GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO S25 – QUESTION BANK SOLUTION

Subject Name & Code:

FUNDAMENTALS OF MACHINE DESIGN (3141907)

Unit 9: Design Against Fluctuating Loads

Repeated Questions

Q1. What is endurance limit/strength? Discuss the factors affecting it.
(Appeared in: SUM 25 (Q5b), WIN 23 (Q4a-i), WIN 22 (Q5c), SUM 24 (Q5b-OR))

Answer:
(04 marks)

Endurance limit (σ_e) / Endurance strength

- **Definition:** The maximum value of completely reversed stress (alternating stress) that a material can withstand for an infinite number of cycles without fatigue failure.
- For **ferrous materials** (steel, cast iron): Endurance limit is typically 0.4 to 0.6 times ultimate tensile strength (σ_{ult}).
- For **non-ferrous materials** (aluminium, copper): No true endurance limit; fatigue strength is specified at a given number of cycles (e.g., 10^8 cycles).

Factors affecting endurance limit (Corrected endurance limit formula)

$\sigma_e' = \sigma_e \times K_a \times K_b \times K_c \times K_d \times K_e \times K_f$
where σ_e = endurance limit of a standard polished rotating-beam specimen.

Factor	Symbol	Description	Typical values
Surface finish factor	K_a	Depends on surface condition (ground, machined, as-forged)	0.9 (ground) to 0.5 (forged)
Size factor	K_b	Larger diameter reduces endurance limit (stress gradient effect)	0.85 (d > 50 mm) to 1.0 (d ≤ 8 mm)
Load factor	K_c	Different loading modes: bending, axial, torsion	Bending = 1.0, Axial = 0.7–0.85, Torsion = 0.58

Factor	Symbol	Description	Typical values
Temperature factor	K_d	Elevated temperature reduces strength	0.9 at 300°C, lower at higher T
Reliability factor	K_e	Accounts for scatter in fatigue data	1.0 (50% reliability), 0.897 (90%), 0.814 (99%)
Miscellaneous effects	K_f	Corrosion, residual stresses, fretting	Depends on condition

Real-world application: Design of shafts, connecting rods, springs, and any component subjected to cyclic loading uses the corrected endurance limit to compute fatigue life.

Q2. Explain Soderberg, Goodman, and Gerber theories/lines/diagrams.
(Appeared in: SUM 22 (Q5a), WIN 24 (Q5c), WIN 23 (Q4c & Q5b))

Answer:
(03-07 marks)

These are **failure criteria for fatigue under fluctuating stresses** with non-zero mean stress.

1. Soderberg Line (Conservative):

- Connects σ_e (endurance limit) on amplitude axis to S_yt (yield strength) on mean stress axis
- **Equation:** $\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{FS}$
- **Assumption:** Yielding at yield strength
- **Most conservative** of the three
- **Used for:** Ductile materials where yielding is failure criterion

2. Goodman Line (Modified Goodman):

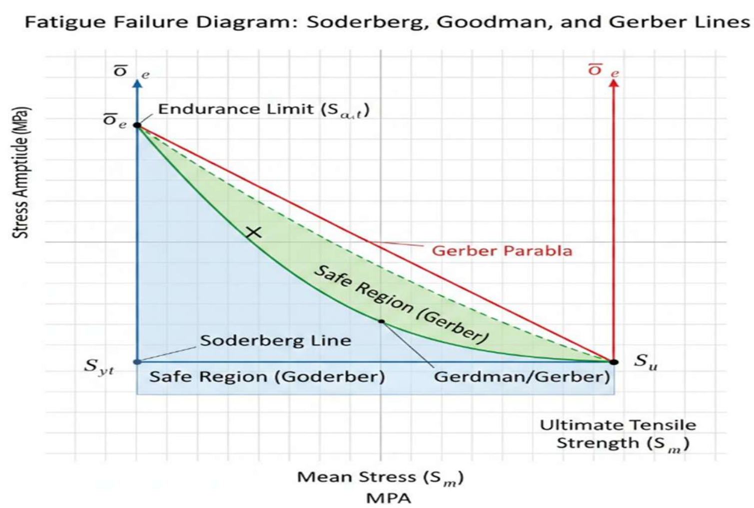
- Connects σ_e on amplitude axis to S_u (ultimate strength) on mean stress axis
- **Equation:** $\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_u} = \frac{1}{FS}$
- **Assumption:** Fracture at ultimate strength
- **Moderately conservative**
- **Most commonly used** in machine design

3. Gerber Parabola:

- Parabolic curve connecting σ_e to S_u
- **Equation:** $\frac{\sigma_a}{\sigma_e} + (\frac{\sigma_m}{S_u})^2 = \frac{1}{FS}$
- **Less conservative** than Goodman
- **Used for:** Reliable data available, critical applications

Comparison:

- **Most to least conservative:** Soderberg → Goodman → Gerber
- **For zero mean stress (σ_m=0):** All give same factor of safety
- **For σ_m > 0:** Gerber gives highest FS, Soderberg lowest



Q3. What is stress concentration? State reasons and methods to reduce it. (Appeared in: SUM 25 (Q4b), WIN 24 (Q5c), SUM 24 (Q5a), SUM 23 (Q5b), WIN 22 (Q5b))

Answer:
(04-07 marks)

Stress concentration

- **Definition:** Localised increase in stress in a material due to sudden changes in cross-section, discontinuities, or defects.
- **Stress concentration factor (K_t):** K_t = σ_max / σ_nominal

Reasons (causes) of stress concentration:

1. **Sharp corners or notches** – High stress at the re-entrant corner.
2. **Holes** – Stress rises around the edge of a hole (K_t ≈ 3 for a small hole in a wide plate).
3. **Keyways and splines** – Sharp edges create stress raisers.

4. **Threads** – Root of threads has high K_t (2–3).
5. **Oil holes and cross-holes** – Intersections cause high local stress.
6. **Rapid change in cross-section** – Abrupt diameter change (shoulder).
7. **Cracks, scratches, or surface defects** – Act as notch.

Methods to reduce stress concentration:

Method	Explanation	Sketch concept
Use fillets (rounded corners)	Increase radius at internal corners	Radius $R \geq 0.25 \times$ smaller diameter
Undercutting	Provide a relief groove near shoulder	Common on shafts near bearing seats
Increase radius gradually	Use large transition radius	Elliptical or parabolic fillets
Add stress relief holes	Drill small holes at ends of cracks/slots	Used in aircraft fuselage
Remove unnecessary material	Use multiple small holes instead of one large	For weight reduction while reducing K_t
Improve surface finish	Polishing, grinding to remove scratches	Reduces surface stress raisers
Use cold working	Shot peening creates compressive residual stress	Offsets tensile stress concentration

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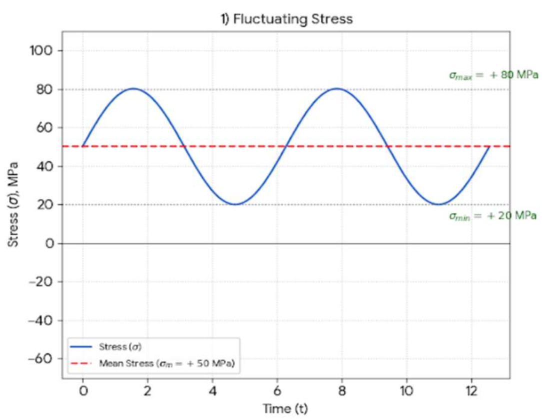
Other Important Questions

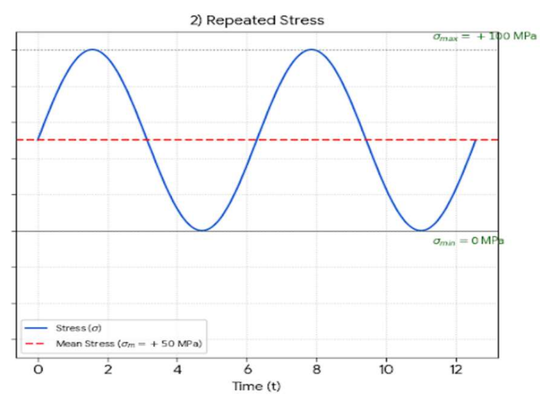
Q4. Explain with figure a. Fluctuating stresses b. Repeated stresses c. Reversed stresses (Appeared in: WIN 22 (Q5a))

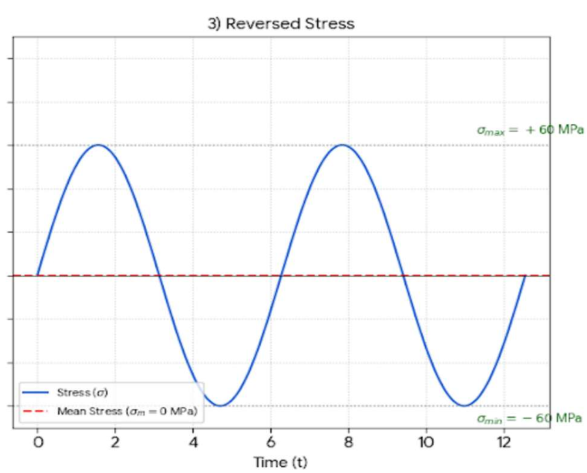
Answer:
(03 marks)

Types of fluctuating stresses (stress-time curves)

Type	Definition	Stress-time waveform	Typical example
Reversed stress	Alternates equally between tension and compression ($\sigma_m = 0$)	Symmetrical sine wave about zero	Rotating shaft, bending of a bicycle pedal crank
Repeated stress	Varies from zero to a maximum ($\sigma_m = \sigma_a$)	Sinusoidal from 0 to σ_max	Punch press, tensile test (unloading to zero)
Fluctuating stress	Varies between two non-zero values ($\sigma_m \neq 0$, $\sigma_a \neq 0$)	Asymmetric sine wave	Connecting rod, gear tooth (tension to less tension)







Q5. Explain different types of Fatigue stresses with stress–time curves.
(Appeared in: SUM 23 (Q5a))

Answer:
(03 marks)

Fatigue stresses are variable stresses that can cause failure after many cycles. Types based on stress-time pattern:

- **Completely reversed (alternating)** – $\sigma_m = 0$, $\sigma_a = \sigma_{max}$.
- **Pulsating (repeated)** – $\sigma_{min} = 0$, $\sigma_m = \sigma_a = \sigma_{max}/2$.
- **Fluctuating (sinusoidal with mean)** – $\sigma_m \neq 0$, $\sigma_a \neq 0$.
- **Random (irregular)** – Non-periodic stress (e.g., automotive suspension).

Q6. Explain cumulative fatigue damage with Minor's Equation.
(Appeared in: SUM 23 (Q5b))

Answer:
(04 marks)

Cumulative fatigue damage (Miner's rule)

When a component is subjected to varying stress levels, each stress cycle consumes a fraction of the total fatigue life. Miner's linear damage rule states that failure occurs when the sum of damage fractions equals 1.

$\Sigma (n_i / N_i) = 1$
where:
n_i = number of cycles applied at stress level i,
N_i = number of cycles to failure at that stress level (from S-N curve).

Assumptions:

- Damage accumulates linearly.
- No effect of loading sequence (order of stresses does not matter).
- Each cycle consumes the same amount of damage regardless of prior history (simplification).

Limitations:

- Does not account for overload effects (shakedown, residual stress).
- Sequence effect is ignored (e.g., high stress first reduces life more than low stress first).
- Conservative for many materials; sometimes failure occurs at $\Sigma(n_i/N_i)$ between 0.5 and 2.

Real-world application: Used in design of aircraft components, wind turbine blades, vehicle suspensions where loads vary randomly. Miner's rule allows estimation of total life from a load spectrum.

Q7. Derive Soderberg's equation and state its application to different types of loadings. (Appeared in: WIN 22 (Q5c))

Answer:
(07 marks)

Derivation of Soderberg Equation:

Step 1: Stress Components
Consider fluctuating stress with:

- Mean stress = σ_m
- Stress amplitude = σ_a
- Maximum stress = $\sigma_m + \sigma_a$
- Minimum stress = $\sigma_m - \sigma_a$

Step 2: Soderberg Line Definition
Straight line connecting:

- Point A: (0, σ_e) on σ_a-axis (completely reversed fatigue limit)
- Point B: (S_yt, 0) on σ_m-axis (yield strength)

Step 3: Equation of Line
Using two-point form: $\frac{\sigma_a - 0}{\sigma_e - 0} = \frac{S_{yt} - \sigma_m}{S_{yt} - 0}$

$$\frac{\sigma_a}{\sigma_e} = 1 - \frac{\sigma_m}{S_{yt}}$$

Step 4: Include Factor of Safety (FS)
For design, divide stresses by FS:

$$\frac{\sigma_a}{\sigma_e/FS} + \frac{\sigma_m}{S_{yt}/FS} = 1$$

$$\frac{\sigma_a}{\sigma_e} \times FS + \frac{\sigma_m}{S_{yt}} \times FS = 1$$

$$\boxed{\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{FS}}$$

Alternative form:

$$FS = \frac{1}{\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_{yt}}}$$

Step 5: Application to Different Loadings

A. Completely Reversed Loading (σ_m = 0):

$$FS = \frac{\sigma_e}{\sigma_a}$$

B. Repeated Loading (σ_min = 0):
Then: $\sigma_m = \sigma_a$ = σ_max/2

$$FS = \frac{1}{\frac{\sigma_{max}}{2\sigma_e} + \frac{\sigma_{max}}{2S_{yt}}} = \frac{2}{\frac{\sigma_{max}}{\sigma_e} + \frac{\sigma_{max}}{S_{yt}}}$$

C. Fluctuating Loading (General Case):
Given σ_max and σ_min:

- σ_m = (σ_max + σ_min)/2
- σ_a = (σ_max - σ_min)/2
  Substitute into main equation

D. Combined Loading (Bending + Torsion):
Use equivalent stresses:

- Equivalent mean stress: $\sigma_{m(eq)} = \sqrt{\sigma_{bm}^2 + 3\tau_m^2}$
- Equivalent amplitude: $\sigma_{a(eq)} = \sqrt{\sigma_{ba}^2 + 3\tau_a^2}$
  Then apply Soderberg equation

Advantages of Soderberg:

- Conservative (safe for ductile materials)
- Simple to apply
- Considers both fatigue and yielding

Limitations:

- Too conservative for many applications
- Doesn't account for stress concentration explicitly (must use K_f)

Q8. A machine component is subjected to fluctuating stress that varies from 40 to 100 N/mm². The corrected endurance limit stress for the machine component is 270 N/mm². The ultimate tensile strength and yield strength of the material are 600 and 450 N/mm²

respectively. Find the factor of safety using (i) Gerber theory (ii) Soderberg line (iii) Goodman line
(Appeared in: WIN 24 (Q5c), WIN 23 (Q4c))

Answer:
(07 marks)

Step 1: Given Data

- σ_max = 100 N/mm²
- σ_min = 40 N/mm²
- σ_e = 270 N/mm² (corrected endurance limit)
- S_u = 600 N/mm² (ultimate strength)
- S_yt = 450 N/mm² (yield strength)

Step 2: Calculate Mean and Amplitude Stresses

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{100 + 40}{2} = 70 \text{ N/mm}^2$$
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{100 - 40}{2} = 30 \text{ N/mm}^2$$

Step 3: (i) Gerber Theory
Equation: $\frac{\sigma_a}{\sigma_e} + (\frac{\sigma_m}{S_u})^2 = \frac{1}{FS}$

$$\frac{30}{270} + (\frac{70}{600})^2 = \frac{1}{FS}$$
$$0.1111 + (0.1167)^2 = \frac{1}{FS}$$
$$0.1111 + 0.01361 = \frac{1}{FS}$$
$$0.12471 = \frac{1}{FS}$$
$$FS = \frac{1}{0.12471} = 8.02$$

Step 4: (ii) Soderberg Line
Equation: $\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{FS}$

$$\frac{30}{270} + \frac{70}{450} = \frac{1}{FS}$$
$$0.1111 + 0.1556 = \frac{1}{FS}$$
$$0.2667 = \frac{1}{FS}$$

$$FS = \frac{1}{0.2667} = 3.75$$

Step 5: (iii) Goodman Line
Equation: $\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_u} = \frac{1}{FS}$

$$\frac{30}{270} + \frac{70}{600} = \frac{1}{FS}$$
$$0.1111 + 0.1167 = \frac{1}{FS}$$
$$0.2278 = \frac{1}{FS}$$
$$FS = \frac{1}{0.2278} = 4.39$$

Step 6: Comparison

- **Gerber:** FS = 8.02 (least conservative)
- **Goodman:** FS = 4.39 (moderate)
- **Soderberg:** FS = 3.75 (most conservative)

Step 7: Conclusion
For design, typically use Goodman (moderately conservative) or Soderberg (very conservative). Gerber is used when experimental data is available.

$$\boxed{\text{Gerber: } FS = 8.02 \text{Goodman: } FS = 4.39 \text{Soderberg: } FS = 3.75}$$

Q9. A steel rod is subjected to a reversed axial load of 200 kN. Find the diameter of rod for factor of safety 2. Neglect column action. The material has ultimate tensile strength of 1000 MPa and yield strength of 900 MPa. The endurance limit in reversed bending may be assumed to be half of ultimate tensile strength. The correction factor may be taken as follows. Ka = Axial load correction factor = 0.7, for machined surface factor = 0.8, size factor = 0.85, stress concentration factor is 1.0
(Appeared in: SUM 24 (Q5c-OR))

Answer:
(07 marks)

Step 1: Given Data

- Load P = 200 kN = 200,000 N (reversed axial, so $\sigma_m = 0$)
- FS = 2
- S_u = 1000 MPa = 1000 N/mm²
- S_yt = 900 MPa = 900 N/mm²

- Endurance limit in bending: $\sigma_{eb} = S_u/2 = 500$ N/mm²
- Correction factors:
  - $K_a = 0.7$ (axial load)
  - $K_b = 0.8$ (machined surface)
  - $K_c = 0.85$ (size factor)
  - $K_t = 1.0$ (stress concentration, given as 1.0 means already accounted or not present)

Step 2: Calculate Modified Endurance Limit
For axial loading with zero mean stress:

$$\sigma_e' = \sigma_{eb} \times K_a \times K_b \times K_c$$
$$\sigma_e' = 500 \times 0.7 \times 0.8 \times 0.85$$
$$\sigma_e' = 500 \times 0.476 = 238 \text{ N/mm}^2$$

Step 3: Calculate Allowable Stress Amplitude
For completely reversed loading ($\sigma_m = 0$):

$$\sigma_a(allowable) = \frac{\sigma_e'}{FS} = \frac{238}{2} = 119 \text{ N/mm}^2$$

Step 4: Calculate Required Area

$$A = \frac{P}{\sigma_a} = \frac{200{,}000}{119} = 1680.7 \text{ mm}^2$$

Step 5: Calculate Diameter

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 1680.7}{\pi}} = \sqrt{2140} = 46.26 \text{ mm}$$

Step 6: Check Static Strength (Yield)
Static stress: $\sigma = \frac{P}{A} = \frac{200{,}000}{\pi/4 \times 46.26^2} = \frac{200{,}000}{1681} = 119 \text{ N/mm}^2$
Allowable static stress: $\frac{S_{yt}}{FS} = \frac{900}{2} = 450 \text{ N/mm}^2 > 119$ ✓ OK

Step 7: Final Answer
Take standard size: **d = 47 mm** or **50 mm** for safety

$$\boxed{d = 47 \text{ mm (minimum), use 50 mm for practical design}}$$

Q10. The endurance strength for a part is 280 MPa while S_u = 630 MPa. It is subjected to a loading as follows: σ_{m1} = 315 MPa, σ_{a1} = 96 MPa for 80% of time; σ_{m2} = 245 MPa, σ_{a2} = 145 MPa for 20% of time. Find the expected life in number of cycles of reversals. Assume K_t = 1.5.
(Appeared in: SUM 23 (Q5c))

Answer:
(07 marks)

Step 1: Given Data

- σ_e = 280 MPa (endurance strength for infinite life, say 10^6 cycles)
- S_u = 630 MPa
- Loading 1: 80% time: σ_{m1} = 315 MPa, σ_{a1} = 96 MPa
- Loading 2: 20% time: σ_{m2} = 245 MPa, σ_{a2} = 145 MPa
- K_t = 1.5 (stress concentration factor)

Step 2: Apply Goodman Equation with K_t
Modified Goodman: $\frac{K_t\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_u} = \frac{1}{FS}$

For life estimation, set FS = 1 (failure condition):

$$\frac{K_t\sigma_a}{\sigma_e} + \frac{\sigma_m}{S_u} = 1$$

Step 3: Calculate Equivalent Stress for Each Loading

Loading 1:

$$\frac{1.5 \times 96}{280} + \frac{315}{630} = \frac{144}{280} + 0.5 = 0.5143 + 0.5 = 1.0143 > 1$$

This indicates failure would occur quickly under this loading alone.

Loading 2:

$$\frac{1.5 \times 145}{280} + \frac{245}{630} = \frac{217.5}{280} + 0.3889 = 0.7768 + 0.3889 = 1.1657 > 1$$

Also > 1, so both loadings would cause failure independently.

Step 4: Need S-N Curve Data
Assume typical steel S-N curve: $\sigma = \sigma_e(N/N_0)^{-b}$
where for steel: b ≈ 0.1 (slope in log-log plot)
$N_0 = 10^6$ cycles (endurance limit cycles)

Or use approximation: For $\sigma > \sigma_e$, life is finite.

Step 5: Find Cycles to Failure for Each Stress Level
We need the alternating stress equivalent. Use modified Goodman to find equivalent completely reversed stress:

From Goodman: $\sigma_{ar} = \frac{\sigma_a}{1-\frac{\sigma_m}{S_u}}$

Loading 1:

$$\sigma_{ar1} = \frac{96}{1-\frac{315}{630}} = \frac{96}{1-0.5} = \frac{96}{0.5} = 192 \text{ MPa}$$

Loading 2:

$$\sigma_{ar2} = \frac{145}{1-\frac{245}{630}} = \frac{145}{1-0.3889} = \frac{145}{0.6111} = 237.3 \text{ MPa}$$

Step 6: Apply Stress Concentration
With $K_t = 1.5$, actual stresses are higher:

Effective alternating stresses:
σ_eff1 = K_t × σ_ar1 = 1.5 × 192 = 288 MPa
σ_eff2 = K_t × σ_ar2 = 1.5 × 237.3 = 356 MPa

Step 7: Estimate Life from S-N Curve
Assume Basquin's equation: σ_a^m N = constant
For steel, m ≈ 10 (typical)

From given: at σ_e = 280 MPa, N = 10^6 cycles

$$\sigma^m N = \text{constant} = (280)^{10} \times 10^6$$

For Loading 1: (288 MPa)

$$N_1 = \frac{(280)^{10} \times 10^6}{(288)^{10}} = 10^6 \times (\frac{280}{288})^{10}$$
$$\frac{280}{288} = 0.9722, (0.9722)^{10} = 0.756$$
$$N_1 = 10^6 \times 0.756 = 7.56 \times 10^5 \text{ cycles}$$

For Loading 2: (356 MPa)

$$N_2 = \frac{(280)^{10} \times 10^6}{(356)^{10}} = 10^6 \times (\frac{280}{356})^{10}$$
$$\frac{280}{356} = 0.7865, (0.7865)^{10} = 0.092$$
$$N_2 = 10^6 \times 0.092 = 9.2 \times 10^4 \text{ cycles}$$

Step 8: Apply Miner's Rule
Let total life = N_total cycles
Fraction of cycles at each loading:

- $n_1 = 0.8N\_total$
- $n_2 = 0.2N\_total$

Miner's rule: $\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1$

$$\frac{0.8N_{total}}{7.56 \times 10^5} + \frac{0.2N_{total}}{9.2 \times 10^4} = 1$$
$$N_{total}(\frac{0.8}{7.56 \times 10^5} + \frac{0.2}{9.2 \times 10^4}) = 1$$
$$N_{total}(1.058 \times 10^{-6} + 2.174 \times 10^{-6}) = 1$$
$$N_{total} \times 3.232 \times 10^{-6} = 1$$
$$N_{total} = \frac{1}{3.232 \times 10^{-6}} = 3.094 \times 10^5 \text{ cycles}$$

Step 9: Final Answer

$$\boxed{\text{Expected life} = 3.09 \times 10^5 \text{ cycles}}$$

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