GUJARAT TECHNOLOGICAL UNIVERSITY

BE-3 SEMESTER – OLD PAPER – S22 TO W25 – QUESTION BANK ANSWER

Subject Name & Code:
FLUID FLOW OPERATIONS (3130502)

(**Disclaimer:** The purpose of these AI-generated responses is just education and reference. Utilise them to grasp topics and structure, but always rewrite in your own words and double-check)

Unit 2: Fluid Dynamics – Continuity, Bernoulli, Manometers, Momentum

Q2 – Derive the equation of continuity. (7 marks)
Appeared in: S23 (Q2c OR, 07 marks) → Highest: 7 marks
Ans:
Definition: Continuity equation states that **mass flow rate remains constant** along a stream tube in steady flow (mass is conserved).
Assumptions:

- Steady flow ($\partial\rho/\partial t = 0$)
- No sources or sinks
- Flow is one-dimensional across cross-section

Derivation (Control volume approach):
Consider a stream tube with varying cross-section area A_1 and A_2, velocities V_1 and V_2, densities ρ_1 and ρ_2.
Step 1 – Mass entering at section 1 per unit time:
$\dot{m}_1 = \rho_1 A_1 V_1$
Step 2 – Mass leaving at section 2 per unit time:
$\dot{m}_2 = \rho_2 A_2 V_2$
Step 3 – For steady flow, mass inflow = mass outflow:
$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
Step 4 – General form (differential):
Take infinitesimal control volume of length dx, area A.
Net mass flux = $\partial(\rho AV)/\partial x \cdot dx = 0$
$\Rightarrow \partial(\rho AV)/\partial x = 0$
For **incompressible flow** (ρ constant):
$A_1 V_1 = A_2 V_2$ or $Q = A\ V =$ constant
Rectangular coordinates (3D form):
$\partial\rho/\partial t + \partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z = 0$
For steady incompressible:
$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$
[DG PROMPT]
Title: Continuity Equation – Stream tube
Description: Draw a converging tube. Left section label (1) with area A_1, velocity V_1, density ρ_1. Right section (2) with A_2, V_2, ρ_2. Dashed streamlines along boundaries. Arrow from left to right. Write equation $\rho_1A_1V_1 = \rho_2A_2V_2$ below. No dimensional proportions needed.
Real-world application:

- Pipe flow calculations – finding velocity at reduced diameter

- Hydraulic systems – designing nozzle diameters
- Cardiovascular flow – blood velocity in arteries

Numerical example:
Water flows in pipe: $A_1 = 0.01\ m^2$, $V_1 = 2$ m/s, $A_2 = 0.005\ m^2$. Find V_2.
Solution: $A_1V_1 = A_2V_2 \rightarrow 0.01\times2 = 0.005\times V_2 \rightarrow V_2 = 4$ m/s.

Q3(a) – Define mass velocity and average velocity relation. (7 marks)
Appeared in: S24 (Q3c, 07 marks) → Highest: 7 marks
Ans:
Definitions:

- **Average velocity ($\bar{V}$):** Volumetric flow rate divided by cross-sectional area.
  $\bar{V} = Q / A = (\int V\, dA) / A$
- **Mass velocity (G):** Mass flow rate per unit cross-sectional area.
  $G = \dot{m} / A = \rho \bar{V}$ (for incompressible flow)

Relation between G and $\bar{V}$:
For incompressible fluid (ρ constant):
$G = \rho \bar{V}$
For compressible fluid:
$G = \rho \bar{V}$ but ρ varies along flow → G still used in **mass flow rate** expressions.
Derivation of relation:
$\dot{m} = \int \rho V\, dA$ (over cross-section).
If ρ = constant and V varies, define $\bar{V}$ such that:
$\dot{m} = \rho A \bar{V} \Rightarrow \bar{V} = (1/A) \int V\, dA$
Then $G = \dot{m}/A = \rho \bar{V}$
For laminar flow (parabolic profile):
$V(r) = V_{max} [1 - (r/R)^2]$
$\bar{V} = V_{max} / 2$
$G = \rho (V_{max} / 2)$
For turbulent flow (flatter profile):
$\bar{V} \approx 0.8\ V_{max}$ to $0.85\ V_{max}$
Real-world application:

- Chemical reactor design – G determines residence time.
- Heat exchanger – G used in Reynolds number for compact surfaces.
- Pipe sizing – from required G, find area and diameter.

Example:
Water ($\rho = 1000\ kg/m^3$) flows at $\bar{V} = 2$ m/s in pipe of $A = 0.005\ m^2$.
$G = 1000 \times 2 = 2000\ kg/(s\cdot m^2)$.
$\dot{m} = G \times A = 2000 \times 0.005 = 10$ kg/s.

Q3(b) – Define/Explain: Potential flow, Streamline, Stream tubes. (3 marks)
Appeared in: S22 (Q3a, 03 marks) → Highest: 3 marks
Ans:
Potential flow:

- Flow of an **ideal fluid** (inviscid, incompressible) that is **irrotational** (curl V = 0).
- Velocity field derived from a potential function φ: $V = \nabla\varphi$.
- Governed by Laplace's equation $\nabla^2\varphi = 0$.

Streamline:

- A line whose tangent at any point is in the direction of the **instantaneous velocity vector**.
- In steady flow, streamlines coincide with **pathlines**.

- Equation: $dx/u = dy/v = dz/w$.

Stream tube:

- A bundle of streamlines forming a **tubular surface**.
- No flow crosses the lateral surface.
- Used to apply continuity and Bernoulli equations.

Real-world application:

- Aerodynamics – potential flow over airfoils (approximation).
- Groundwater flow – streamlines show flow direction.

Q4(a) – Derive/Explain: Kinetic energy correction factor (α). (3 marks)
Appeared in: W24 (Q3a OR, 03 marks) → Highest: 3 marks
Ans:
Definition: Kinetic energy correction factor α is the ratio of **actual kinetic energy per unit mass** based on velocity profile to **kinetic energy computed using average velocity**.
$\alpha = (\int V^3\, dA) / (\bar{V}^3 A)$
Derivation:
Actual KE flux through cross-section = $\int (\frac{1}{2} V^2)(\rho V\, dA) = \frac{1}{2} \rho \int V^3\, dA$
KE flux using average velocity = $\frac{1}{2} \rho \bar{V}^3 A$
Therefore, $\alpha = [\frac{1}{2} \rho \int V^3\, dA] / [\frac{1}{2} \rho \bar{V}^3 A] = (\int V^3\, dA) / (\bar{V}^3 A)$
For laminar flow (parabolic profile): $\alpha = 2.0$
For turbulent flow: $\alpha \approx 1.05 – 1.10$ (≈ 1.0 for engineering calculations)
Real-world application:

- Bernoulli equation with friction – α corrects the kinetic head term.
- Pipe flow energy balance – significant only for laminar flow.

Q4(b) – Derive/Explain: Momentum correction factor (β). (3 marks)
Appeared in: W24 (Q2a, 03 marks), W22 (Q2a, 03 marks) → Highest: 3 marks
Ans:
Definition: Momentum correction factor β is the ratio of **actual momentum flux** to **momentum flux computed using average velocity**.
$\beta = (\int V^2\, dA) / (\bar{V}^2 A)$
Derivation:
Actual momentum flux = $\int V (\rho V\, dA) = \rho \int V^2\, dA$
Momentum flux using average velocity = $\rho \bar{V}^2 A$
Therefore, $\beta = (\int V^2\, dA) / (\bar{V}^2 A)$
For laminar flow (parabolic profile): $\beta = 4/3 \approx 1.33$
For turbulent flow: $\beta \approx 1.02 – 1.05$ (≈ 1.0 in most problems)
Real-world application:

- Momentum equation for pipe bends and nozzles.
- Force on reducers – β corrects for non-uniform velocity.

Other Important Questions
OQ1 – Derive the working equation of a U-tube manometer. (7 marks)
Appeared in: S23 (Q1c, 07 marks) → Highest: 7 marks
Ans:
Definition: A U-tube manometer measures pressure difference between two points by balancing a fluid column of known density.
Construction:

- Glass tube bent into U shape.
- Filled with manometric fluid (mercury, water, oil).

- Two ends connected to pressure sources P_1 and P_2.

Derivation of equation:

Let:

- ρ_m = density of manometer fluid
- ρ_f = density of fluid above manometer fluid (if any)
- h_1, h_2 = vertical heights from reference line (say, lower meniscus)
- Δh = difference in manometer fluid levels

Step 1 – Pressure at point A (left limb bottom):

$P\_A = P_1 + \rho\_f\, g\, h_1$

Step 2 – Pressure at point B (right limb bottom, same horizontal level):

$P\_B = P_2 + \rho\_f\, g\, h_2 + \rho\_m\, g\, \Delta h$ (if fluid above is same ρ_f)

Step 3 – Equality of pressures at same level in same fluid (A and B):

$P_1 + \rho\_f\, g\, h_1 = P_2 + \rho\_f\, g\, h_2 + \rho\_m\, g\, \Delta h$

Step 4 – Rearranged:

$P_1 – P_2 = \rho\_m\, g\, \Delta h + \rho\_f\, g\, (h_2 – h_1)$

Special case – same fluid above meniscus ($h_1 = h_2$):

$P_1 – P_2 = \rho\_m\, g\, \Delta h$

[DG PROMPT]

Title: U-tube manometer

Description: Draw U-shaped glass tube. Left top: P_1, right top: P_2. Left leg: height h_1 from reference line. Right leg: height h_2. Lower curved section filled with manometer fluid (shaded). Vertical difference between menisci = Δh. Arrows pointing downward for gravity. Labels: Manometer fluid density ρ_m, Fluid above density ρf.

Real-world application:

- Laboratory pressure measurements (wind tunnels, pipelines).
- HVAC systems – filter pressure drop.

Example:

Mercury manometer ($\rho_m = 13600$ kg/m³), Δh = 0.1 m, same fluid air above (neglect).
$\Delta P = 13600 \times 9.81 \times 0.1 = 13341.6$ Pa.

OQ2 – Develop equation for pressure difference for inclined tube manometer. (7 marks)

Appeared in: S22 (Q2c, 07 marks) → Highest: 7 marks

Ans:

Definition: Inclined tube manometer (draft gauge) amplifies small pressure differences by measuring length along an inclined tube.

Derivation:

Let:

- θ = angle of inclination from horizontal
- L = length of liquid column along the inclined tube
- Vertical height difference = $h = L \sin\theta$
- ρ_m = density of manometer fluid
- ρ_f = density of fluid above (often negligible for gas)

Pressure balance (same level):

$P_1 – P_2 = \rho\_m\, g\, h = \rho\_m\, g\, L \sin\theta$

Advantage: For small ΔP, L becomes large and easy to read. Sensitivity = $1/\sin\theta$.

Example: θ = 10°, sin θ = 0.1736. For same ΔP, $L = h / 0.1736 \approx 5.76\, h$ → amplification 5.76×.

[DG PROMPT]

Title: Inclined tube manometer

Description: Draw a reservoir on left (large cross-section) connected to a transparent inclined tube (angle θ). Right end open to atmosphere or P_2. Manometer liquid rises length L along tube. Mark vertical height $h = L \sin\theta$. Label P_1 (reservoir side) and P_2 (tube end). Arrow for angle θ.

Real-world application:

- Measuring low pressures (e.g., furnace draft, fan pressure).
- Aircraft airspeed indicators (low speed).

OQ3 – Derive Bernoulli's equation for flow with friction through inclined stream tube. (7 marks)

Appeared in: S24 (Q2c, 07 marks) → Highest: 7 marks

Ans:

Assumptions:

- Steady flow
- Incompressible fluid (ρ constant)
- Flow along a stream tube with friction (viscous losses included)

Derivation from Euler's equation with friction term:

Consider an infinitesimal element of length ds, area dA, inclined at angle θ to horizontal. Forces:

- Pressure forces: $P\,dA - (P + dP)\,dA = -\,dP\,dA$
- Gravity component: $-\rho g\,dA\,ds \sin\theta = -\rho g\,dA\,dz$ (since $dz = ds \sin\theta$)
- **Friction force:** $-\tau_w P_w\,ds$ (where P_w = wetted perimeter, τ_w = wall shear stress)

Newton's 2nd law (momentum):

$\rho\,dA\,ds\,V\,dV = -\,dP\,dA - \rho g\,dA\,dz - \tau_w P_w\,ds$

Divide by $\rho\,dA$:

$V\,dV + (dP/\rho) + g\,dz + (\tau_w P_w\,ds)/(\rho\,dA) = 0$

Define **head loss due to friction** $dh_f = (\tau_w P_w\,ds)/(\rho g\,dA) = (4\tau_w\,ds)/(\rho g D)$ for pipe ($P_w = \pi D$, $dA = \pi D^2/4 \Rightarrow P_w/dA = 4/D$).

Integrate from section 1 to 2:

$\int V\,dV = (V_2^2 - V_1^2)/2$

$\int dP/\rho = (P_2 - P_1)/\rho$ (ρ constant)

$\int g\,dz = g(z_2 - z_1)$

$\int dh_f = h_f$ (total friction head loss)

Final Bernoulli equation with friction:

$(P_1/\rho g) + (V_1^2/2g) + z_1 = (P_2/\rho g) + (V_2^2/2g) + z_2 + h_f$

[DG PROMPT]

Title: Inclined stream tube with friction

Description: Draw a tapering tube sloping upward from left to right. Label section 1 (left) with P_1, V_1, z_1. Section 2 (right) with P_2, V_2, z_2. Dashed streamlines inside. Show friction arrows at wall opposing flow. Write equation below.

Real-world application:

- Pipe flow with pumps – h_f calculated via Darcy-Weisbach.
- Hydraulic grade line analysis.

OQ4 – Explain pump work in Bernoulli's equation. (4 marks)
Appeared in: S22 (Q4b, 04 marks) → Highest: 4 marks
Ans:
Concept: Pump adds **mechanical energy** to the fluid. In Bernoulli's equation, pump work appears as an **additional head term (h_pump)** on the left side.
Modified Bernoulli equation:
$(P_1/\rho g) + (V_1^2/2g) + z_1 + h_{pump} = (P_2/\rho g) + (V_2^2/2g) + z_2 + h_f$
Where:
h_{pump} = (Pump power) / (ρ g Q) (ideal, neglecting losses)
Sign convention:

- **Pump work positive** when added to fluid.
- **Turbine work negative** (extracted).

Real-world application:

- Sizing pump for pipeline – h_pump must overcome friction + elevation + pressure difference.

Example:
If $h_f = 10$ m, $z_2 - z_1 = 5$ m, and $P_2 = P_1$, $V_2 = V_1$, then $h_{pump} = 15$ m.

OQ5 – Describe correction for friction in Bernoulli's equation. (4 marks)
Appeared in: S22 (Q4b OR, 04 marks) → Highest: 4 marks
Ans:
Friction correction accounts for **energy loss** due to viscous shear. In Bernoulli's equation (originally inviscid), we add a **head loss term h_f** on the right side.
Sources of friction loss:

- **Major losses:** Due to pipe wall friction – $h_{f,major} = f\,(L/D)\,(V^2/2g)$ [Darcy-Weisbach]
- **Minor losses:** Due to fittings, bends, valves – $h_{f,minor} = K\,(V^2/2g)$

Corrected Bernoulli equation:
$(P_1/\rho g) + (V_1^2/2g) + z_1 = (P_2/\rho g) + (V_2^2/2g) + z_2 + h_{f,major} + \Sigma\, h_{f,minor}$
Friction factor (f):

- Laminar flow: $f = 64/Re$
- Turbulent: Moody chart or Colebrook equation

Real-world application:

- Pipeline design – h_f determines pump power.
- Natural gas transmission – friction loss reduces pressure.

OQ6 – Explain form friction losses in Bernoulli's equation with example. (3 marks)
Appeared in: W25 (Q5a, 03 marks) → Highest: 3 marks
Ans:
Form friction losses (pressure drag) occur due to **flow separation** and **wake formation** around objects, not due to wall shear. Also called **minor losses**.
Cause: Sudden expansion, contraction, bends, valves, obstructions.
Equation: $h_{form} = K\,(V^2/2g)$
Where K = loss coefficient (experimental).
Example – Sudden expansion:
$K = [1 - (A_1/A_2)]^2$
If $A_1/A_2 = 0.25$, then $K = (1 - 0.25)^2 = 0.5625$.
For $V_1 = 5$ m/s, $g = 9.81$ m/s²:
$h_{form} = 0.5625 \times (5^2)/(2\times 9.81) = 0.5625 \times 1.274 = 0.716$ m

Real-world application:

- Energy loss at pipe entrance, diffusers.
- Heat exchanger headers – minimizing K reduces pumping cost.

OQ7 – Derive angular momentum equation / macroscopic momentum balance. (No marks specified – provide detailed derivation, assume 7 marks for completeness)
Ans:
Definition: The angular momentum equation states that the **net torque** on a control volume equals the **rate of change of angular momentum** within the CV plus the **net flux of angular momentum** across the control surface.
For steady flow:
$\Sigma T = \Sigma (\dot{m} \times r \times V)_{out} - \Sigma (\dot{m} \times r \times V)_{in}$
Derivation from Reynolds Transport Theorem:
Let B = angular momentum $H = r \times (m V)$.
Intensive property $\beta = r \times V$.
Reynolds Transport Theorem:
$dH_{sys}/dt = \partial/\partial t \int_{CV} (r \times V) \rho \, dV + \int_{CS} (r \times V) \rho (V\cdot\hat{n}) \, dA$
For steady flow, $\partial/\partial t = 0$.
Torque $T = dH_{sys}/dt = \int_{CS} (r \times V) \rho (V\cdot\hat{n}) \, dA$
For discrete inlets/outlets (one-dimensional flow):
$T = \Sigma (\dot{m} \, r \, V_\theta)_{out} - \Sigma (\dot{m} \, r \, V_\theta)_{in}$
where V_θ = tangential velocity component.
Euler's turbomachinery equation (special case):
Torque $T = \dot{m} (r_2 V_{\theta 2} - r_1 V_{\theta 1})$
Power $= T \omega = \dot{m} (r_2 V_{\theta 2} \omega - r_1 V_{\theta 1} \omega) = \dot{m} (U_2 V_{\theta 2} - U_1 V_{\theta 1})$ where $U = r\omega$.
[DG PROMPT]
Title: Angular momentum – control volume
Description: Draw a turbine rotor (circular shape). Inlet at radius r_1 with velocity V_1 (tangential component $V_{\theta 1}$). Outlet at radius r_2 with $V_{\theta 2}$. Arrows for torque T and angular velocity ω. Label control surface.
Real-world application:

- Pump and turbine design (velocity triangles).
- Jet engines – torque on spools.
- Wind turbine – blade torque calculation.

****