

For hollow shaft:

$$T = \frac{\pi \tau}{16} d_o^3 (1 - k^4)$$

Step 6: Calculate Outer Diameter

$$\begin{aligned} 1.2456 \times 10^6 &= \frac{\pi \times 50}{16} d_o^3 (1 - 0.75^4) \\ 1.2456 \times 10^6 &= 9.8175 d_o^3 (1 - 0.3164) \\ 1.2456 \times 10^6 &= 9.8175 d_o^3 \times 0.6836 \\ 1.2456 \times 10^6 &= 6.712 d_o^3 \\ d_o^3 &= \frac{1.2456 \times 10^6}{6.712} = 185,600 \\ d_o &= \sqrt[3]{185,600} = 57.04 \text{ mm} \end{aligned}$$

Take $d_o = 58 \text{ mm}$

Step 7: Inner Diameter

$$d_i = 0.75 \times d_o = 0.75 \times 58 = 43.5 \text{ mm}$$

Take $d_i = 44 \text{ mm}$

Step 8: Comparison

- Solid shaft: 52 mm diameter
- Hollow shaft: 58 mm OD, 44 mm ID
- Material saving = Area ratio:

$$\text{Solid area} = \frac{\pi}{4} \times 52^2 = 2124 \text{ mm}^2$$

$$\text{Hollow area} = \frac{\pi}{4} (58^2 - 44^2) = 0.7854 \times (3364 - 1936) = 1120 \text{ mm}^2$$

$$\text{Material saving} = \frac{2124 - 1120}{2124} \times 100 = 47.3\%$$

Final Answers:

$$d_{\text{solid}} = 52 \text{ mm}$$

$$d_{\text{hollo}} = 58 \text{ mm OD, 44 mm ID (47\% material saving)}$$

Q6. 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of

safety of 2.

(Appeared in: SUM 25 (Q5c))

Answer:

(07 marks)

Step 1: Given Data

- Shaft diameter, $d = 45 \text{ mm}$
- Shaft yield strength, $S_{yt}(\text{shaft}) = 400 \text{ MPa}$
- Key dimensions: width $b = 14 \text{ mm}$, thickness $t = 9 \text{ mm}$
- Key yield strength, $S_{yt}(\text{key}) = 340 \text{ MPa}$
- Factor of safety, $FOS = 2$
- Theory: Maximum shear stress (Tresca)

Step 2: Allowable Stresses

For shaft:

$$\tau_{allow(\text{shaft})} = \frac{S_{yt(\text{shaft})}}{2 \times FOS} = \frac{400}{2 \times 2} = 100 \text{ MPa}$$

For key:

$$\tau_{allow(\text{key})} = \frac{S_{yt(\text{key})}}{2 \times FOS} = \frac{340}{2 \times 2} = 85 \text{ MPa}$$

$$\sigma_{c(allow)} = \frac{S_{yt(\text{key})}}{FOS} = \frac{340}{2} = 170 \text{ MPa}$$

Step 3: Maximum Permissible Torque from Shaft

$$T_{max} = \frac{\pi d^3 \tau_{allow(\text{shaft})}}{16} = \frac{\pi \times 45^3 \times 100}{16}$$

Step 4: Key Length from Shear Consideration

For key in shear:

$$T = \tau_{allow(\text{key})} \times b \times L \times \frac{d}{2}$$

$$1,790,438 = 85 \times 14 \times L \times \frac{45}{2}$$

$$1,790,438 = 85 \times 14 \times L \times 22.5$$

$$1,790,438 = 26,775 \times L$$

$$L_{shear} = \frac{1,790,438}{26,775} = 66.86 \text{ mm}$$

Step 5: Key Length from Crushing Consideration

For key in crushing (assuming one side contact):

$$\begin{aligned}
 T &= \sigma_{c(allow)} \times \frac{t}{2} \times L \times \frac{d}{2} \\
 1,790,438 &= 170 \times \frac{9}{2} \times L \times \frac{45}{2} \\
 1,790,438 &= 170 \times 4.5 \times L \times 22.5 \\
 1,790,438 &= 170 \times 101.25 \times L \\
 1,790,438 &= 17,212.5 \times L \\
 L_{crush} &= \frac{1,790,438}{17,212.5} = 104.0 \text{ mm}
 \end{aligned}$$

Step 6: Select Key Length

Take the larger value for safety: **L = 104 mm**

Check Practicality:

- Hub length for gear/sprocket typically $1.5d = 67.5 \text{ mm}$
- $104 \text{ mm} > 67.5 \text{ mm} \rightarrow$ Key would protrude
- Solution: Use two keys staggered, or increase key width/thickness

Alternative: Redesign key for $L \approx 68 \text{ mm}$

From shear equation with $L = 68 \text{ mm}$:

From crushing with $L = 68 \text{ mm}$:

Increase key thickness to match:

From crushing equation:

$$\begin{aligned}
 1,790,438 &= 170 \times \frac{t}{2} \times 68 \times 22.5 \\
 1,790,438 &= 170 \times t \times 34 \times 22.5 \\
 1,790,438 &= 130,050 \times t \\
 t &= \frac{1,790,438}{130,050} = 13.77 \text{ mm}
 \end{aligned}$$

Use standard: **t = 14 mm**

Step 7: Final Answer

For given key (14×9), required length = **104 mm**

But practical design: Use **key 14 mm × 14 mm × 68 mm**

$L = 104 \text{ mm}$ for 14×9 key, or use 14×14×68 mm key for practical fit

Q7. What is spline? Explain different types of splines.

(Appeared in: SUM 23 (Q3a))

Answer:

(03 marks)

Types of splines:

Type	Description	Application
Straight (parallel) spline	Rectangular teeth parallel to shaft axis	Low-cost, general purpose (e.g., gearboxes)
Involute spline	Teeth with involute profile (similar to gear teeth)	High torque, self-centering (automotive transmissions)
Serration (triangular spline)	Small triangular teeth, fine pitch	Light duty, thin sections
Helical spline	Teeth at an angle – allows axial movement with rotation	Constant velocity joints

Advantages over keys: Even stress distribution, higher torque capacity, permits axial sliding, better centring.

Q8. Differentiate between shaft, spindle and axle.

(Appeared in: SUM 22 (Q3a), WIN 23 (Q3a), WIN 24 (Q3a))

Answer:

(03 marks)

Term	Definition	Loads carried	Rotation	Example
Shaft	Rotating member that transmits power	Bending + Torsion	Rotates	Transmission shaft, motor shaft
Spindle	Short shaft, often with precise bearing supports	Mostly bending (less torque)	Rotates	Machine tool spindle, grinding wheel spindle

Term	Definition	Loads carried	Rotation	Example
Axle	Stationary or rotating member that supports wheels	Only bending (no torque)	May rotate or be fixed	Railway axle, front wheel axle of car (non-driving)

Key points:

- Shaft → torque + bending.
- Axle → only bending (no torque transmission).
- Spindle → precision, high stiffness, often hollow.

Real-world example: Rear axle of a car (if driving wheel – it's a shaft; if non-driving – it's an axle).

Q9. A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously; it is subjected to an axial thrust of 10 kN, and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.

(Appeared in: SUM 25 (Q4c, 07 marks))

Answer:

(07 marks)

Step 1: Given Data

- Outer diameter, $d_o = 40 \text{ mm} = 0.04 \text{ m}$
- Inner diameter, $d_i = 25 \text{ mm} = 0.025 \text{ m}$
- Twisting moment (Torque),
- Axial thrust, $P = 10 \text{ kN} = 10,000 \text{ N}$ (compressive)
- Bending moment,

Step 2: Calculate Section Properties

Cross-sectional area:

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (40^2 - 25^2) = \frac{\pi}{4} (1600 - 625)$$

$$A = \frac{\pi}{4} \times 975 = 765.77 \text{ mm}^2$$

Moment of inertia (for bending):

$$\begin{aligned}
 I &= \frac{\pi}{64} (d_o^4 - d_i^4) = \frac{\pi}{64} (40^4 - 25^4) \\
 40^4 &= 2,560,000, 25^4 = 390,625 \\
 d_o^4 - d_i^4 &= 2,560,000 - 390,625 = 2,169,375 \text{ mm}^4 \\
 I &= \frac{\pi}{64} \times 2,169,375 = 0.04909 \times 2,169,375 = 106,500 \text{ mm}^4
 \end{aligned}$$

Polar moment of inertia (for torsion):

$$\begin{aligned}
 J &= \frac{\pi}{32} (d_o^4 - d_i^4) = \frac{\pi}{32} \times 2,169,375 = 0.098175 \times 2,169,375 \\
 J &= 213,000 \text{ mm}^4 \text{ (Note: } J = 2I \text{ for circular sections)}
 \end{aligned}$$

Section modulus for bending:

$$Z = \frac{I}{y_{max}} = \frac{I}{d_o/2} = \frac{106,500}{20} = 5,325 \text{ mm}^3$$

Step 3: Calculate Individual Stresses

A. Direct compressive stress (from axial thrust):

$$\sigma_d = \frac{P}{A} = \frac{10,000}{765.77} = 13.06 \text{ N/mm}^2 \text{ (compressive)}$$

B. Bending stress (maximum at outer fiber):

$$\sigma_b = \frac{M}{Z} = \frac{80,000}{5,325} = 15.02 \text{ N/mm}^2$$

- On top fiber: compressive (adds to axial compression)
- On bottom fiber: tensile (opposes axial compression)

C. Torsional shear stress (maximum at outer surface):

$$\begin{aligned}
 \tau &= \frac{T \cdot r}{J} = \frac{T \cdot (d_o/2)}{J} = \frac{120,000 \times 20}{213,000} \\
 \tau &= \frac{2,400,000}{213,000} = 11.27 \text{ N/mm}^2
 \end{aligned}$$

Step 4: Combined Stresses at Critical Points

Point A (Top fiber - maximum compression):

- Direct stress: $\sigma_d = -13.06 \text{ N/mm}^2$ (compressive)
- Bending stress: $\sigma_b = -15.02 \text{ N/mm}^2$ (compressive, at top)

- Total normal stress: $\sigma_x = -13.06 - 15.02 = -28.08 \text{ N/mm}^2$
- Shear stress: $\tau_{xy} = 11.27 \text{ N/mm}^2$ (same everywhere on surface)

Point B (Bottom fiber):

- Direct stress: $\sigma_d = -13.06 \text{ N/mm}^2$
- Bending stress: $\sigma_b = +15.02 \text{ N/mm}^2$ (tensile, at bottom)
- Total normal stress: $\sigma_x = -13.06 + 15.02 = +1.96 \text{ N/mm}^2$ (tensile)
- Shear stress: $\tau_{xy} = 11.27 \text{ N/mm}^2$

Step 5: Maximum Compressive Stress

Clearly, Point A has maximum compression:

$$\sigma_{c(max)} = 28.08 \text{ N/mm}^2 \text{ compressive}$$

Step 6: Maximum Shear Stress

Using Mohr's circle or maximum shear stress formula:

For Point A: $\sigma_x = -28.08$, $\sigma_y = 0$, $\tau_{xy} = 11.27$

Maximum shear stress:

$$\begin{aligned}\tau_{max} &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \tau_{max} &= \sqrt{\left(\frac{-28.08 - 0}{2}\right)^2 + (11.27)^2} \\ \tau_{max} &= \sqrt{(-14.04)^2 + (11.27)^2} \\ \tau_{max} &= \sqrt{197.12 + 127.01} = \sqrt{324.13} \\ \tau_{max} &= 18.00 \text{ N/mm}^2\end{aligned}$$

For Point B: $\sigma_x = 1.96$, $\sigma_y = 0$, $\tau_{xy} = 11.27$

$$\begin{aligned}\tau_{max} &= \sqrt{\left(\frac{1.96 - 0}{2}\right)^2 + (11.27)^2} = \sqrt{(0.98)^2 + 127.01} \\ \tau_{max} &= \sqrt{0.96 + 127.01} = \sqrt{127.97} = 11.31 \text{ N/mm}^2\end{aligned}$$

The maximum shear stress occurs at Point A:

$$\tau_{max} = 18.00 \text{ N/mm}^2$$

Step 7: Principal Stresses at Point A (for reference)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-28.08 + 0}{2} \pm 18.00 = -14.04 \pm 18.00$$

$$\sigma_1 = -14.04 + 18.00 = 3.96 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_2 = -14.04 - 18.00 = -32.04 \text{ N/mm}^2 \text{ (compressive)}$$

Step 8: Summary of Results

1. **Maximum compressive stress:** $\sigma_{c(max)} = 28.08 \text{ N/mm}^2$ (at top fiber)
2. **Maximum shear stress:** $\tau_{max} = 18.00 \text{ N/mm}^2$ (at top fiber)
3. **Maximum principal compressive stress:** $\sigma_2 = 32.04 \text{ N/mm}^2$
4. **Maximum principal tensile stress:** $\sigma_1 = 3.96 \text{ N/mm}^2$

Note: The maximum compressive principal stress (32.04 N/mm^2) is slightly higher than the maximum compressive normal stress (28.08 N/mm^2) because it considers the combined effect of normal and shear stresses through Mohr's circle transformation.

Maximum compressive stress = 28.08 N/mm^2

Maximum shear stress = 18.00 N/mm^2
