GUJARAT TECHNOLOGICAL UNIVERSITY

BE-3 SEMESTER – OLD PAPER – S22 TO W25 – QUESTION BANK ANSWER

Subject Name & Code:

FLUID FLOW OPERATIONS (3130502)

(Disclaimer: The purpose of these AI-generated responses is just education and reference. Utilise them to grasp topics and structure, but always rewrite in your own words and double-check)

Unit 3: Flow of Incompressible Fluids in Conduits and Thin Layers

Q1 – Derive Hagen-Poiseuille equation. (7 marks)
Appeared in: S24 (Q3c OR, 07 marks), S23 (Q2c, 07 marks), W24 (Q2b, 04 marks), W22 (Q2b, 04 marks) → Highest: 7 marks
Ans:
Definition: Hagen-Poiseuille equation gives the **pressure drop** for steady, fully developed, **laminar flow** of an incompressible Newtonian fluid through a horizontal circular pipe.
Assumptions:

- Steady, laminar flow (Re < 2000)
- Horizontal pipe (no gravity effect)
- Incompressible, Newtonian fluid
- Fully developed (velocity profile constant along length)
- No slip at wall

Derivation (force balance method):
Consider a cylindrical fluid element of radius **r** and length **L** inside the pipe.
Step 1 – Forces on the element:

- Pressure force upstream: $P_1 \times (\pi r^2)$
- Pressure force downstream: $P_2 \times (\pi r^2)$ (acts opposite)
- Shear stress (τ) on cylindrical surface: $\tau \times (2\pi r \times L)$ (resists motion)

For steady flow (no acceleration), net force = 0:
$(P_1 – P_2)\, \pi r^2 – \tau\, (2\pi r\, L) = 0$
Step 2 – Express τ from Newton's law of viscosity:
$\tau = –\mu\, (du/dr)$ (negative sign because u decreases as r increases)
Substitute:
$(P_1 – P_2)\, \pi r^2 = –\mu\, (du/dr)\, (2\pi r\, L)$
Simplify:
$(P_1 – P_2)\, r = –2\mu\, L\, (du/dr)$
Step 3 – Separate variables and integrate:
$du = – [(P_1 – P_2)/(2\mu\, L)]\, r\, dr$
Integrate:
$u(r) = – [(P_1 – P_2)/(4\mu\, L)]\, r^2 + C$
Step 4 – Apply boundary condition (no slip at wall): at r = R (pipe radius), u = 0
$0 = – [(P_1 – P_2)/(4\mu\, L)]\, R^2 + C \Rightarrow C = [(P_1 – P_2)/(4\mu\, L)]\, R^2$
Thus velocity profile:
$u(r) = [(P_1 – P_2)/(4\mu\, L)]\, (R^2 – r^2)$ (Parabolic)
Step 5 – Maximum velocity (at r = 0):

u_{max} = $[(P_1 - P_2)/(4\mu L)] R^2$
Step 6 – Average velocity:
$\bar{V} = (1/A) \int u \, dA = (1/\pi R^2) \int_0^R u(r) \, 2\pi r \, dr = (P_1 - P_2) R^2 / (8\mu L)$
Step 7 – Hagen-Poiseuille equation (pressure drop):
$\Delta P = P_1 - P_2 = (8 \mu L \bar{V}) / R^2 = (32 \mu L \bar{V}) / D^2$ (since D = 2R)
In terms of volumetric flow rate $Q = \bar{V} \times \pi R^2$:
$\Delta P = (128 \mu L Q) / (\pi D^4)$
[DG PROMPT]
Title: Force balance for Hagen-Poiseuille derivation
Description: Draw a horizontal circular pipe. Inside, show a cylindrical fluid element of radius r and length L. Left end: pressure P_1 acting rightward. Right end: pressure P_2 acting leftward. On the curved surface, draw small arrows representing shear stress τ opposing flow. Label radius R (pipe wall), r (element radius). Add parabolic velocity profile across cross-section.
Real-world application:

- Capillary viscometers – measure viscosity from flow rate.
- Microfluidics, blood flow in small arteries.
- Lubrication theory (oil films).

Numerical example:
Water ($\mu = 0.001$ Pa·s) flows through D = 0.01 m, L = 1 m, $\bar{V} = 0.1$ m/s. Find ΔP.
$\Delta P = (32 \times 0.001 \times 1 \times 0.1) / (0.01)^2 = (0.0032) / (0.0001) = 32$ Pa.

Q2 – Define hydraulic radius / equivalent diameter. (4 marks)
Appeared in: S24 (Q3a, 03 marks), W24 (Q4b, 04 marks) → Highest: 4 marks
Ans:
Hydraulic radius (R_h):

- Definition: Ratio of **cross-sectional area (A)** of flow to **wetted perimeter (P_w)**.
  R_h = A / P_w
- Units: meters (length)

Equivalent (hydraulic) diameter (D_h):

- Used for non-circular ducts to apply circular pipe equations.
  D_h = 4 R_h = 4A / P_w

For common shapes:

Shape	Area (A)	Wetted perimeter (P_w)	D_h
Circular pipe (full)	πD^2/4	πD	D
Rectangular duct (a×b)	a × b	2(a+b)	2ab/(a+b)
Annulus (outer R, inner r)	π(R^2−r^2)	2π(R+r)	2(R−r)

Real-world application:

- HVAC ducts – pressure drop calculations.
- Heat exchangers (non-circular channels).
- Open channel flow (hydraulic radius used in Manning's equation).

Example: Rectangular duct 0.2 m × 0.1 m.
A = 0.02 m^2, P_w = 0.6 m, D_h = 4×0.02/0.6 = 0.1333 m.

Q3 – Show that average velocity = ½ maximum velocity for laminar flow in circular pipe. (7 marks)
Appeared in: W25 (Q4c, 07 marks) → Highest: 7 marks
Ans:
Given: Fully developed laminar flow in circular pipe. Velocity profile (from Hagen-Poiseuille):
u(r) = u_max [1 – (r/R)^2]
where u_max = ΔP R^2/(4μL)
Step 1 – Average velocity definition:
$\bar{V} = (1/A) \int u \, dA = (1/\pi R^2) \int_0^R u(r) \cdot 2\pi r \, dr$
Step 2 – Substitute u(r):
$\bar{V} = (2/R^2) \int_0^R u_{max} [1 – (r/R)^2] \, r \, dr$
Step 3 – Integrate:
$\int_0^R r \, dr = R^2/2$
$\int_0^R (r^3/R^2) \, dr = (1/R^2) \times (R^4/4) = R^2/4$
So $\bar{V} = (2u_{max}/R^2) [R^2/2 – R^2/4] = (2u_{max}/R^2) \times (R^2/4) = u_{max}/2$
Thus $\bar{V} = u_{max} / 2$
Real-world application:

- Flow rate calculation from centreline velocity (using Pitot tube).
- Verification of numerical simulations.

Numerical example:
If u_max = 2 m/s, then $\bar{V}$ = 1 m/s. Volumetric flow $Q = \bar{V} \times \pi R^2$.

Q4 – Discuss friction loss in sudden expansion/contraction of pipe. (7 marks)
Appeared in: W25 (Q5c, 07 marks), S23 (Q5c OR, 07 marks) → Highest: 7 marks
Ans:
Sudden expansion: Flow from smaller pipe (area A_1) to larger pipe (area A_2).
Head loss formula (Borda-Carnot equation):
$h_{L,exp} = (V_1 – V_2)^2 / (2g) = V_1^2/(2g) [1 – (A_1/A_2)]^2$
Derivation (momentum + energy):

- Pressure rise due to deceleration.
- Energy loss due to turbulent eddies in the expansion corner.

Sudden contraction: Flow from large pipe (A_1) to small pipe (A_2).
Head loss: $h_{L,cont} = K_c (V_2^2/2g)$
where $K_c = 0.5 [1 – (A_2/A_1)]$ for sharp entrance (approx.)
Loss coefficient values:

A_2/A_1	K_expansion (based on V_1)	K_contraction (based on V_2)
0.1	0.81	0.41
0.5	0.25	0.22
1.0	0.00	0.00

Physical mechanism:

- Expansion: Flow separates at the corner, forms a **recirculation zone** (dead zone). Kinetic energy converted to heat.
- Contraction: Flow accelerates smoothly (vena contracta forms downstream of contraction), then decelerates, causing loss.

[DG PROMPT]

Title: Sudden expansion and contraction
Description: Draw two side-by-side sketches. Left: sudden expansion – small pipe (A_1) on left joining larger pipe (A_2) on right. Show recirculation eddy in the corner near the step. Right: sudden contraction – large pipe (A_1) on left, smaller pipe (A_2) on right. Show vena contracta (narrowest flow cross-section) just downstream of contraction. Label velocities V_1, V_2 and head loss h_L.
Real-world application:

- Heat exchanger headers – minimize losses by gradual tapers.
- Pipe fittings (reducers, expanders).
- Valve design.

Numerical example (expansion):
$A_1 = 0.005$ m², $A_2 = 0.02$ m², $V_1 = 4$ m/s.
h_L = (4^2/2×9.81) × [1 – (0.005/0.02)]^2 = (16/19.62) × (1 – 0.25)^2 = 0.815 × 0.5625 = 0.458 m

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Q5 – Explain velocity distribution for laminar flow of Newtonian fluid in circular channel. (4 marks)
Appeared in: W25 (Q5b, 04 marks) → Highest: 4 marks
Ans:
Velocity distribution (from Hagen-Poiseuille):
u(r) = u_max [1 – (r/R)^2] where u_max = ΔP R^2/(4μL)
Characteristics:

- **Parabolic profile** (quadratic in r).
- Maximum at centre (r=0).
- Zero at wall (r=R) – no-slip condition.
- **Average velocity** = u_max/2.
- Shear stress τ = –μ (du/dr) = (2μ u_max / R^2) r → linear from zero at centre to maximum at wall.

[DG PROMPT]
Title: Laminar velocity profile in circular pipe
Description: Draw a circular cross-section (circle). Inside, draw concentric parabolic curve – highest at centre, touching wall at zero. Label r=0 (centre), r=R (wall), u_max at centre. Add arrow indicating direction of flow (out of page or horizontal).
Real-world application:

- Capillary tube viscometers – measure u(r) optically.
- Polymer extrusion – melt velocity distribution affects product quality.

OQ1 – Discuss friction factor for flow through channels of non-circular cross-section. (7 marks)
Appeared in: W23 (Q3c OR, 07 marks) → Highest: 7 marks
Ans:
Concept: For non-circular ducts, use **hydraulic diameter** D_h = 4A/P_w to apply circular pipe correlations.
Friction factor (Darcy) definition:
f = (ΔP × D_h) / (½ ρ V̄^2 L)
Laminar flow (Re_h < 2000):
f = C / Re_h where Re_h = ρ V̄ D_h / μ
Constant C depends on cross-section shape:

Cross-section	C (f × Re_h)

Cross-section	C (f × Re_h)
Circular	64
Square	57
Rectangular (a/b = 2)	62
Rectangular (a/b = 4)	73
Annulus (concentric)	96 (for inner wall fixed)
Parallel plates	96

Turbulent flow:
Use Moody chart with D_h and **relative roughness** ε/D_h. Approximate correlations:

- Smooth turbulent: f = 0.316 Re_h^{-0.25} (Blasius) for Re_h up to 10^5.
- Rough turbulent: Colebrook equation valid with D_h.

Limitations:

- D_h works well for turbulent flow but less accurate for laminar with sharp corners.
- Secondary flows in rectangular ducts affect friction.

Real-world application:

- Compact heat exchangers (square/rectangular channels).
- Fuel rods in nuclear reactors (annular flow).
- Microchannels (non-circular).

Numerical example:
Square duct side 0.1 m, D_h = 0.1 m, water at 1 m/s, μ=0.001 Pa·s, ρ=1000 kg/m³.
Re = 1000×0.1×1/0.001 = 100,000 (turbulent). f ≈ 0.018 (from Moody).

OQ2 – Effect of roughness for flow through pipe. (4 marks)
Appeared in: W23 (Q3b OR, 04 marks) → Highest: 4 marks
Ans:
Surface roughness (ε) – average height of irregularities on pipe wall.
Effect on friction factor (f):

Flow regime	Effect of roughness
Laminar (Re < 2000)	No effect – f = 64/Re independent of ε.
Smooth turbulent (Re moderate, ε small)	No effect – wall irregularities buried inside viscous sublayer.
Transitional	f begins to increase with ε.
Fully rough turbulent (Re large)	f independent of Re, depends only on ε/D – hydraulic rough regime. f = f(ε/D) via Moody chart.

Moody chart regions:

- Smooth pipe: Colebrook equation reduces to Prandtl's smooth law.
- Rough pipe: $f = 1/(1.14 + 2\log(D/\varepsilon))^2$.

Critical Reynolds number for roughness effect:
Re > 65 (D/ε) for onset of fully rough regime.
Real-world application:

- Aging pipes – increased roughness raises pumping cost.
- Selecting pipe material (steel vs PVC).
- Hydraulic design of water distribution systems.

Example: ε = 0.05 mm, D = 50 mm → ε/D = 0.001. At Re=10^6, f ≈ 0.019 (smooth) vs f ≈ 0.0205 (rough).

OQ3 – Define friction factor and establish relation between skin friction parameters. (3 marks)
Appeared in: W23 (Q3a, 03 marks) → Highest: 3 marks
Ans:
Definition of Darcy friction factor (f):
f = (ΔP × D) / (½ ρ $\bar{V}$^2 L) for circular pipe.
It represents ratio of wall shear stress to dynamic pressure: f = 8 τ_w / (ρ $\bar{V}$^2).
Relation between skin friction parameters:

Parameter	Symbol	Relation with f
Wall shear stress	τ_w	τ_w = (f/8) ρ $\bar{V}$^2
Fanning friction factor	C_f	C_f = f/4 = τ_w / (½ ρ $\bar{V}$^2)
Shear velocity	u_*	u_* = √(τ_w/ρ) = $\bar{V}$ √(f/8)

Laminar flow relation: f = 64/Re.
Turbulent flow: f determined from Colebrook or Moody.
Real-world application:

- Pressure drop calculations in pipelines.
- Pump sizing.

OQ4 – Explain fully developed flow and transition length for laminar/turbulent flow. (7 marks)
Appeared in: W24 (Q5c, 07 marks), W22 (Q5c, 07 marks) → Highest: 7 marks
Ans:
Fully developed flow:

- Velocity profile **no longer changes** along the flow direction (∂u/∂x = 0).
- Wall shear stress and pressure gradient become constant.
- For laminar: parabolic profile.
- For turbulent: flatter profile (log-law).

Hydrodynamic entrance length (transition length, Le):
Distance from pipe inlet to point where fully developed flow begins.

Flow regime	L_e (approximate)	Formula
Laminar	L_e / D ≈ 0.05 Re	L_e = 0.05 Re D
Turbulent	L_e / D ≈ 10 to 20 (independent of Re)	L_e ≈ 10D to 20D

Example: For Re = 1000 (laminar), L_e = 0.05×1000×D = 50D.

For turbulent, Re = 10^5, L_e ≈ 15D (much shorter despite higher Re).
Physical reason:
- Laminar: slow diffusion of momentum → long entrance.
- Turbulent: eddy mixing rapidly flattens profile → short entrance.

[DG PROMPT]
Title: Entrance region and fully developed flow
Description: Draw a horizontal pipe. At inlet, uniform velocity profile (flat). Moving downstream, draw boundary layers growing from walls. At L_e, dashed lines meet. Downstream of L_e, draw parabolic (laminar) or flatter (turbulent) profile. Label "Entrance region" and "Fully developed region".
Real-world application:
- Heat exchangers – in short tubes, entrance region enhances heat transfer.
- Pipe flow measurements – place instruments after L_e for accurate friction factor.

OQ5 – Explain separation of boundary layers in diverging channel. (3 marks)
Appeared in: W24 (Q3a, 03 marks), W22 (Q3a, 03 marks) → Highest: 3 marks
Ans:
Boundary layer separation occurs in a diverging channel (diffuser) due to **adverse pressure gradient** (pressure increases in flow direction).
Mechanism:
- In a diffuser, velocity decreases, pressure rises.
- Low-momentum fluid near the wall cannot overcome the pressure rise.
- Flow reverses near the wall → separation point where wall shear τ_w = 0.

Consequences:
- Formation of **recirculation zones** (wake).
- Increased pressure loss (form drag).
- Reduced diffuser efficiency.

Delay separation by:
- Small divergence angle (≤ 7° for conical diffusers).
- Boundary layer suction or trip wires.

[DG PROMPT]
Title: Boundary layer separation in diverging channel
Description: Draw a diverging channel (width increasing from left to right). Inlet uniform flow. Near wall, draw thin boundary layer. At a certain point, show streamline detaching from wall and curling back to form an eddy. Label "Separation point", "Recirculation zone", "Adverse pressure gradient".
Real-world application:
- Diffuser design for wind tunnels, pumps.
- Aircraft intake ducts.

OQ6 – Numerical on friction loss / pressure drop in pipes (Multiple papers)
Since specific numerical data not provided in CQB, below are **representative solved examples** as per GTU pattern.

Numerical 1 (S22, Q5b – 04 marks): Water flow in pipe
Typical question: Water flows through a pipe of diameter 0.1 m, length 100 m, average velocity 2 m/s. Friction factor f = 0.02. Find head loss due to friction and pressure drop.
Ans:
Given:
D = 0.1 m, L = 100 m, $\bar{V}$ = 2 m/s, f = 0.02, ρ = 1000 kg/m³, g = 9.81 m/s².

Formula (Darcy-Weisbach):
$h_f = f (L/D) (\bar{V}^2/2g)$
$\Delta P = \rho g h_f$
Solution:
$h_f = 0.02 \times (100/0.1) \times (2^2/(2\times9.81)) = 0.02 \times 1000 \times (4/19.62) = 20 \times 0.2039 = 4.078$ m
$\Delta P = 1000 \times 9.81 \times 4.078 = 40000$ Pa ≈ 40 kPa
Final answer: h_f = **4.08 m**, ΔP = **40 kPa**

Numerical 2 (S22, Q5c – 07 marks): Crude oil pumping
Typical question: Crude oil (μ = 0.1 Pa·s, ρ = 850 kg/m³) flows at 0.05 m³/s through a 0.2 m diameter pipe of length 500 m. Find pumping power required if pump efficiency is 70%.
Ans:
Given: Q = 0.01 m³/s, D = 0.2 m, L = 500 m, μ = 0.1 Pa·s, ρ = 850 kg/m³, η = 0.7.
Step 1 – Velocity:
$\bar{V} = Q / A = 0.01 / (\pi\times0.2^2/4) = 0.01 / 0.031416 = 0.318$ m/s
Step 2 – Reynolds number:
$Re = \rho \bar{V} D / \mu = 850 \times 0.318 \times 0.2 / 0.1 = 540.6$ (laminar)
Step 3 – Friction factor (laminar):
$f = 64/Re = 64/540.6 = 0.1184$
Step 4 – Head loss (Darcy-Weisbach):
$h_f = f (L/D) (\bar{V}^2/2g) = 0.1184 \times (500/0.2) \times (0.318^2/19.62) = 0.1184 \times 2500 \times (0.101/19.62) = 296 \times 0.00515 = 1.524$ m
Step 5 – Pump power (hydraulic):
$P_{hyd} = \rho g Q h_f = 850 \times 9.81 \times 0.01 \times 1.524 = 127.1$ W
Step 6 – Motor power:
$P_{motor} = P_{hyd} / \eta = 127.1 / 0.7 = 181.6$ W
Final answer: Pump power required = **182 W** (approx.)

Numerical 3 (W24, Q2c – 07 marks): Sugar syrup flow
Typical question: Sugar syrup (μ = 0.5 Pa·s, ρ = 1200 kg/m³) flows laminarly in a 0.05 m diameter pipe at average velocity 0.05 m/s. Pipe length = 20 m. Find pressure drop.
Ans:
Given: μ = 0.5 Pa·s, ρ = 1200 kg/m³, D = 0.05 m, $\bar{V}$ = 0.05 m/s, L = 20 m.
Check Re: $Re = 1200\times0.05\times0.05 / 0.5 = 3$ (laminar).
Using Hagen-Poiseuille: $\Delta P = (32 \mu L \bar{V}) / D^2 = (32 \times 0.5 \times 20 \times 0.05) / (0.05)^2 = (32\times0.5\times1) / 0.0025 = 16 / 0.0025 = 6400$ Pa.
Final answer: ΔP = **6.4 kPa**

Numerical 4 (W23, Q3b – 04 marks): Water flow in pipe
Typical question: Water flows at 1.5 m/s in a 0.15 m diameter smooth pipe. Find friction factor using Blasius equation.
Ans:
Given: $\bar{V}$ = 1.5 m/s, D = 0.15 m, water ($\nu = 1\times10^{-6}$ m²/s).
$Re = \bar{V} D / \nu = 1.5 \times 0.15 / 1e\text{-}6 = 225{,}000$ (turbulent, smooth pipe).
Blasius ($Re < 10^5$? 225,000 > 10^5, but Blasius valid up to 10^5. For 2.25×10^5 use other correlation. For answer bank, we use Blasius as approximate:
$f = 0.316 / Re^{0.25} = 0.316 / (225000)^{0.25}$.
$Re^{0.25} = (2.25e5)^{0.25} = (2.25)^{0.25} \times (1e5)^{0.25} = 1.224 \times 10 = 12.24$.
$f = 0.316 / 12.24 = 0.0258$.
Final answer: f ≈ **0.0258**

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