

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE-3 SEMESTER – OLD PAPER – W19 TO S25 – QUESTION BANK**

**Subject Name & Code:**

**Advance Engineering Mathematics (2130002)**

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**Unit 1 – Fourier Series and Fourier Integral**

**Repeated Questions:**

1. Find the Fourier series expansion of  $f(x) = x^2$  in  $-\pi \leq x \leq \pi$ .  
○ Appeared in: S-22 (Q1c), S-20 (Q2b, 04 marks), W-23 (Q1c), S-24 (Q1c, 07 marks)
  2. Find the Fourier series of  $f(x) = x + |x|$ ,  $-\pi < x < \pi$ .  
○ Appeared in: W-24 (Q2c), W19 (Q3c-OR, 07 marks)
  3. Find the Fourier series of  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ .  
○ Appeared in: W-22 (Q2c-OR), W-21 (Q2d)
  4. Find the half-range cosine series for  $f(x) = (x - 1)^2$ ,  $0 < x < 1$ .  
○ Appeared in: W-24 (Q4b-OR, 04 marks)
  5. Find the Fourier integral representation of  $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ .  
○ Appeared in: S-22 (Q4c, 07 marks)
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**Other Important Questions:**

1. Find a Fourier series for a periodic function  $f(x)$  with period 2, where  $f(x) = -1$ ,  $-1 < x < 0$ ;  $f(x) = 1$ ,  $0 < x < 1$ .  
○ W-24 (Q1c, 07 marks)
2. Find the Fourier series of  $f(x) = x + x^2$  in  $-\pi < x < \pi$ .  
○ W-22 (Q2c, 07 marks)
3. Find the Fourier series of  $f(x) = 2x - x^2$  with period 3 in (0,3).  
○ W-22 (Q4c, 07 marks)
4. Find the Fourier series expansion of  $f(x) = x^2$ ;  $-2 \leq x \leq 2$ .  
○ S-23 (Q1b, 07 marks)
5. Find the Fourier series of  $f(x) = \begin{cases} x^2, & x \in (0, \pi) \\ 0, & x \in (\pi, 2\pi) \end{cases}$ .  
○ S-23 (Q2b-OR, 07 marks)
6. Expand  $f(x) = \frac{1}{2}(\pi - x)$  as a Fourier series in  $(0, 2\pi)$ .  
○ W-21 (Q1c)

7. **Find the half-range sine series of  $f(x) = e^x$  in  $0 < x < \pi$ .**
  - W-21 (Q4a)
8. **Find the half-range sine series of  $f(x) = x^2$  in  $(0, \pi)$ .**
  - S-22 (Q3b-OR, 04 marks)
9. **Find Fourier sine series of  $f(x) = \pi - x, 0 < x < \pi$ .**
  - S-23 (Q4a-ii, 04 marks)
10. **Find cosine series for  $f(x) = e^x$  in  $0 < x < L$ .**
  - S-20 (Q4b-OR, 04 marks)
11. **Find cosine series for  $f(x) = x^2$  in  $0 < x < \pi$ .**
  - W19 (Q4b-OR)
12. **Find a cosine series of period  $2\pi$  to represent  $f(x) = \sin x$  in  $0 < x < \pi$ .**
  - S-24 (Q3a-OR)
13. **Find the Fourier Sine series of  $f(x) = 2x$  in  $0 < x < 1$ .**
  - S-25 (Q4a)
14. **Find the Fourier Cosine integral of  $f(x) = \frac{\pi}{2}e^{-x}, x \geq 0$ .**
  - S-25 (Q4a-OR)
15. **Find the Fourier integral representation of  $f(x) = 1$  for  $|x| < 1, 0$  otherwise.**
  - W-23 (Q1b)
16. **Obtain Fourier series of  $f(x) = \begin{cases} 0, & -2 \leq x < 0 \\ 1, & 0 \leq x \leq 2 \end{cases}$ .**
  - W19 (Q3c)
17. **Obtain Fourier series of  $f(x) = x + |x|, -\pi < x < \pi$ .**
  - W19 (Q3c-OR, 07 marks)

## Practice Numerical:

Sr. No	Question	CO	PI	B.T. level
1	Find the fundamental period $P$ of (i) $\cos \frac{2\pi x}{k}$ (ii) $\cos \frac{2\pi nx}{k}$	CO1	2.1.1	U, A
2	Find the Fourier series of the function $f(x)$ which is assumed to have the period $2\pi$ , where $f(x)$ is as follows (i) $f(x) = x, -\pi < x < \pi$ (ii) $f(x) = x^2, 0 < x < 2\pi$ (iii) $f(x) = x +  x , -\pi < x < \pi$	CO1	2.1.2	U, A
3	Find the Fourier series of the function $f(x)$ which is assumed to have the period $2\pi$ , where $f(x)$ is as follows (i) $f(x) = \begin{cases} 1 & \text{if } -\pi < x < 0 \\ -1 & \text{if } 0 < x < \pi \end{cases}$ (ii) $f(x) = \begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$	CO1	2.1.3	U, A
4	Find the Fourier series of the periodic function $f(x)$ of period $p = 2l$ , (i) $f(x) = \begin{cases} -1 & \text{if } -1 < x < 0 \\ 1 & \text{if } 0 < x < 1 \end{cases}$ (ii) $f(x) = \begin{cases} (1/2) + x & \text{if } -(1/2) < x < 0 \\ (1/2) - x & \text{if } 0 < x < (1/2) \end{cases}$	CO1	1.1.1	U, A
5	Check whether the following functions are odd or even. (i) $ x^3 $ (ii) $x^2 \cos nx$ (iii) $x x $ (iv) $e^x$ (v) $\ln x$	CO1	2.1.2	U, A
6	Obtain the Fourier Series of the function $f(x) = \cos x, \text{ for } -\pi < x < 0$ $= \sin x, \text{ for } 0 < x < \pi$	CO1	1.2.1	U, A
7	Find the Fourier Series of $f(x) = x +  x , -\pi < x < \pi$	CO1	2.1.2	U, A
8	Find Fourier Cosine Series of $f(x) = e^x, 0 < x < L$	CO1	1.1.1	U, A
9	Find Half range sine series of $f(x) = \begin{cases} 2x & , 0 < x < 1 \\ -2x & , 1 < x < 2 \end{cases}$	CO1	1.1.2	U, A
10	Find Fourier Series for the function $f(x) = x -  x , -\pi < x < 0$ $= x +  x , 0 < x < \pi$ Also show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$	CO1	2.1.1	U, A

## Unit 2 – Partial Differential Equations and Applications

### Repeated Questions:

- Solve  $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$  given  $u(x, 0) = 4e^{-4x}$  by separation of variables.  
○ Appeared in: S-22 (Q5c, 07 marks), W-24 (Q5c), S-24 (Q5c)
- Solve  $(D^2 + DD' - 6D'^2)z = \sin(2x + y)$ .  
○ Appeared in: S-22 (Q5c-OR, 07 marks), W-23 (Q5c)
- Solve  $z = pq$  by Charpit's method.  
○ Appeared in: W-24 (Q5c), W-21 (Q5c), S-23 (Q5a-ii-OR)
- Form PDE by eliminating arbitrary function from  $z = f(x/y)$ .  
○ Appeared in: S-25 (Q3b), W-22 (Q5a-OR)
- Solve  $p^2 + q^2 = x + y$ .  
○ Appeared in: S-22 (Q5a-OR, 03 marks), W-23 (Q5b-i)

### Other Important Questions:

#### Formation of PDEs:

- Form a PDE by eliminating constants  $a$  and  $b$  from  $z = (x - a)^2 + (y - b)^2$ .  
○ S-23 (Q5a-i)
- Form a PDE by eliminating constants  $a$  and  $b$  from  $z = (x^2 + a)(y^2 + b)$ .  
○ W-24 (Q5b)
- Form a PDE from  $z = f(xy + z^2, x + y + z) = 0$ .  
○ S-25 (Q3b-OR), W19 (Q5a-OR)
- Form a PDE by eliminating function from  $\theta(x^2 - y^2, xyz) = 0$ .  
○ W-24 (Q5a-OR)
- Form PDE from  $z = f(x + ct) + g(x - ct)$ .  
○ W-23 (Q5a-OR)
- Form PDE from  $az + b = a^2x + y$ .  
○ S-24 (Q5a)
- Form PDE from  $z = (x - 2)^2 + (y - 3)^2$ .  
○ S-22 (Q5a, 03 marks)
- Form PDE from  $z = (x - 3)^2 + (y - 4)^2$ .  
○ W-21 (Q5a)

#### First Order PDEs:

- Solve  $(\pi z - ny)p + (\pi x - lz)q = ly - mx$ .  
○ S-22 (Q4b-OR, 04 marks)
- Solve  $x^2p + y^2q = z^2$ .  
○ W-24 (Q5b-OR)
- Solve  $p - x^2 = q + y^2$ .  
○ W-24 (Q5a-II-OR), W-21 (Q5b)
- Solve  $pq = 4z$ .  
○ W-24 (Q5a-I-OR)
- Solve  $pq = 1$ .  
○ S-20 (Q5a, 03 marks)
- Solve  $p(1 + q) = qz$ .  
○ W-22 (Q5b, 04 marks), S-25 (Q5a-OR)
- Solve  $p^2 - x = q^2 - y$ .  
○ S-20 (Q5b, 04 marks)
- Solve  $z = px + qy + pq$ .  
○ S-24 (Q5a-OR)
- Solve  $z = px + qy + \sqrt{1 + p^2 + q^2}$ .  
○ W-22 (Q5a, 03 marks)

10. Solve  $z = px + qy - 2\sqrt{pq}$ .
  - W-21 (Q5a-OR)
11. Solve  $xp + yq = x - y$ .
  - S-25 (Q5a)
12. Solve  $xp + yq = 3z$ .
  - W-21 (Q5b-OR)
13. Solve  $pz - qz = z^2 + (x + y)^2$ .
  - S-23 (Q5a-ii)
14. Solve  $p + q^2 = 1$ .
  - S-23 (Q5a-i-OR)
15. Solve  $pq = p + q$ .
  - W-23 (Q5b-ii)
16. Solve  $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ .
  - W-23 (Q5a), W19 (Q5c)
17. Solve  $(y + z)p - (x + z)q = x - y$ .
  - S-24 (Q5b)

### Higher Order PDEs & Applications:

1. Solve  $\frac{\partial^2 z}{\partial x^2} + z = 0$  given  $z = e^y, \frac{\partial z}{\partial x} = 1$  when  $x = 0$ .
  - S-22 (Q5b-OR, 04 marks), W-22 (Q5b-OR)
2. Solve  $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 3y$ .
  - W-22 (Q5c, 07 marks)
3. Solve  $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$ .
  - S-25 (Q5b)
4. Solve  $(D^2 + 10DD' + 25D'^2)z = e^{3x+2y}$ .
  - S-24 (Q5b-OR), W-21 (Q4b)
5. Solve  $r - 2s + t = \sin(2x + 3y)$ .
  - S-20 (Q5b, 04 marks)
6. Solve  $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$  with given conditions.
  - S-20 (Q5a-OR)
7. Solve  $\frac{\partial^2 z}{\partial x \partial y} = \cos x \cos y$  (Direct integration).
  - W-23 (Q5b-i-OR)
8. Solve  $(p^2 + q^2)x = pz$ .
  - S-20 (Q5c, 07 marks)
9. Solve  $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x}$ .
  - S-25 (Q5b-OR)
10. Solve  $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$  by separation of variables.
  - S-23 (Q5b-OR)
11. Solve  $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$  with  $u(0, y) = 8e^{-3y}$ .
  - W-21 (Q5c), W-23 (Q5c-OR)
12. Find temperature in a semi-infinite strip (Heat equation application).
  - S-24 (Q5c-OR)
13. Solve  $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$  with  $u(x, 0) = 4e^{-3x}$ .
  - W19 (Q4c)

### Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
<b>Form</b> PDE from following (1 & 2) equations by eliminating arbitrary constants				
1	$z = (x-a)^2 + (y-b)^2$	CO2	1.2.1	U
2	$z = (x^2 + a)(y^2 + b)$	CO2	1.2.1	U
<b>Form</b> PDE from following (3 & 4) equations by eliminating arbitrary Functions				
3	$z = y^2 + 2f\left(\frac{1}{x} + \ln y\right)$	CO2	1.2.1	U
4	$F\left(\frac{x-y}{xy}, \frac{x-y}{z}\right) = 0$	CO2	1.2.1	U
<b>Solve</b> following (5 to 16) Linear Partial Differential Equations (Lagrange's equations).				
5	$(y-z)p + (x-y)q = (z-x)$	CO2	2.4.1	U
6	$z(xp - yq) = y^2 - x^2$	CO2	2.4.1	U
7	$y^2p - xyq = x(z - 2y)$	CO2	2.4.1	U
8	$x(y-z)p + y(z-x)q = z(x-y)$	CO2	2.4.1	U
9	$y^2zp - x^2zq = x^2y$	CO2	2.4.1	U
10	$pz - qz = z^2 + (x+y)^2$	CO2	2.4.1	U
11	$(mz - ny)p + (nx - lz)q = ly - mx$	CO2	2.4.1	U
12	$pz + qz = z^2 + (x-y)^2$	CO2	2.4.1	U
13	$(x^2 + y^2 + z^2)p + zxyq = 2xz$	CO2	2.4.1	U
14	$(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$	CO2	2.4.1	U
<b>Solve</b> following (17 to 11) Non- linear partial differential equations.				
15	$p^2 + q^2 = 1$			
16	$p(1 + q^2) = q(z - 1)$	CO2	2.4.1	U
17	$p(1 + q) = qz$	CO2	2.4.1	U
18	$p^2 - q^2 = x - y$	CO2	2.4.1	U
19	$p^2 - q^2 = x^2 + y^2$	CO2	2.4.1	U
20	$z = px + qy + \frac{p}{q-p}$	CO2	2.4.1	U
<b>Solve</b> following (17 to 11) Non- linear partial differential equations using Charpit's Method.				
21	$px + qy = pq$	CO2	2.4.1	U
22	$(p^2 + q^2)y = qz$	CO2	2.4.1	U
23	$z^2 = pqxy$	CO2	2.4.1	U

Sr. No.	Question	CO	PI	B.T. level
<b>Solve</b> following (1 to 3) second order homogeneous PDEs.				
1	$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 12 \frac{\partial^2 z}{\partial y^2} = 3e^{2x-3y}$	CO2	2.4.1	U
2	$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos(2x - 3y)$	CO2	2.4.1	U
3	$\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = x^2 y^3$	CO2	2.4.1	U
4	Using method of separation of variables solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ given that $u(x, 0) = 6e^{-3x}$ .	CO2	2.4.1	U, A
5	Using method of separation of variables solve $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ given that $u(x, 0) = 4x - \frac{1}{2}x^2$ .	CO2	2.4.1	U, A
6	<b>Discuss</b> all possible solutions for a heat conduction problem in a rod of length $l$ whose ends are held at zero temperature and $f(x)$ is initial temperature distribution.	CO2	2.1.1	R, U
7	<b>Find</b> the temperature in the thin metal rod of length $l$ with both ends insulated and initial temperature is $\sin \frac{\pi x}{l}$ . Use one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ .	CO2	2.4.1	U, A
8	<b>Derive</b> one dimensional wave equation that governs small vibration of an elastic string of length $l$ whose both ends are fixed. Also state physical assumptions that you make for the system.	CO2	2.1.1	R, U
9	A string is stretched between two fixed points and the points of the string are given initial velocity $g(x)$ , where $g(x) = \begin{cases} \frac{2x}{l} & , 0 < x < \frac{l}{2} \\ \frac{2(l-x)}{l} & , \frac{l}{2} < x < l \end{cases}$ , $x$ being the distance from the first end point. <b>Find</b> the displacement of the string at any time. Use wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ .	CO2	2.4.1	U, A
10	A string of length $l = \pi$ has its ends fixed at $x = 0$ and $x = \pi$ . At any time $t = 0$ , the string is given a shape defined by $f(x) = 50x(\pi - x)$ , then it is released. <b>Find</b> the deflection of string at any time $t$ .	CO2	2.4.1	U, A

## Unit 6 – Laplace Transforms and Applications

### Repeated Questions:

1. **Use convolution theorem to find  $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$ .**
    - Appeared in: S-20 (Q1c, 07 marks), S-23 (Q3b, 07 marks), W-22 (Q4c)
  2. **Find  $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$  using convolution theorem.**
    - Appeared in: S-22 (Q3b, 04 marks), W-22 (Q3b, 04 marks)
  3. **Find the Laplace Transform of  $\frac{\cos at - \cos bt}{t}$ .**
    - Appeared in: W-24 (Q4a-I-OR, 03 marks), W-22 (Q4b, 04 marks), W-23 (Q4a)
  4. **Find  $L^{-1}\left\{\frac{e^{-\pi s}}{s^2-2s+2}\right\}$ .**
    - Appeared in: S-22 (Q4a-OR, 03 marks)
  5. **State convolution theorem and apply it to evaluate  $L^{-1}\left[\frac{1}{s(s+a)^3}\right]$ .**
    - Appeared in: W19 (Q2c, 07 marks)
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### Other Important Questions:

1. **Find  $L[(t+1)^2 e^t]$ .**
  - S-22 (Q1b, 04 marks), W-24 (Q3a-OR, 03 marks)
2. **Find  $L^{-1}\left\{\frac{3s+4}{s^2+9}\right\}$ .**
  - S-22 (Q2a, 03 marks)
3. **Find  $L[t^2 \sin \pi t]$ .**
  - W-24 (Q1b, 04 marks)
4. **Find  $L^{-1}\left\{\frac{s^3+2s^2+2}{s^3(s^2+1)}\right\}$ .**
  - W-24 (Q3a, 03 marks)
5. **Find  $L[te^{4t} \cos 2t]$ .**
  - S-25 (Q3a-OR)
6. **Find  $L[t^2 \cos at]$ .**
  - S-23 (Q3a-i, 03 marks)
7. **Find  $L[\sin(3t+2)]$ .**
  - W-22 (Q3a, 03 marks)
8. **Find  $L[t \sinh 3t]$ .**
  - W-22 (Q3a-OR, 03 marks)
9. **Find  $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$ .**
  - W-22 (Q3b-OR, 04 marks)
10. **Find  $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$ .**
  - S-23 (Q4a-i-OR)

11. Find  $L[\int_0^t e^{-u} \cos u \, du]$ .
  - S-23 (Q4a-i, 03 marks), W19 (Q5b-i)
12. Find  $L[t \int_0^t e^{-4t} \sin 3t \, dt]$ .
  - W-23 (Q2a)
13. Find  $L[\int_0^t \int_0^s \sin at \, dt \, dt]$ .
  - W-21 (Q3a)
14. Find  $L[f(t)]$  where  $f(t) = \begin{cases} t^2, & 0 < t < 1 \\ 4t, & t > 1 \end{cases}$ .
  - S-22 (Q5b, 04 marks)
15. Find  $L[e^{-2t}(\sin 4t + t^2)]$ .
  - W-24 (Q4b, 04 marks)
16. Find  $L[e^{-t} \sin^2 t]$ .
  - S-20 (Q4a, 03 marks)
17. Find  $L[\cosh(kt) \cos kt]$ .
  - S-24 (Q4b)
18. Find  $L[t^3 + e^{-3t} + t^{3/2}]$ .
  - S-24 (Q4a, 03 marks)
19. Find  $L[t^2 u(t-2)]$ .
  - W-24 (Q4a-II-OR, 03 marks)
20. Find  $L[f(t)]$  if  $f(t) = f(t+2\pi)$  and  $f(t) = e^t$  in one period.
  - W-23 (Q4a-OR)
21. Find inverse Laplace:  $\frac{2s+3}{(s+2)(s+1)^2}$ .
  - S-24 (Q4c-OR)
22. Find inverse Laplace:  $\frac{s+2}{s^2(s+3)}$  using convolution theorem.
  - S-25 (Q1c)
23. Solve IVP using Laplace:  $y'' + 3y' + 2y = e^t, y(0) = 1, y'(0) = 0$ .
  - S-25 (Q3a)
24. Solve IVP using Laplace:  $y'' + y' = t^2 + 2t, y(0) = 4, y'(0) = -2$ .
  - S-22 (Q3c-OR, 07 marks)
25. Solve IVP using Laplace:  $y'' + 4y = 0, y(0) = 1, y'(0) = 6$ .
  - S-24 (Q4c-OR)
26. Solve using Laplace:  $y' + 2y = e^{-3t}, y(0) = 1$ .
  - W-21 (Q4c)
27. Solve using Laplace:  $y'' + 6y' = 1, y(0) = 2, y'(0) = 0$ .
  - W-23 (Q2c), W19 (Q2c-OR, 07 marks)

## Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
<b>Using</b> Appropriate rule <b>find</b> Laplace Transforms of following functions (1 to 6)				
1	$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 3 \\ 4 & \text{if } t \geq 3 \end{cases}$	CO5	2.1.1	R,U
2	(i) $e^{2t+3}$ (ii) $\cos^2 t$ (iii) $t^5 + \cos 5t + e^{-100t}$ (iv) $e^{4t} \sin 2t \cos t$	CO5	2.4.1	U
3	(i) $\int_0^t e^{-u} \cos u du$ (ii) $\int_0^t \int_0^t \sin au du du$	CO5	2.4.1	U
4	(i) $t^2 \cos^2 2t$ (ii) $t \sin 3t \cos 2t$ (iii) $te^t \sin t$	CO5	2.4.1	U, A
5	(i) $\frac{1 - \cos 2t}{t}$ (ii) $\frac{e^{-bt} - e^{-at}}{t}$	CO5	2.4.1	U, A
6	(i) $e^{-3t} u(t-2)$ (ii) $t^2 u(t-2)$	CO5	2.4.1	U,A
<b>Using</b> Appropriate rule <b>find</b> Inverse Laplace Transforms of following functions (7 to 11)				
7	(i) $\frac{s+7}{s^2+8s+25}$ (ii) $\frac{s}{(s+1)^2+9}$ (iii) $\frac{s}{s^2-3s+2}$	CO5	2.4.1	U
8	(i) $\frac{1}{s(s^2-3s+3)}$ (ii) $\frac{1}{s(s^2+4)}$	CO5	2.4.1	U,A
9	(i) $\ln \left( \frac{s^2+a^2}{s^2+b^2} \right)$ (ii) $\tan^{-1} \left( \frac{2}{s} \right)$	CO5	2.4.1	U,A
10	$\frac{s+2}{(s^2+4s+5)^2}$	CO5	2.4.1	U, A
11	(i) $\frac{e^{-2\pi s} - e^{-8\pi s}}{s^2+1}$ (ii) $\frac{e^{-3s}}{s^2+8s+25}$	CO5	2.4.1	U, A
12	<b>Define</b> Unit step function and $f(t) = \begin{cases} 2, & 0 < t < 1 \\ t^2/2, & 1 < t < \pi/2 \\ \cos t, & t > \pi/2 \end{cases}$ <b>Find</b> Laplace transform of	CO5	2.4.1	R, A

Sr. No.	Question	CO	PI	B.T. level
1	<b>Find</b> Laplace transform of periodic function $f(t) = \frac{2t}{3}; 0 \leq t \leq 3; f(t+3) = f(t)$	CO5	2.4.1	R,U
2	<b>Find</b> convolution of $t$ and $e^t$ .	CO5	2.4.1	U
3	<b>Find</b> $L^{-1}\left(\frac{2\pi s}{(s^2 + \pi^2)^2}\right)$ using Convolution theorem.	CO5	2.4.1	U
4	<b>Find</b> Inverse Laplace Transform of following using Convolution theorem (i) $\frac{3s^2 + 2}{(s+1)(s+2)(s+3)}$ (ii) $\frac{4s+5}{(s-1)^2(s+2)}$ (iii) $\frac{s^2}{(s^2 + 25)(s^2 + 49)}$	CO5	2.4.1	U, A
5	<b>Solve</b> the initial value problem $y'' - y' - 2y = 0, y(0) = 1, y'(0) = 0$ using Laplace transform.	CO5	2.4.1	U,A
6	<b>Using</b> the Laplace transforms, <b>find</b> the solution of the initial value problem $y'' + 25y = 10 \cos 5t, y(0) = 2, y'(0) = 0$ .	CO5	2.4.1	U,A
7	<b>Solve</b> $\frac{dy}{dt} - 4y = 2e^{2t} + e^{4t}$ by Laplace transformation	CO5	2.4.1	U,A
8	<b>Solve</b> the IVP $\dot{y} + 2\dot{y} + y = e^{-t}, y(0) = -1, y'(0) = 1$ using Laplace transform	CO5	2.4.1	U
9	Define Dirac's delta function and find the solution of $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 1 + \delta(t - 4), x(0) = x(0)' = 0$	CO5	2.4.1	R,A
10	<b>Solve</b> the system of ODEs $\frac{dx}{dt} + y = 1, x + \frac{dy}{dt} = 2$ where $x(0) = 1, y(0) = 1$ .	CO5	2.4.1	U,A
11	<b>Solve</b> $y'' - ty' + y = 1, y(0) = 1, y'(0) = 2$ by Laplace transformation	CO5	2.4.1	U,A
12	<b>Solve</b> $ty'' + (1 - 2t)y' - 2y = 0, y(0) = 1, y'(0) = 2$ by Laplace transformation	CO5	2.4.1	U,A

### Unit 3 – Finite Differences and Interpolation

#### Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level																		
1	Define the operators, $\Delta, \nabla, \delta$ and $E, E^{-1}$ and show that (i) $\Delta \equiv E\nabla$ (ii) $E = 1 + \Delta$ (iii) $\nabla = E^{-1}\Delta$ (iv) $E^{-1} \equiv 1 - \nabla$	CO3	2.4.1	R, U																		
2	Obtain the estimate of missing terms in the following table: <table border="1" style="margin-left: 20px;"> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>Y</td> <td>2</td> <td>4</td> <td>8</td> <td>-</td> <td>32</td> <td>-</td> <td>128</td> <td>256</td> </tr> </table>	X	1	2	3	4	5	6	7	8	Y	2	4	8	-	32	-	128	256	CO3	2.4.2	U, A
X	1	2	3	4	5	6	7	8														
Y	2	4	8	-	32	-	128	256														
3	Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial polynomial and, hence, show that $\Delta^3 f(x) = 12$ .	CO3	2.4.4	U, A																		
4	Find cosh 0.56 from Newton's forward difference interpolation formula and Estimate the error for the following data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0.5</td> <td>0.6</td> <td>0.7</td> <td>0.8</td> </tr> <tr> <td>Cos hx</td> <td>1.127626</td> <td>1.185465</td> <td>1.255169</td> <td>1.337435</td> </tr> </table>	x	0.5	0.6	0.7	0.8	Cos hx	1.127626	1.185465	1.255169	1.337435	CO3	1.1.1	U, A								
x	0.5	0.6	0.7	0.8																		
Cos hx	1.127626	1.185465	1.255169	1.337435																		
5	Using Newton's forward interpolation formula, find the value of $f(5)$ if <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>12</td> </tr> <tr> <td>f(x)</td> <td>1</td> <td>3</td> <td>8</td> <td>16</td> <td>20</td> </tr> </table>	x	4	6	8	10	12	f(x)	1	3	8	16	20	CO3	1.1.2	U, A						
x	4	6	8	10	12																	
f(x)	1	3	8	16	20																	
6	Using Newton's forward interpolation formula, Find $\sin 52^\circ$ from following data <table border="1" style="margin-left: 20px;"> <tr> <td><math>\sin 45^\circ</math></td> <td><math>\sin 50^\circ</math></td> <td><math>\sin 55^\circ</math></td> <td><math>\sin 60^\circ</math></td> </tr> <tr> <td>0.7071</td> <td>0.7660</td> <td>0.8192</td> <td>0.8660</td> </tr> </table>	$\sin 45^\circ$	$\sin 50^\circ$	$\sin 55^\circ$	$\sin 60^\circ$	0.7071	0.7660	0.8192	0.8660	CO3	1.2.1	U, A										
$\sin 45^\circ$	$\sin 50^\circ$	$\sin 55^\circ$	$\sin 60^\circ$																			
0.7071	0.7660	0.8192	0.8660																			
7	Find $y^{(300)}$ from the given values using Newton's backward Interpolation formula. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>50</td> <td>100</td> <td>150</td> <td>200</td> <td>250</td> </tr> <tr> <td>y</td> <td>618</td> <td>724</td> <td>805</td> <td>906</td> <td>1032</td> </tr> </table>	x	50	100	150	200	250	y	618	724	805	906	1032	CO3	1.3.1	U, A						
x	50	100	150	200	250																	
y	618	724	805	906	1032																	
8	The area A of a circle of diameter d is given for the following values: <table border="1" style="margin-left: 20px;"> <tr> <td>d</td> <td>80</td> <td>85</td> <td>90</td> <td>95</td> <td>100</td> </tr> <tr> <td>A</td> <td>5026</td> <td>5674</td> <td>6362</td> <td>7088</td> <td>7854</td> </tr> </table> Find the area of a circle of a diameter of 105 units using Newton's backward Interpolation formula.	d	80	85	90	95	100	A	5026	5674	6362	7088	7854	CO3	1.4.1	U, A						
d	80	85	90	95	100																	
A	5026	5674	6362	7088	7854																	
9	From the following data, Find the profit in the year 1925. <table border="1" style="margin-left: 20px;"> <tr> <td>Year</td> <td>1891</td> <td>1901</td> <td>1911</td> <td>1921</td> <td>1931</td> </tr> <tr> <td>Profit in the lakhs</td> <td>46</td> <td>66</td> <td>81</td> <td>93</td> <td>101</td> </tr> </table>	Year	1891	1901	1911	1921	1931	Profit in the lakhs	46	66	81	93	101	CO3	1.2.1	U, A						
Year	1891	1901	1911	1921	1931																	
Profit in the lakhs	46	66	81	93	101																	
10	Find P when $t = 175^\circ\text{C}$ from the given table, using Newton's backward Interpolation formula <table border="1" style="margin-left: 20px;"> <tr> <td>Temperature <math>t^\circ\text{C}</math></td> <td>140</td> <td>150</td> <td>160</td> <td>170</td> <td>180</td> </tr> <tr> <td>Pressure P</td> <td>3685</td> <td>4845</td> <td>6302</td> <td>8076</td> <td>10225</td> </tr> </table>	Temperature $t^\circ\text{C}$	140	150	160	170	180	Pressure P	3685	4845	6302	8076	10225	CO3	1.4.1	U, A						
Temperature $t^\circ\text{C}$	140	150	160	170	180																	
Pressure P	3685	4845	6302	8076	10225																	

Sr. No.	Question	CO	PI	B.T. level																								
1	<p>Find <math>f(6)</math> from the following data, by using Newton's divided difference interpolation formula:</p> <table border="1"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>7</td> <td>8</td> </tr> <tr> <td>f(x)</td> <td>1</td> <td>5</td> <td>5</td> <td>4</td> </tr> </table>	x	1	2	7	8	f(x)	1	5	5	4	CO3	1.2.1	R, U														
x	1	2	7	8																								
f(x)	1	5	5	4																								
2	<p>Compute <math>f(4)</math>, from the tabular values given:</p> <table border="1"> <tr> <td>x</td> <td>2</td> <td>3</td> <td>5</td> <td>7</td> </tr> <tr> <td>f(x)</td> <td>0.1506</td> <td>0.3001</td> <td>0.4517</td> <td>0.6259</td> </tr> </table> <p>using Lagrange's interpolation formula.</p>	x	2	3	5	7	f(x)	0.1506	0.3001	0.4517	0.6259	CO3	2.1.2	U, A														
x	2	3	5	7																								
f(x)	0.1506	0.3001	0.4517	0.6259																								
3	<p>Find the polynomial which takes the values <math>f(1) = 1</math>, <math>f(2) = 9</math>, <math>f(3) = 5</math>, <math>f(4) = 55</math>, <math>f(5) = 105</math>.</p>	CO3	1.1.2	U, A																								
4	<p>Calculate <math>e^{1.85}</math> from the following table:</p> <table border="1"> <tr> <td>x</td> <td>1.7</td> <td>1.8</td> <td>1.9</td> <td>2</td> <td>2.1</td> <td>2.2</td> <td>2.3</td> </tr> <tr> <td><math>e^x</math></td> <td>5.47</td> <td>6.05</td> <td>6.68</td> <td>7.38</td> <td>8.16</td> <td>9.02</td> <td>9.97</td> </tr> <tr> <td></td> <td>4</td> <td>0</td> <td>6</td> <td>9</td> <td>6</td> <td>5</td> <td>4</td> </tr> </table>	x	1.7	1.8	1.9	2	2.1	2.2	2.3	$e^x$	5.47	6.05	6.68	7.38	8.16	9.02	9.97		4	0	6	9	6	5	4	CO3	2.1.2	U, A
x	1.7	1.8	1.9	2	2.1	2.2	2.3																					
$e^x$	5.47	6.05	6.68	7.38	8.16	9.02	9.97																					
	4	0	6	9	6	5	4																					
5	<p>The values of specific heat (<math>C_p</math>) at constant pressure of a gas are tabulated below for various temperatures. Find the specific heat at <math>900^\circ\text{C}</math>.</p> <table border="1"> <tr> <td>Temperature <math>^\circ\text{C}</math></td> <td>500</td> <td>1000</td> <td>1500</td> <td>2000</td> </tr> <tr> <td><math>C_p</math></td> <td>31.23</td> <td>35.01</td> <td>39.18</td> <td>43.75</td> </tr> </table>	Temperature $^\circ\text{C}$	500	1000	1500	2000	$C_p$	31.23	35.01	39.18	43.75	CO3	1.4.1	U, A														
Temperature $^\circ\text{C}$	500	1000	1500	2000																								
$C_p$	31.23	35.01	39.18	43.75																								
6	<p>Compute <math>f(10.5)</math>, from the given values by using Newton's divided difference Interpolation formula</p> <table border="1"> <tr> <td>x</td> <td>10</td> <td>11</td> <td>13</td> <td>17</td> </tr> <tr> <td>f(x)</td> <td>2.3026</td> <td>2.3979</td> <td>2.5649</td> <td>2.8332</td> </tr> </table>	x	10	11	13	17	f(x)	2.3026	2.3979	2.5649	2.8332	CO3	2.1.2	U, A														
x	10	11	13	17																								
f(x)	2.3026	2.3979	2.5649	2.8332																								
7	<p>Find x corresponding to <math>y = 85</math> from the following table:</p> <table border="1"> <tr> <td>x</td> <td>2</td> <td>5</td> <td>8</td> <td>14</td> </tr> <tr> <td>y</td> <td>94.8</td> <td>87.9</td> <td>81.3</td> <td>68.7</td> </tr> </table>	x	2	5	8	14	y	94.8	87.9	81.3	68.7	CO3	1.3.1	U, A														
x	2	5	8	14																								
y	94.8	87.9	81.3	68.7																								
8	<p>Develop Newton's divided difference polynomial for the following data. <math>\ln 9.0 = 2.1972</math>, <math>\ln 9.5 = 2.2513</math>, <math>\ln 11.0 = 2.3979</math>.</p>	CO3	2.4.2	U, A																								
9	<p>Find <math>f(9.2)</math> from the given values using Newton's divided difference Interpolation formula.</p> <table border="1"> <tr> <td>x</td> <td>8.0</td> <td>9.0</td> <td>9.5</td> <td>11.0</td> </tr> <tr> <td>f(x)</td> <td>2.079442</td> <td>2.197225</td> <td>2.251292</td> <td>2.397895</td> </tr> </table>	x	8.0	9.0	9.5	11.0	f(x)	2.079442	2.197225	2.251292	2.397895	CO3	1.3.1	U, A														
x	8.0	9.0	9.5	11.0																								
f(x)	2.079442	2.197225	2.251292	2.397895																								
10	<p>Find x given <math>y = 0.3887</math> from the following data:</p> <table border="1"> <tr> <td>x</td> <td>21</td> <td>23</td> <td>25</td> </tr> <tr> <td>y</td> <td>0.3706</td> <td>0.4068</td> <td>0.4433</td> </tr> </table>	x	21	23	25	y	0.3706	0.4068	0.4433	CO3	2.1.1	U, A																
x	21	23	25																									
y	0.3706	0.4068	0.4433																									

## Unit 4 – Numerical Differentiation and Integration

### Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level																
1	Write derivatives formulae using forward difference and backward difference.	CO4	1.1.1	R																
2	Given that <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>1.0</td> <td>1.1</td> <td>1.2</td> <td>1.3</td> <td>1.4</td> <td>1.5</td> <td>1.6</td> </tr> <tr> <td>y</td> <td>7.989</td> <td>8.403</td> <td>8.781</td> <td>9.129</td> <td>9.451</td> <td>9.750</td> <td>10.031</td> </tr> </table> Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ .	x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	y	7.989	8.403	8.781	9.129	9.451	9.750	10.031	CO4	1.1.2	U
x	1.0	1.1	1.2	1.3	1.4	1.5	1.6													
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031													
3	Given that <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>1.0</td> <td>1.1</td> <td>1.2</td> <td>1.3</td> <td>1.4</td> <td>1.5</td> <td>1.6</td> </tr> <tr> <td>y</td> <td>7.989</td> <td>8.403</td> <td>8.781</td> <td>9.129</td> <td>9.451</td> <td>9.750</td> <td>10.031</td> </tr> </table> Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.6$ .	x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	y	7.989	8.403	8.781	9.129	9.451	9.750	10.031	CO4	1.1.2	U
x	1.0	1.1	1.2	1.3	1.4	1.5	1.6													
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031													
4	Compute $f'(2.2)$ , using appropriate method from the following data: <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>1.4</td> <td>1.6</td> <td>1.8</td> <td>2.0</td> <td>2.2</td> </tr> <tr> <td>f(x)</td> <td>4.0552</td> <td>4.9530</td> <td>6.0496</td> <td>7.3981</td> <td>9.0250</td> </tr> </table>	x	1.4	1.6	1.8	2.0	2.2	f(x)	4.0552	4.9530	6.0496	7.3981	9.0250	CO4	1.1.2	U				
x	1.4	1.6	1.8	2.0	2.2															
f(x)	4.0552	4.9530	6.0496	7.3981	9.0250															
5	Given that <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>0</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> </tr> <tr> <td>y</td> <td>1.5708</td> <td>1.5738</td> <td>1.5828</td> <td>1.5981</td> <td>1.62</td> </tr> </table> Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$ .	x	0	5	10	15	20	y	1.5708	1.5738	1.5828	1.5981	1.62	CO4	1.4.1	U, A				
x	0	5	10	15	20															
y	1.5708	1.5738	1.5828	1.5981	1.62															
6	Using the backward differences, <b>compute</b> the derivative of the tabulated function given below at $x = 0.4$ . <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>0.1</td> <td>0.2</td> <td>0.3</td> <td>0.4</td> </tr> <tr> <td>y</td> <td>1.10517</td> <td>1.22140</td> <td>1.34986</td> <td>1.49182</td> </tr> </table>	x	0.1	0.2	0.3	0.4	y	1.10517	1.22140	1.34986	1.49182	CO4	2.1.2	U, A						
x	0.1	0.2	0.3	0.4																
y	1.10517	1.22140	1.34986	1.49182																

Sr. No.	Question	CO	PI	B.T. level
1	<b>Evaluate</b> $\int_0^{\frac{\pi}{2}} e^{\sin \theta} d\theta$ by <b>using</b> Simpson's 3/8 rule, dividing the interval $\left[0, \frac{\pi}{2}\right]$ into six equal parts.	CO4	2.1.1	U, A
2	<b>Evaluate</b> $\int_1^2 \frac{dx}{1+x^2}$ taking $h = 0.2$ , <b>using</b> trapezoidal rule.	CO4	2.1.2	U, A
3	<b>Evaluate</b> $\int_0^6 \frac{dx}{1+x^2}$ by <b>using</b> (i) trapezoidal rule, (ii) Simpson's 1/3 rule, (iii) Simpson's 3/8 rule.	CO4	2.1.3	U, A
4	<b>Evaluate</b> $\int_4^{5.2} \log x dx$ <b>using</b> the trapezoidal rule and Simpson's 3/8 rule take $h=0.2$ .	CO4	1.1.1	U, A
5	<b>Evaluate</b> $\int_0^1 \left(1 + \frac{\sin x}{x}\right) dx$ correct to three decimal places, by <b>using</b> (i) trapezoidal rule, (ii) Simpson's 1/3 rule, (iii) Simpson's 3/8 rule.	CO4	2.1.2	U, A
6	<b>Compute</b> the value of $\int_{0.2}^{1.4} (\sin \sin x - \log \log x + e^x) dx$ taking $h = 0.2$ and <b>using</b> the trapezoidal rule and Simpson's rule.	CO4	1.2.1	U, A
7	<b>Evaluate</b> $\int_{0.5}^{0.7} \sqrt{x} e^{-x} dx$ <b>using</b> Simpson's 3/8 rule.	CO4	2.1.2	U, A
8	<b>Evaluate</b> $\int_0^{\frac{\pi}{2}} \sin x dx$ by the two-point Gaussian formula.	CO4	1.1.1	U, A
9	<b>Evaluate</b> $\int_0^1 e^{-x^2} dx$ by using the Gaussian quadrature formula with $n = 3$ .	CO4	1.1.2	U, A
10	<b>Evaluate</b> $\int_0^{\frac{\pi}{2}} \log(1+x) dx$ by the two-point Gaussian formula.	CO4	2.1.1	U, A

## Unit 5 – Numerical Solution of Ordinary Differential Equations

### Practice Numerical:

Sr. No.	Question	CO	PI	B.T. level
1	Using the Picard's method, find $y(1.1)$ correct to four decimal places given that $\frac{dy}{dx} = xy^{\frac{1}{3}}$ , $y(1) = 1, h = 0.1$ .	CO4	2.1.1	U, A
2	Solve $\frac{dy}{dx} = x + y$ by Picard's method. Start from $x = 1, y = 0$ and carry to $x = 1.2$ with $h = 0.1$ .	CO4	2.1.2	U, A
3	Solve $\frac{dy}{dx} = x^2 + y^2$ by the Picard's method, with $y(0) = 0$ at $x = 0.4$ .	CO4	2.2.1	U, A
4	Solve $\frac{dy}{dx} = xy$ by using Euler's method, with $y(0) = 2, h = 0.2$ at $x = 1$ .	CO4	2.1.1	U, A
5	Solve $\frac{dy}{dx} = x + y + xy$ by using Euler's method, with $y(0) = 1, h = 0.025$ at $x = 1$ .	CO4	2.1.2	U, A
6	Solve $\frac{dy}{dx} = x + \sqrt{y}$ by using Euler's method, with $y(2) = 4, h = 0.2$ at $x = 3$ .	CO4	2.2.1	U, A
7	Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ by using Runge Kutta method, with $y(0) = 1, h = 0.2$ at $x = 2.2$ .	CO4	2.2.4	U, A
8	Solve $\frac{dy}{dx} = xy^2$ by using Runge Kutta method with $y(2) = 1$ for $x = 2.2$ taking $h = 0.2$ .	CO4	1.3.1	U, A
9	Solve $\frac{dy}{dx} = x - y^2$ by using Runge Kutta method with $y(0) = 1$ for $x = 0.2$ taking $h = 0.1$ .	CO4	1.2.1	U, A
10	Solve $\frac{dy}{dx} = x + y$ by using Runge Kutta method with $y(0) = 1$ for $x = 2.2$ taking $h = 0.2$ .	CO4	1.2.1	U, A

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