GUJARAT TECHNOLOGICAL UNIVERSITY

BE-3 SEMESTER – OLD PAPER – S22 TO W25 – QUESTION BANK ANSWER

Subject Name & Code:

FLUID FLOW OPERATIONS (3130502)

Unit 4: Flow of Compressible Fluids

Q1 – Define Mach number and explain its significance. (3 marks)
Appeared in: S24 (Q4a, 03 marks), S23 (Q4a, 03 marks), S22 (Q3a OR, 03 marks) → Highest: 3 marks
Ans:
Definition: Mach number (Ma) is the ratio of flow velocity (V) to the speed of sound (c) in the same medium at the same state.
$Ma = V / c$
Significance:

- **Ma < 0.3** → Incompressible flow (density changes <5%) – use Bernoulli's equation.
- **0.3 < Ma < 0.8** → Subsonic compressible flow – density variations matter.
- **Ma ≈ 1** → Transonic flow – shock waves begin.
- **Ma > 1** → Supersonic flow – expansion fans, oblique shocks.
- Determines flow regime for nozzle/diffuser design.

Real-world application:

- Aircraft design (subsonic vs supersonic).
- Rocket nozzle performance.
- Gas pipeline sizing (choking at Ma=1).

Q2 – Explain isentropic flow of compressible fluid through nozzles. (7 marks)
Appeared in: S23 (Q4b, 04 marks), S22 (Q3c OR, 07 marks), W25 (Q4c, 07 marks) → Highest: 7 marks
Ans:
Definition: Isentropic flow is **adiabatic and reversible** (no friction, no heat transfer, no shocks). For a nozzle, it converts **pressure energy into kinetic energy** (velocity increases, pressure and temperature drop).
Key equations for isentropic nozzle flow (ideal gas):

1. **Continuity:** $\dot{m} = \rho A V$ = constant.
2. **Energy (Steady flow):** $h_1 + V_1^2/2 = h_2 + V_2^2/2 = h_0$ (stagnation enthalpy) → for ideal gas: $C_p T_0 = C_p T + V^2/2$.
3. **Isentropic relations:** P/ρ^k = constant, $T/P^{(k-1)/k}$ = constant.

Velocity at any section:
$V = \sqrt{2 C_p (T_0 - T)} = \sqrt{2 (k/(k-1)) R T_0 (1 - (P/P_0)^{(k-1)/k})}$
Mass flow rate through nozzle:
$\dot{m} = P_0 A / \sqrt{T_0} \times \sqrt{(k/R) (2/(k+1))^{(k+1)/(k-1)}}$ (at throat when choked)
Nozzle types based on back pressure (P_b):

Condition	Flow at throat	Exit flow

Condition	Flow at throat	Exit flow
Pb> Pcritical	Subsonic (< Ma=1)	Subsonic
Pb= P critical	Sonic (Ma=1)	Subsonic or supersonic (depending on nozzle shape)
P b< Pcritical	Sonic (Ma=1) (choked)	Supersonic (convergent-divergent nozzle)

[DG PROMPT]
Title: Convergent-divergent nozzle – isentropic flow
Description: Draw a nozzle symmetric about horizontal axis. Left side: converging section (decreasing area) labelled "Convergent". Middle: throat (minimum area) labelled "Throat (Ma=1)". Right side: diverging section (increasing area) labelled "Divergent". Arrows from left to right. Label inlet: P_0, T_0, V≈0 (stagnation). Throat: Ma=1, P, *T*. Exit: P_e, T_e, V_e (supersonic). Below, plot P/P_0 vs distance along nozzle showing pressure drop.
Real-world application:

- Rocket engine nozzles (supersonic exhaust).
- Steam turbine nozzles.
- Supersonic wind tunnels.

Numerical example:
Air (k=1.4, R=287 J/kg·K) flows isentropically through a nozzle. Stagnation conditions: P_0=500 kPa, T_0=400 K. Find velocity at a section where P=300 kPa.
Solution: $V = \sqrt{[2\times(1.4/0.4)\times287\times400\times(1 - (300/500)^{0.4/1.4})]} = \sqrt{[2\times3.5\times287\times400\times(1 - 0.6^{0.2857})]} = \sqrt{[803600\times(1 - 0.876)]} = \sqrt{[803600\times0.124]} = \sqrt{99646} \approx 316$ m/s.

Q3 – Derive expression for critical pressure ratio (r_c) for isentropic flow. (4 marks)
Appeared in: W25 (Q4b, 04 marks) → Highest: 4 marks
Ans:
Definition: Critical pressure ratio is the ratio of pressure at throat (where Ma=1) to stagnation pressure: r_c = P^*/P_0.
Derivation from isentropic relations:
For isentropic flow: $T_0/T = 1 + [(k-1)/2]\, Ma^2$.
At Ma=1 (throat): $T_0/T^* = 1 + (k-1)/2 = (k+1)/2 \rightarrow T^* = T_0 \times 2/(k+1)$.
From isentropic P-T relation: $(P_0/P) = (T_0/T)^{k/(k-1)}$
Substitute $T_0/T^* = (k+1)/2$:
$P_0/P^* = [(k+1)/2]^{k/(k-1)}$
Therefore:
$$\mathbf{P^*/P_0 = [2/(k+1)]^{k/(k-1)}}$$
For common gases:

- Air (k=1.4) → $P^*/P_0 = [2/2.4]^{1.4/0.4} = (0.8333)^{3.5} = 0.5283$.
- Monatomic (k=1.67) → $P^*/P_0 \approx 0.487$.
- Steam (k=1.3) → $P^*/P_0 \approx 0.546$.

Real-world application:

- Nozzle design – choking occurs when back pressure ≤ P_0 × r_c.
- Orifice meters – critical flow condition for mass flow measurement.

Example: Air nozzle with P_0=500 kPa. Critical pressure = 500 × 0.5283 = 264.15 kPa. If back pressure < 264 kPa, flow is choked.

OQ1 – Derive expression for effect of cross-sectional area on velocity for isentropic flow through nozzle. (3 marks)
Appeared in: W25 (Q4a, 03 marks) → Highest: 3 marks
Ans:
Derivation from continuity and momentum (area-velocity relation):
Start with continuity: $\rho A V = \text{constant} \Rightarrow d\rho/\rho + dA/A + dV/V = 0$ …(1)
Euler's equation for isentropic flow: $dP/\rho + V\,dV = 0$ …(2)
Isentropic relation: $dP = c^2 d\rho$ where c = speed of sound = $\sqrt{(kP/\rho)} = \sqrt{(kRT)}$.
From (2): $c^2 d\rho = -\rho V dV \Rightarrow d\rho/\rho = -(V/c^2) dV = -Ma^2 dV/V$ …(3)
Substitute (3) into (1): $-Ma^2 dV/V + dA/A + dV/V = 0$
$\Rightarrow (1 - Ma^2)(dV/V) + dA/A = 0$
Final relation:
$\mathbf{dA/A = (Ma^2 - 1)\, dV/V}$
Implications:

- **$Ma < 1$ (subsonic):** (Ma^2–1) negative → dA and dV opposite signs.
    - Convergent nozzle ($dA < 0$) → $dV > 0$ (velocity increases).
    - Divergent diffuser ($dA > 0$) → $dV < 0$ (velocity decreases).
- **$Ma > 1$ (supersonic):** (Ma^2–1) positive → dA and dV same sign.
    - Convergent nozzle ($dA < 0$) → $dV < 0$ (velocity decreases – not used).
    - Divergent nozzle ($dA > 0$) → $dV > 0$ (velocity increases).

Real-world application:

- Explains why supersonic nozzles must be divergent.
- Design of C-D nozzles for rockets.

OQ2 – Explain asterisk condition and stagnation condition. Derive stagnation temperature expression. (4 marks)
Appeared in: W24 (Q5b, 04 marks), W22 (Q5b, 04 marks) → Highest: 4 marks
Ans:
Stagnation condition (denoted by subscript 0):

- State of fluid brought to **rest isentropically** (velocity reduced to zero).
- Stagnation temperature (T_0), stagnation pressure (P_0), stagnation density (ρ_0).

**Asterisk condition (denoted by superscript):*

- State at which **Mach number = 1** (sonic condition).
- Usually occurs at nozzle throat.
- Denoted as T*, P*, ρ*, A*.

Derivation of stagnation temperature expression (from energy equation):
Steady flow energy equation (adiabatic, no work):
$h_0 = h + V^2/2$. For ideal gas: $h = C_p T$.
$C_p T_0 = C_p T + V^2/2$
$\Rightarrow T_0 = T + V^2/(2C_p)$
Since $C_p = kR/(k-1)$ and $V = Ma\cdot\sqrt{(kRT)}$:
$V^2/(2C_p) = Ma^2\cdot kRT / [2\cdot kR/(k-1)] = Ma^2\cdot T\cdot(k-1)/2$
Thus:
$\mathbf{T_0/T = 1 + [(k-1)/2]\, Ma^2}$
For asterisk condition (Ma=1): $T_0/T^* = 1 + (k-1)/2 = (k+1)/2$.
Real-world application:

- Gas turbine engine – stagnation temperature measured by thermocouple with recovery factor.
- Nozzle design – T* determines throat conditions.

Numerical example:

Air at Ma=0.8, T=300 K, k=1.4. Find T_0.
$T_0 = 300 \times [1 + 0.4/2 \times 0.8^2] = 300 \times [1 + 0.2\times0.64] = 300 \times 1.128 = 338.4$ K.

OQ3 – Explain concept of isothermal friction flow with diagram. (4 marks)
Appeared in: W24 (Q3b, 04 marks), W22 (Q3b, 04 marks) → Highest: 4 marks
Ans:
Definition: Isothermal friction flow is **compressible flow in a constant-area duct** with **friction** (wall shear) but **constant temperature** (heat transfer maintains isothermal condition).
Assumptions:

- Steady, one-dimensional flow.
- Constant temperature (T = constant).
- Friction present (non-isentropic).
- Ideal gas.

Key characteristics:

- Pressure decreases along duct due to friction.
- Velocity increases (since $\dot{m} = \rho AV$, ρ decreases as P decreases, so V increases).
- Mach number increases along duct.
- **Choking** occurs when Ma reaches 1 (maximum length L_{max}).

Governing equations:

- Continuity: ρV = constant.
- Equation of state: $P = \rho RT$ (R constant).
- Momentum with friction: $dP + \rho V\, dV + (4f/D)\, (\tfrac{1}{2}\rho V^2)\, dx = 0$.

Fanno line (for adiabatic friction flow) is replaced by isothermal line.
[DG PROMPT]
Title: Isothermal friction flow in constant-area duct
Description: Draw a horizontal constant-area pipe. Left end: inlet with P_1, V_1, Ma_1 (<1). Right end: outlet with P_2 (<P_1), V_2 (>V_1), Ma_2 (>Ma_1). Along the pipe, add small wavy lines at wall to represent friction. Plot below: Pressure (P) decreasing along x, Velocity (V) increasing along x, Mach number (Ma) increasing towards 1. Label choking point where Ma=1.
Real-world application:

- Long gas pipelines (natural gas transmission) – approximately isothermal due to ground heat transfer.
- Pneumatic conveying systems.

OQ4 – Barometric equation with nomenclature. (3 marks)
Appeared in: S22 (Q4a, 03 marks) → Highest: 3 marks
Ans:
Definition: Barometric equation gives **pressure variation with altitude** in an isothermal atmosphere (constant temperature).
Derivation (hydrostatic + ideal gas):
Hydrostatic: $dP/dz = -\rho g$.
Ideal gas: $\rho = P/(RT)$.
Combine: $dP/dz = -(P/(RT))\, g \Rightarrow dP/P = -(g/RT)\, dz$.
Integrate from z=0 ($P=P_0$) to z (P=P):
$\ln(P/P_0) = -(g/RT)\, z$
Barometric equation:
$$\mathbf{P = P_0 \exp(-gz / RT)}$$
Nomenclature:

- P = pressure at altitude z (Pa)
- P_0 = pressure at reference (z=0, usually sea level) (Pa)
- g = acceleration due to gravity (≈9.81 m/s^2)
- R = specific gas constant (J/kg·K)
- T = absolute temperature (K) – assumed constant
- z = altitude (m)

Real-world application:

- Altitude correction for aircraft instruments.
- Weather forecasting (pressure at different heights).
- Calibration of barometers.

Example:

Sea level P_0 = 101325 Pa, T = 288 K, R for air = 287 J/kg·K. Find P at z = 1000 m.
P = 101325 × exp(–9.81×1000/(287×288)) = 101325 × exp(–9810/82656) = 101325 × exp(–0.1187) = 101325 × 0.888 = 89976 Pa.

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