GUJARAT TECHNOLOGICAL UNIVERSITY

BE-3 SEMESTER – OLD PAPER – S19 TO S24 – QUESTION BANK

Subject Name & Code:

Advance Engineering Mathematics (130002)

Unit 1 – Fourier Series and Fourier Integral

Repeated Questions:

1. **Find the half-range cosine series for $f(x) = x^2, 0 < x < 1$.**
    - Appeared in: S-24 (Q2a, 07 marks)
2. **Find the Fourier series of $f(x) = x + x^2, -\pi < x < \pi$.**
    - Appeared in: S-24 (Q2b, 07 marks), S-21 (Q3a, 07 marks)
3. **Find the half-range sine series of $f(x) = e^x, 0 < x < \pi$.**
    - Appeared in: W-21 (Q1a-ii, 04 marks), S-22 (Q2a-i, 03 marks)
4. **Find the Fourier cosine series for $f(x) = x^2, 0 < x < c$.**
    - Appeared in: W-21 (Q3a-ii, 04 marks)
5. **Express $f(x) = e^x, 0 < x < 1$ as a half-range Fourier cosine series with period 2.**
    - Appeared in: S-21 (Q3a, 07 marks)
6. **Find the Fourier series of $f(x) = x + |x|, -\pi < x < \pi$.**
    - Appeared in: S-23 (Q1a, 07 marks), S-22 (Q2b, 07 marks)

Other Important Questions:

1. **Obtain Fourier series of $f(x) = x^2$ in the interval (0,4).**
    - Appeared in: W-19 (Q1b, 07 marks)
2. **Find Fourier series of $f(x) = e^{ax}$ in $(0,2\pi), a > 0$.**
    - Appeared in: W-19 (Q3a, 07 marks)
3. **Find Fourier series of $f(x) = \begin{cases} x & 0 \le x \le 2 \\ 4 - x & 2 \le x \le 4 \end{cases}$.**
    - Appeared in: W-19 (Q3b, 07 marks)
4. **Find Fourier series of $f(x) = x^2 + x, -\pi \le x \le \pi$.**
    - Appeared in: S-19 (Q3a, 07 marks)
5. **Find Fourier series of $f(x) = \begin{cases} -\pi & 0 < x < \pi \\ x - \pi & \pi < x < 2\pi \end{cases}$.**

- Appeared in: S-19 (Q3b, 07 marks)

6. **Find Fourier series with period 3 for $f(x) = 2x - x^2$ in (0,3).**
   - Appeared in: S-19 (Q3a, 07 marks)
7. **Obtain half-range Fourier cosine series for $f(x) = c - x$ in (0,c).**
   - Appeared in: S-19 (Q3b, 07 marks)
8. **Find half-range cosine series for $f(x) = x, 0 < x < 3$.**
   - Appeared in: S-23 (Q3a-ii, 04 marks)
9. **Find half-range cosine series for $f(x) = 1, 0 \leq x \leq 1$.**
   - Appeared in: W18 (Q2a-i, 04 marks)
10. **Obtain half-range sine series for $f(x) = 2x, 0 < x < 1$.**
    - Appeared in: W-22 (Q3a-ii, 04 marks)
11. **Obtain Fourier series for $f(x) = x - x^2, -1 < x < 1$.**
    - Appeared in: W18 (Q2b, 07 marks)
12. **Obtain Fourier series for $f(x) = |\sin x|, -\pi < x < \pi$.**
    - Appeared in: W18 (Q3b, 07 marks)
13. **Show that $\int_0^\infty \frac{\cos \omega x}{\omega^2+1} d\omega = \frac{\pi}{2} e^{-x}$ for $x > 0$.**
    - Appeared in: W-23 (Q3b-ii, 05 marks)
14. **Express $f(x) = \sin x, 0 \leq x \leq \pi; 0, x > \pi$ as Fourier sine integral.**
    - Appeared in: S-21 (Q5b-ii, 05 marks)
15. **Find Fourier integral representation of $f(x) = \begin{cases} 2 & |x| < 2 \\ 0 & |x| > 2 \end{cases}$.**
    - Appeared in: W-21 (Q5b, 07 marks)
16. **Find Fourier integral representation of $f(x) = 1, |x| < 1; 0, |x| > 1$ and evaluate $\int_0^\infty \frac{\sin \omega}{\omega} d\omega$.**
    - Appeared in: W-22 (Q5b, 07 marks)
17. **Obtain Fourier series of $f(x) = \frac{1}{2}(\pi - x), 0 \leq x \leq 2\pi$.**
    - Appeared in: S-23 (Q5b, 07 marks), W-23 (Q4b, 07 marks)
18. **Obtain Fourier series of $f(x) = (\frac{\pi - x}{2})^2, 0 \leq x \leq 2\pi$.**
    - Appeared in: W-21 (Q5b, 07 marks)
19. **Find Fourier series of $f(x) = \sqrt{1 - \cos x}$ in [0, 2π].**
    - Appeared in: S-21 (Q3b, 07 marks)

Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
1	**Find** the fundamental period p of (i) $\cos\frac{2\pi x}{k}$ (ii) $\cos\frac{2\pi n x}{k}$	CO1	2.1.1	U, A
2	**Find** the Fourier series of the function $f(x)$ which is assumed to have the period 2π, where $f(x)$ is as follows (i) $f(x)=x, \quad -\pi<x<\pi$ (ii) $f(x)=x^2, \quad 0<x<2\pi$ (iii) $f(x)=x+\|x\|, \quad -\pi<x<\pi$	CO1	2.1.2	U, A
3	**Find** the Fourier series of the function $f(x)$ which is assumed to have the period 2π, where $f(x)$ is as follows (i) $f(x)=\begin{cases}1 & if \quad -\pi<x<0\\ -1 & if \quad 0<x<\pi\end{cases}$ (ii) $f(x)=\begin{cases}x & if \quad -\pi/2<x<\pi/2\\ 0 & if \quad \pi/2<x<3\pi/2\end{cases}$	CO1	2.1.3	U, A
4	**Find** the Fourier series of the periodic function $f(x)$ of period $p=2l$, (i) $f(x)=\begin{cases}-1 & if \quad -1<x<0\\ 1 & if \quad 0<x<1\end{cases}$ (ii) $f(x)=\begin{cases}(1/2)+x & -(1/2)<x<0\\ (1/2)-x & 0<x<(1/2)\end{cases}$	CO1	1.1.1	U, A
5	**Check** whether the following functions are odd or even. (i) $\|x^3\|$ (ii) $x^2\cos nx$ (iii) $x\|x\|$ (iv) e^x (v) $\ln x$	CO1	2.1.2	U, A
6	**Obtain** the Fourier Series of the function $f(x)=cosx, \quad for \; -\pi<x<0$; $=sinx, \quad for \; 0<x<\pi$	CO1	1.2.1	U, A
7	**Find** the Fourier Series of $f(x)=x+\|x\|, \quad -\pi<x<\pi$	CO1	2.1.2	U, A
8	**Find** Fourier Cosine Series of $f(x)=e^x, \quad 0<x<L$	CO1	1.1.1	U, A
9	**Find** Half range sine series of $f(x)=\{2x \quad , \; 0<x<1\,4$; $-2x \quad , \; 1<x<2$	CO1	1.1.2	U, A
10	**Find** Fourier Series for the function $f(x)=x-\|x\| \; , \quad -\pi<x<0$; $=x+\|x\| \; , \quad 0<x<\pi$. Also show that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\frac{\pi}{4}$	CO1	2.1.1	U, A

Unit 2 – Partial Differential Equations and Applications

Topics: Formation of PDEs, first-order PDEs (linear, nonlinear), second-order linear PDEs with constant coefficients, classification, separation of variables, heat/wave equations, D'Alembert's solution.

Repeated Questions:

1. **Solve** $p^2 + q^2 = 1$**.**
   - Appeared in: S-24 (Q5a-i, 04 marks), W18 (Q4a-ii, 03 marks)
2. **Solve** $p^2 - q^2 = x - y$**.**
   - Appeared in: S-24 (Q3a-ii, 03 marks), W-22 (Q4a-i, 03 marks)
3. **Solve** $yzp - xzq = xy$**.**
   - Appeared in: S-24 (Q5a-i, 04 marks)
4. **Solve** $(y + z)p + (z + x)q = x + y$**.**
   - Appeared in: W18 (Q2a-ii, 03 marks), S-22 (Q5a-i, 03 marks)
5. **Solve** $pz - qz = z^2 + (x + y)^2$**.**
   - Appeared in: W-21 (Q4b-ii, 04 marks), S-23 (Q4b-ii, 04 marks)
6. **Solve** $p^2 + q^2 = npq$**.**
   - Appeared in: W-21 (Q4b-i, 03 marks), S-23 (Q4b-i, 03 marks)
7. **Form PDE by eliminating arbitrary functions from** $xyz = \phi(x + y + z)$**.**
   - Appeared in: W-21 (Q5a-i, 03 marks), S-23 (Q5a-i, 03 marks)
8. **Solve** $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ **or similar using separation of variables.**
   - Appeared in: S-19 (Q2a, 07 marks), W-23 (Q5b, 07 marks)

Other Important Questions:

1. **Solve** $(p - q)(z - px - qy) = 1$**.**
   - Appeared in: S-24 (Q3a-ii, 03 marks)
2. **Solve** $(D^2 + 10DD' + 25D'^2)z = e^{3x+2y}$**.**
   - Appeared in: S-24 (Q5b, 07 marks)
3. **Solve** $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ **using separation of variables.**
   - Appeared in: S-24 (Q5b, 07 marks)
4. **Solve** $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ **with** $u(0, y) = 3e^{-y} - 5e^{-5y}$**.**
   - Appeared in: S-21 (Q2a, 07 marks)
5. **Form PDE by eliminating arbitrary functions from** $z = f(x^2 - y^2)$**.**
   - Appeared in: S-21 (Q5a-i, 03 marks)
6. **Form PDE by eliminating arbitrary constants from** $z = (x - a)^2 + (y - b)^2$**.**
   - Appeared in: W-23 (Q5a, 07 marks), S-22 (Q5a-i, 03 marks)
7. **Solve** $z = px + qy + n\sqrt{1 + p^2 + q^2}$**.**
   - Appeared in: S-21 (Q5a-i, 03 marks)
8. **Solve** $(mz - ny)p + (nx - lz)q = ly - mx$**.**
   - Appeared in: S-21 (Q5b-ii, 04 marks)
9. **Form PDE by eliminating arbitrary functions from** $f(x^2 + y^2, z - xy) = 0$**.**
   - Appeared in: W-23 (Q5a, 07 marks)
10. **Form PDE by eliminating arbitrary functions from** $z = xy + f(x^2 + y^2)$**.**
    - Appeared in: W-22 (Q5a-i, 03 marks)
11. **Solve** $p(1 + q) = qz$**.**
    - Appeared in: W-22 (Q4a-ii, 04 marks)
12. **Solve** $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ **with** $u(x, 0) = 4e^{-3x}$**.**
    - Appeared in: S-23 (Q4b, 07 marks), W-22 (Q4b, 07 marks)
13. **Solve** $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ **with** $u(x, 0) = 6e^{-3x}$**.**
    - Appeared in: S-22 (Q5b, 07 marks)
14. **Solve** $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$**.**

- Appeared in: W-19 (Q5b, 07 marks)

15. **Solve** $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2\,\partial y} + 4\frac{\partial^3 z}{\partial x\,\partial y^2} = 2\sin(3x + 2y)$**.**
    - Appeared in: S-19 (Q5b-i, 07 marks)
16. **Solve** $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$**.**
    - Appeared in: S-19 (Q5b-ii, 07 marks)
17. **Solve** $2r + 5s + 2t = 0$**.**
    - Appeared in: W18 (Q3a-i, 04 marks)
18. **Solve** $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ **using separation of variables.**
    - Appeared in: W18 (Q4b-i, 04 marks)
19. **Solve** $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$**.**
    - Appeared in: S-19 (Q5a-ii, 04 marks)
20. **Solve** $p + q = pq$**.**
    - Appeared in: S-19 (Q5a-i, 03 marks)
21. **Form PDE from** $z = y^2 + 2f(\frac{1}{x} + \log\, y)$**.**
    - Appeared in: S-19 (Q5a-i, 03 marks)

Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
Form PDE from following (1 & 2) equations by eliminating arbitrary constants				
1	$z=(x-a)^2+(y-b)^2$	CO2	1.2.1	U
2	$z=(x^2+a)(y^2+b)$	CO2	1.2.1	U
Form PDE from following (3 & 4) equations by eliminating arbitrary Functions				
3	$z=y^2+2f\left(\frac{1}{x}+lny\right)$	CO2	1.2.1	U
4	$F\left(\frac{x-y}{xy},\frac{x-y}{z}\right)=0$	CO2	1.2.1	U
Solve following (5 to 16) Linear Partial Differential Equations (Lagrange's equations).				
5	$(y-z)p+(x-y)q=(z-x)$	CO2	2.4.1	U
6	$z(xp-yq)=y^2-x^2$	CO2	2.4.1	U
7	$y^2p-xyq=x(z-2y)$	CO2	2.4.1	U
8	$x(y-z)p+y(z-x)q=z(x-y)$	CO2	2.4.1	U
9	$y^2zp-x^2zq=x^2y$	CO2	2.4.1	U
10	$pz-qz=z^2+(x+y)^2$	CO2	2.4.1	U
11	$(mz-ny)p+(nx-lz)q=ly-mx$	CO2	2.4.1	U
12	$pz+qz=z^2+(x-y)^2$	CO2	2.4.1	U
13	$(x^2+y^2+z^2)p+zxyq=2xz$	CO2	2.4.1	U
14	$(x^2-yz)p+(y^2-zx)q=(z^2-xy)$	CO2	2.4.1	U
Solve following (17 to 11) Non- linear partial differential equations.				
15	$p^2+q^2=1$			
16	$p(1+q^2)=q(z-1)$	CO2	2.4.1	U
17	$p(1+q)=qz$	CO2	2.4.1	U
18	$p^2-q^2=x-y$	CO2	2.4.1	U
19	$p^2-q^2=x^2+y^2$	CO2	2.4.1	U
20	$z=px+qy+\frac{p}{q-p}$	CO2	2.4.1	U
Solve following (17 to 11) Non- linear partial differential equations using Charpit's Method.				
21	$px+qy=pq$	CO2	2.4.1	U
22	$(p^2+q^2)y=qz$	CO2	2.4.1	U
23	$z^2=pqxy$	CO2	2.4.1	U

Sr. No.	Question	CO	PI	B.T. level
Solve following (1 to 3) second order homogeneous PDEs.				
1	$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-12\frac{\partial^2 z}{\partial y^2}=3e^{2x-3y}$	CO2	2.4.1	U
2	$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}=\cos(2x-3y)$	CO2	2.4.1	U
3	$\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y^2}=x^2y^3$	CO2	2.4.1	U
4	Using method of separation of variables solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ given that $u(x,0)=6e^{-3x}$.	CO2	2.4.1	U, A
5	Using method of separation of variables solve $\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$given that $u(x,0)=4x-\frac{1}{2}x^2$.	CO2	2.4.1	U,A
6	**Discuss** all possible solutions for a heat conduction problem in a rod of length *l* whose ends are held at zero temperature and *f(x)* is initial temperature distribution.	CO2	2.1.1	R,U
7	**Find** the temperature in the thin metal rod of length l with both ends insulated and initial temperature is $sin\frac{\pi x}{l}$. Use one-dimensional heat equation$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.	CO2	2.4.1	U,A
8	**Derive** one dimensional wave equation that governs small vibration of an elastic string of length *l* whose both ends are fixed. Also state physical assumptions that you make for the system.	CO2	2.1.1	R, U
9	A string is streched between two fixed points and the points of the string are given initial velocity$g(x)$, where $g(x)=\{\frac{2x}{l}\quad,0<x<\frac{l}{2}\ \ \frac{2(l-x)}{l}\quad,\frac{l}{2}<x<l$, $x$being the distance from the first end point. **Find** the displacement of the string at any time. Use wave equation$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$.	CO2	2.4.1	U,A
10	A string of length $l=\pi$ has its ends fixed at $x=0$ and $x=\pi$. At any time $t=0$, the string is given a shape defined by $f(x)=50x(\pi-x)$, then it is realesd. **Find** the deflection of string at any time t.	CO2	2.4.1	U,A

Unit 6 – Laplace Transforms and Applications

Topics: Definition, properties, shifting theorems, transforms of derivatives/integrals, inverse transforms, convolution theorem, applications to ODEs and PDEs.

Repeated Questions:

1. **Find Laplace transform of** $e^{-3t}(2\cos\ 5t - 3\sin\ 5t)$ **using first shifting theorem.**
   - Appeared in: S-24 (Q1b-i, 04 marks), S-23 (Q2b-i, 04 marks), W-23 (Q3a-i, 04 marks)
2. **Find inverse Laplace transform of** $\frac{3s+7}{s^2-2s-3}$**.**
   - Appeared in: W-21 (Q2b-ii, 04 marks), S-21 (Q4b-i, 07 marks), S-23 (Q2b-ii, 04 marks)
3. **Find inverse Laplace transform of** $\log(1+\frac{\omega^2}{s^2})$**.**
   - Appeared in: S-24 (Q4a-i, 04 marks), S-22 (Q4a-ii, 04 marks)
4. **Find inverse Laplace transform of** $\frac{s+2}{s^2+4s+8}$ **or similar forms.**
   - Appeared in: S-24 (Q1b-ii, 03 marks)
5. **Find inverse Laplace transform using convolution theorem:** $\frac{1}{(s+1)(s^2+1)}$**.**
   - Appeared in: S-24 (Q4b, 07 marks), S-21 (Q4b-ii, 04 marks)
6. **Find Laplace transform of** $\frac{\sin\ 2t}{t}$**.**
   - Appeared in: S-24 (Q4a-ii, 03 marks)

Other Important Questions:

1. **Find Laplace transform of** $\sinh^5 t$**.**
   - Appeared in: W-21 (Q2b-i, 03 marks)
2. **Find inverse Laplace transform of** $s\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$**.**
   - Appeared in: W-21 (Q4a, 07 marks)
3. **Find Laplace transform of** $e^t(1+\sqrt{t})^4$**.**
   - Appeared in: W-19 (Q4a-i, 03 marks)
4. **Find inverse Laplace transform of** $\frac{2s+2}{s^2+2s+1}$**.**
   - Appeared in: W-19 (Q4a-ii, 04 marks)
5. **Find inverse Laplace transform of** $\frac{1}{(s-2)(s+2)^2}$ **using convolution theorem.**
   - Appeared in: W-19 (Q4b, 07 marks)
6. **Find Laplace transform of** $e^{-3t}u(t-2)$**.**

- Appeared in: W-19 (Q4a-i, 03 marks)

7. **Find inverse Laplace transform of** $\frac{e^{-2s}}{(s+4)^3}$.
   - Appeared in: W-19 (Q4a-ii, 04 marks)
8. **Solve** $y'' + 3y' + 2y = e^t, y(0) = 1, y'(0) = 0$ **using Laplace transform.**
   - Appeared in: W-21 (Q4a, 07 marks)
9. **Solve** $y'' - 3y' + 2y = 12e^{-2t}, y(0) = 2, y'(0) = 6$ **using Laplace transform.**
   - Appeared in: W-19 (Q4b, 07 marks)
10. **Find Laplace transform of** $\frac{e^{-2t}\sin 2t\cosh t}{t}$.
    - Appeared in: S-21 (Q4a-ii, 04 marks)
11. **Find inverse Laplace transform of** $\ln(1 + \frac{1}{s^2})$.
    - Appeared in: S-21 (Q4b-ii, 07 marks)
12. **Solve** $y' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1$ **using Laplace transform.**
    - Appeared in: S-21 (Q4a, 07 marks)
13. **Find Laplace transform of** $t\sin^2 3t$.
    - Appeared in: S-21 (Q4b-i, 03 marks)
14. **State convolution theorem and use it to find inverse Laplace transform of** $\frac{1}{(s+1)(s^2+1)}$.
    - Appeared in: S-21 (Q4b-ii, 04 marks)
15. **Find Laplace transform of** $\frac{\cos at - c \quad bt}{t}$.
    - Appeared in: W18 (Q1a-ii, 03 marks)
16. **Find inverse Laplace transform of** $\frac{3(s^2-1)^2}{s^5}$.
    - Appeared in: W18 (Q1b-ii, 03 marks)
17. **State convolution theorem and use it to find inverse Laplace transform of** $\frac{s}{(s^2+a^2)(s^2+b^2)}$.
    - Appeared in: W18 (Q3b, 07 marks)
18. **Find inverse Laplace transform of** $\frac{5s^2-2s-19}{(s+3)(s-1)^2}$.
    - Appeared in: W18 (Q4a-ii, 03 marks)
19. **Find inverse Laplace transform of** $\frac{5s+3}{(s-1)(s^2+2s+5)}$.
    - Appeared in: S-23 (Q4a, 07 marks)
20. **Find inverse Laplace transform of** $\frac{s+2}{(s^2+4s+5)^2}$ **using convolution theorem.**

- Appeared in: S-23 (Q5a, 07 marks), S-22 (Q4b, 07 marks)

21. **Find Laplace transform of** $\cos\ 3t \cdot \cos\ 2t \cdot \cos\ t$**.**
    - Appeared in: S-23 (Q5a-ii, 04 marks)
22. **Find inverse Laplace transform of** $\tan^{-1}(\frac{2}{s})$**.**
    - Appeared in: W-22 (Q5a-ii, 04 marks)
23. **Find Laplace transform of** $\frac{1-\text{co}\quad t}{t}$**.**
    - Appeared in: W-22 (Q5a-ii, 04 marks)
24. **Find Laplace transform of** $t^2 e^{3t} \sin\ 4t$**.**
    - Appeared in: W-22 (Q4a-i, 03 marks)
25. **State convolution theorem and use it to find inverse Laplace transform of** $\frac{1}{s(s^2+4)}$**.**
    - Appeared in: W-22 (Q4a-ii, 04 marks), W-23 (Q4a, 07 marks)
26. **Solve** $y'' + 2y' + 5y = e^{-t} \sin\ t, y(0) = 0, y'(0) = 1$ **using Laplace transform.**
    - Appeared in: W-22 (Q4b, 07 marks)
27. **Find inverse Laplace transform of** $\frac{s^3}{s^4-81}$**.**
    - Appeared in: W-22 (Q2b-ii, 04 marks)
28. **State second shifting theorem and use it to find inverse Laplace transform of** $\frac{e^{-3s}}{s^2+8s+25}$**.**
    - Appeared in: W-22 (Q5a-i, 03 marks)
29. **Find Laplace transform of** $\frac{e^{-t} \sin\ t}{t}$**.**
    - Appeared in: S-22 (Q4a-i, 03 marks)
30. **Find inverse Laplace transform of** $\ln \frac{s^2+1}{(s+1)(s-2)^2}$**.**
    - Appeared in: S-22 (Q1a-ii, 05 marks)

Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
Using Appropriate rule **find** Laplace Transforms of folllowing functions (1 to 6)				
1	$f(t)=\begin{cases}0 & if\ 0\le t<3\\ 4 & if\ t\ge 3\end{cases}$	CO5	2.1.1	R,U
2	(i) e^{2t+3} (ii) $\cos^2 t$ (iii) $t^5+\cos 5t+e^{-100t}$ (iv) $e^{4t}\sin 2t\cos t$	CO5	2.4.1	U
3	(i) $\int_0^t e^{-u}\cos u\,du$ (ii) $\int_0^t\int_0^t \sin au\,du\,du$	CO5	2.4.1	U
4	(i) $t^2\cos^2 2t$ (ii) $t\sin 3t\cos 2t$ (iii) $te^t sint$	CO5	2.4.1	U, A
5	(i) $\frac{1-\cos 2t}{t}$ (ii) $\frac{e^{-bt}-e^{-at}}{t}$	CO5	2.4.1	U, A
6	(i) $e^{-3t}u(t-2)$ (ii) $t^2u(t-2)$	CO5	2.4.1	U,A
Using Appropriate rule **find** Inverse Laplace Transforms of following functions (7 to 11)				
7	(i) $\frac{s+7}{s^2+8s+25}$ (ii) $\frac{s}{(s+1)^2+9}$ (iii) $\frac{s}{s^2-3s+2}$	CO5	2.4.1	U
8	(i) $\frac{1}{s(s^2-3s+3)}$ (ii) $\frac{1}{s(s^2+4)}$	CO5	2.4.1	U,A
9	(i) $\ln\left(\frac{s^2+a^2}{s^2+b^2}\right)$ (ii) $\tan^{-1}\left(\frac{2}{s}\right)$	CO5	2.4.1	U,A
10	$\frac{s+2}{(s^2+4s+5)^2}$	CO5	2.4.1	U, A
11	(i) $\frac{e^{-2\pi s}-e^{-8\pi s}}{s^2+1}$ (ii) $\frac{e^{-3s}}{s^2+8s+25}$	CO5	2.4.1	U, A
12	**Define** Unit step function and **Find** Laplace transform of $f(t)=\begin{cases}2, & 0<t<1\\ t^2/2, & 1<t<\pi/2\\ \cos t, & t>\pi/2\end{cases}$	CO5	2.4.1	R, A

Sr. No.	Question	CO	PI	B.T. level
1	**Find** Laplace transform of periodic function $f(t)=\frac{2t}{3};0\le t\le 3;\ f(t+3)=f(t)$.	CO5	2.4.1	R,U
2	**Find** convolution of t and e^t.	CO5	2.4.1	U
3	**Find** $L^{-1}\left(\frac{2\pi s}{(s^2+\pi^2)^2}\right)$ using Convolution theorem.	CO5	2.4.1	U
4	Find Inverse Laplace Transform of following using Convolution theorem $(i)\ \frac{3s^2+2}{(s+1)(s+2)(s+3)}$ $(ii)\ \frac{4s+5}{(s-1)^2(s+2)}$ $(iii)\ \frac{s^2}{(s^2+25)(s^2+49)}$	CO5	2.4.1	U, A
5	**Solve** the initial value problem $y''-y'-2y=0,\ y(0)=1,\ y'(0)=0$ using Laplace transform.	CO5	2.4.1	U,A
6	**Using** the Laplace transforms, **find** the solution of the initial value problem $y''+25y=10\cos 5t,\quad y(0)=2,\ y'(0)=0$.	CO5	2.4.1	U,A
7	**Solve** $\frac{dy}{dt}-4y=2e^{2t}+e^{4t}$ by Laplace transformation	CO5	2.4.1	U,A
8	**Solve** the IVP $y''+2y'+y=e^{-t},\ y(0)=-1,\ y'(0)=1$ using Laplace transform	CO5	2.4.1	U
9	Define Dirac's delta function and find the solution of $\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=1+\delta(t-4),\ x(0)=x'(0)=0$	CO5	2.4.1	R,A
10	**Solve** the system of ODEs $\frac{dx}{dt}+y=1,\ x+\frac{dy}{dt}=2$ where $x(0)=1,\ y(0)=1.$	CO5	2.4.1	U,A
11	**Solve** $y''-ty'+y=1,\ y(0)=1,\ y'(0)=2$ by Laplace transformation	CO5	2.4.1	U,A
12	**Solve** $ty''+(1-2t)y'-2y=0,\ y(0)=1,\ y'(0)=2$ by Laplace transformation	CO5	2.4.1	U,A

Unit 3 – Finite Differences and Interpolation

Practice Numerical:

<table>
<tr><th>Sr. No.</th><th>Question</th><th>CO</th><th>PI</th><th>B.T. level</th></tr>
<tr><td>1</td><td>Define the operators, Δ, ∇, δ and E, E^{-1} and show that
(i) $\Delta \equiv E\nabla$ (ii) $E = 1 + \Delta$ (iii) $\nabla = E^{-1}\Delta$ (iv) $E^{-1} \equiv 1 - \nabla$</td><td>CO3</td><td>2.4.1</td><td>R, U</td></tr>
<tr><td>2</td><td>Obtain the estimate of missing terms in the following table:
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>Y</td><td>2</td><td>4</td><td>8</td><td>-</td><td>32</td><td>-</td><td>128</td><td>256</td></tr></table></td><td>CO3</td><td>2.4.2</td><td>U, A</td></tr>
<tr><td>3</td><td>Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial polynomial and ,hence, show that $\Delta^3 f(x) = 12$.</td><td>CO3</td><td>2.4.4</td><td>U, A</td></tr>
<tr><td>4</td><td>Find cosh 0.56 from Newton's forward difference interpolation formula and Estimate the error for the following data.
<table><tr><td>x</td><td>0.5</td><td>0.6</td><td>0.7</td><td>0.8</td></tr><tr><td>Cos hx</td><td>1.127626</td><td>1.185465</td><td>1.255169</td><td>1.337435</td></tr></table></td><td>CO3</td><td>1.1.1</td><td>U, A</td></tr>
<tr><td>5</td><td>Using Newton's forward interpolation formula, find the value of $f(5)$ if
<table><tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr><tr><td>f(x)</td><td>1</td><td>3</td><td>8</td><td>16</td><td>20</td></tr></table></td><td>CO3</td><td>1.1.2</td><td>U, A</td></tr>
<tr><td>6</td><td>Using Newton's forward interpolation formula, Find $\sin 52^\circ$ from following data
<table><tr><td>$\sin 45^\circ$</td><td>$\sin 50^\circ$</td><td>$\sin 55^\circ$</td><td>$\sin 60^\circ$</td></tr><tr><td>0.7071</td><td>0.7660</td><td>0.8192</td><td>0.8660</td></tr></table></td><td>CO3</td><td>1.2.1</td><td>U, A</td></tr>
<tr><td>7</td><td>Find $y(300)$ from the given values using Newton's backward Interpolation formula.
<table><tr><td>x</td><td>50</td><td>100</td><td>150</td><td>200</td><td>250</td></tr><tr><td>y</td><td>618</td><td>724</td><td>805</td><td>906</td><td>1032</td></tr></table></td><td>CO3</td><td>1.3.1</td><td>U, A</td></tr>
<tr><td>8</td><td>The area A of a circle of diameter d is given for the following values:
<table><tr><td>d</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr><tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr></table>Find the area of a circle of a diameter of 105 units using Newton's backward Interpolation formula.</td><td>CO3</td><td>1.4.1</td><td>U, A</td></tr>
<tr><td>9</td><td>From the following data, Find the profit in the year 1925.
<table><tr><td>Year</td><td>1891</td><td>1901</td><td>1911</td><td>1921</td><td>1931</td></tr><tr><td>Profit in the lakhs</td><td>46</td><td>66</td><td>81</td><td>93</td><td>101</td></tr></table></td><td>CO3</td><td>1.2.1</td><td>U, A</td></tr>
<tr><td>10</td><td>Find P when t = 175^0C from the given table, using Newton's backward Interpolation formula
<table><tr><td>Temperature t^0 C</td><td>140</td><td>150</td><td>160</td><td>170</td><td>180</td></tr><tr><td>Pressure P</td><td>3685</td><td>4845</td><td>6302</td><td>8076</td><td>10225</td></tr></table></td><td>CO3</td><td>1.4.1</td><td>U, A</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><th>Sr. No.</th><th>Question</th><th>CO</th><th>PI</th><th>B.T. level</th></tr>
<tr><td>1</td><td>Find $f(6)$ from the following data, by using Newton's divided difference interpolation formula:
<table><tr><td>x</td><td>1</td><td>2</td><td>7</td><td>8</td></tr><tr><td>f(x)</td><td>1</td><td>5</td><td>5</td><td>4</td></tr></table></td><td>CO3</td><td>1.2.1</td><td>R, U</td></tr>
<tr><td>2</td><td>Compute $f(4)$, from the tabular values given:
<table><tr><td>x</td><td>2</td><td>3</td><td>5</td><td>7</td></tr><tr><td>$f(x)$</td><td>0.1506</td><td>0.3001</td><td>0.4517</td><td>0.6259</td></tr></table>
using Lagrange's interpolation formula.</td><td>CO3</td><td>2.1.2</td><td>U, A</td></tr>
<tr><td>3</td><td>Find the polynomial which takes the values $f(1) = 1,\ f(2) = 9,\ f(3) = 5,\ \ f(4) = 55,\ f(5) = 105$.</td><td>CO3</td><td>1.1.2</td><td>U, A</td></tr>
<tr><td>4</td><td>Calculate $e^{1.85}$ from the following table:
<table><tr><td>x</td><td>1.7</td><td>1.8</td><td>1.9</td><td>2</td><td>2.1</td><td>2.2</td><td>2.3</td></tr><tr><td>e^x</td><td>5.474</td><td>6.050</td><td>6.686</td><td>7.389</td><td>8.166</td><td>9.025</td><td>9.974</td></tr></table></td><td>CO3</td><td>2.1.2</td><td>U, A</td></tr>
<tr><td>5</td><td>The values of specific heat (C_p) at constant pressure of a gas are tabulated below for various temperatures. Find the specific heat at $900^{\circ}C$.
<table><tr><td>Temperature $^{\circ}C$</td><td>500</td><td>1000</td><td>1500</td><td>2000</td></tr><tr><td>C_p</td><td>31.23</td><td>35.01</td><td>39.18</td><td>43.75</td></tr></table></td><td>CO3</td><td>1.4.1</td><td>U, A</td></tr>
<tr><td>6</td><td>Compute $f(10.5)$, from the given values by using Newton's divided difference Interpolation formula
<table><tr><td>x</td><td>10</td><td>11</td><td>13</td><td>17</td></tr><tr><td>$f(x)$</td><td>2.3026</td><td>2.3979</td><td>2.5649</td><td>2.8332</td></tr></table></td><td>CO3</td><td>2.1.2</td><td>U, A</td></tr>
<tr><td>7</td><td>Find x corresponding to $y = 85$ from the following table:
<table><tr><td>x</td><td>2</td><td>5</td><td>8</td><td>14</td></tr><tr><td>y</td><td>94.8</td><td>87.9</td><td>81.3</td><td>68.7</td></tr></table></td><td>CO3</td><td>1.3.1</td><td>U, A</td></tr>
<tr><td>8</td><td>Develop Newton's divided difference polynomial for the following data.
ln 9.0 = 2.1972, ln 9.5 = 2.2513, ln 11.0 = 2.3979.</td><td>CO3</td><td>2.4.2</td><td>U, A</td></tr>
<tr><td>9</td><td>Find $f(9.2)$ from the given values using Newton's divided difference Interpolation formula.
<table><tr><td>x</td><td>8.0</td><td>9.0</td><td>9.5</td><td>11.0</td></tr><tr><td>$f(x)$</td><td>2.079442</td><td>2.197225</td><td>2.251292</td><td>2.397895</td></tr></table></td><td>CO3</td><td>1.3.1</td><td>U, A</td></tr>
<tr><td>10</td><td>Find x given y = 0.3887 from the following data:
<table><tr><td>X</td><td>21</td><td>23</td><td>25</td></tr><tr><td>y</td><td>0.3706</td><td>0.4068</td><td>0.4433</td></tr></table></td><td>CO3</td><td>2.1.1</td><td>U, A</td></tr>
</table>

Unit 4 – Numerical Differentiation and Integration

Practice Numerical:

<table>
<tr><th>Sr. No.</th><th>Question</th><th>CO</th><th>PI</th><th>B.T. level</th></tr>
<tr><td>1</td><td>Write derivatives formulae using forward difference and backward difference.</td><td>CO4</td><td>1.1.1</td><td>R</td></tr>
<tr><td>2</td><td>Given that
<table>
<tr><td>x</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td><td>1.6</td></tr>
<tr><td>y</td><td>7.989</td><td>8.403</td><td>8.781</td><td>9.129</td><td>9.451</td><td>9.750</td><td>10.031</td></tr>
</table>
Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1 .</td><td>CO4</td><td>1.1.2</td><td>U</td></tr>
<tr><td>3</td><td>Given that
<table>
<tr><td>x</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td><td>1.6</td></tr>
<tr><td>y</td><td>7.989</td><td>8.403</td><td>8.781</td><td>9.129</td><td>9.451</td><td>9.750</td><td>10.031</td></tr>
</table>
Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.6 .</td><td>CO4</td><td>1.1.2</td><td>U</td></tr>
<tr><td>4</td><td>Compute $f'(2.2)$,using appropriate method from the following data:
<table>
<tr><td>x</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2.0</td><td>2.2</td></tr>
<tr><td>f(x)</td><td>4.0552</td><td>4.9530</td><td>6.0496</td><td>7.3981</td><td>9.0250</td></tr>
</table></td><td>CO4</td><td>1.1.2</td><td>U</td></tr>
<tr><td>5</td><td>Given that
<table>
<tr><td>x</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td></tr>
<tr><td>y</td><td>1.5708</td><td>1.5738</td><td>1.5828</td><td>1.5981</td><td>1.62</td></tr>
</table>
Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 3.</td><td>CO4</td><td>1.4.1</td><td>U, A</td></tr>
<tr><td>6</td><td>Using the backward differences, compute the derivative of the tabulated function given below at x = 0.4.
<table>
<tr><td>x</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td></tr>
<tr><td>y</td><td>1.10517</td><td>1.22140</td><td>1.34986</td><td>1.49182</td></tr>
</table></td><td>CO4</td><td>2.1.2</td><td>U, A</td></tr>
</table>

Sr. No.	Question	CO	PI	B.T. level
1	**Evaluate** $\int_0^{\frac{\pi}{2}} e^{\sin\theta}\, d\theta$ by **using** Simpson's 3/8 rule, dividing the interval $\left[0, \frac{\pi}{2}\right]$ into six equal parts.	CO4	2.1.1	U, A
2	**Evaluate** $\int_1^2 \frac{dx}{1+x^2} dx$ taking $h = 0.2$,**using** trapezoidal rule.	CO4	2.1.2	U, A
3	**Evaluate** $\int_0^6 \frac{dx}{1+x^2} dx$ by **using** (i) trapezoidal rule, (ii) Simpson's 1/3 rule, (iii) Simpson's 3/8 rule.	CO4	2.1.3	U, A
4	**Evaluate** $\int_4^{5.2} logx\, dx$ **using** the trapezoidal rule and Simpson's 3/8 rule take h=0.2.	CO4	1.1.1	U, A
5	**Evaluate** $\int_0^1 (1 + \frac{sinx}{x})dx$ correct to three decimal places, by **using** (i) trapezoidal rule, (ii) Simpson's 1/3 rule, (iii) Simpson's 3/8 rule.	CO4	2.1.2	U, A
6	**Compute** the value of $\int_{0.2}^{1.4}(sin\, sinx - log\, logx + e^x)dx$ taking $h = 0.2$ and **using** the trapezoidal rule and Simpson's rule.	CO4	1.2.1	U, A
7	**Evaluate** $\int_{0.5}^{0.7} \sqrt{x}\, e^{-x} dx$ **using** Simpson's 3/8 rule.	CO4	2.1.2	U, A
8	**Evaluate** $\int_0^{\frac{\pi}{2}} sinxdx$ by the two-point Gaussian formula.	CO4	1.1.1	U, A
9	**Evaluate** $\int_0^1 e^{-x^2}\, dx$ by using the Gaussian quadrature formula with $n = 3$.	CO4	1.1.2	U, A
10	**Evaluate** $\int_0^{\frac{\pi}{2}} log(1 + x)dx$ by the two-point Gaussian formula.	CO4	2.1.1	U, A

Unit 5 – Numerical Solution of Ordinary Differential Equations

Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
1	**Using** the Picard's method , **find** $y(1.1)$ correct to four decimal places given that $\frac{dy}{dx} = xy^{\frac{1}{3}}, y(1) = 1, h = o.1$.	CO4	2.1.1	U, A
2	**Solve** $\frac{dy}{dx} = x + y$ by Picard's method. Start from $x = 1, y = 0$ and carry to $x = 1.2$ with $h = o.1$.	CO4	2.1.2	U, A
3	**Solve** $\frac{dy}{dx} = x^2 + y^2$ by the Picard's method, with $y(0) = 0$ at $x = 0.4$.	CO4	2.2.1	U, A
4	**Solve** $\frac{dy}{dx} = xy$ by **using** Euler's method, with $y(0) = 2, h = 0.2$ at $x = 1$.	CO4	2.1.1	U, A
5	**Solve** $\frac{dy}{dx} = x + y + xy$ by **using** Euler's method, with $y(0) = 1, h = 0.025$ at $x = 1$.	CO4	2.1.2	U, A
6	**Solve** $\frac{dy}{dx} = x + \sqrt{y}$ by **using** Euler's method, with $y(2) = 4, h = 0.2$ at $x = 3$.	CO4	2.2.1	U, A
7	**Solve** $\frac{dy}{dx} = \frac{y-x}{y+x}$ by **using** Runge Kutta method, with $y(0) = 1, h = 0.2$ at $x = 2.2$.	CO4	2.2.4	U, A
8	**Solve** $\frac{dy}{dx} = xy^2$ by **using** Runge Kutta method with $y(2) = 1$ for $x = 2.2$ taking $h = 0.2$.	CO4	1.3.1	U, A
9	**Solve** $\frac{dy}{dx} = x - y^2$ by **using** Runge Kutta method with $y(0) = 1$for $x = 0.2$ taking $h = 0.1$.	CO4	1.2.1	U, A
10	**Solve** $\frac{dy}{dx} = x + y$ by **using** Runge Kutta method with $y(0) = 1$ for $x = 2.2$ taking $h = 0.2$.	CO4	1.2.1	U, A

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