GUJARAT TECHNOLOGICAL UNIVERSITY

BE-3 SEMESTER – OLD PAPER – S22 TO W25 – QUESTION BANK ANSWER

Subject Name & Code:

FLUID FLOW OPERATIONS (3130502)

Unit 5: Dimensional Analysis and Drag

Q1 – Define drag and drag coefficient. (3 marks)

Appeared in: S24 (Q4a, 03 marks), S23 (Q4a OR, 03 marks), W25 (Q4a, 03 marks)
Ans:
Drag (FD):

- Force exerted by a flowing fluid on a body **parallel to the direction of flow** (opposes motion).
- Two components:
  - **Pressure drag** (form drag) – due to flow separation and wake.
  - **Skin friction drag** – due to viscous shear along the surface.

Drag coefficient (CD)

- Dimensionless parameter:
  $C_D = F_D / (\frac{1}{2} \rho V^2 A)$
  where:
  ρ = fluid density,
  V = free-stream velocity,
  A = reference area (frontal area for bluff bodies, planform area for airfoils).

Real-world application:

- Automotive aerodynamics – lower C_D reduces fuel consumption.
- Wind load on buildings, parachute design, sports equipment.

Example: Sphere at Re ~ 1000 → $C_D \approx 0.47$; streamlined airfoil → $C_D \approx 0.05$.

Q2 – Explain Rayleigh method / Buckingham Pi method for dimensional analysis. (7 marks)

Appeared in: S24 (Q1c, 07 marks), S23 (Q4b, 04 marks), W25 (Q3c, 07 marks), W23 (Q5c, 07 marks)
Ans:
Definition: Dimensional analysis reduces the number of variables in a physical problem by grouping them into **dimensionless π-terms**.

Rayleigh Method (for problems with ≤ 4 variables)
Steps:

1. Write the dependent variable as a product of powers of independent variables:
   $Y = k X_1^a X_2^b X_3^c$
2. Replace each variable with its dimensions (MLT or FLT system).
3. Equate exponents of M, L, T on both sides.
4. Solve for a, b, c.

5. Rewrite in dimensionless form.

Example (drag force on sphere):
F = f(ρ, V, D) → F = k ρ^a V^b D^c
Dimensions: [F] = M L T^-2, [ρ] = M L^-3, [V] = L T^-1, [D] = L
Equating exponents gives a=1, b=2, c=2 → F/(ρ V^2 D^2) = constant.
Limitation: Becomes cumbersome when variables > 4.

Buckingham Pi Theorem
Statement: If a physical phenomenon involves **n** variables and **m** fundamental dimensions, the variables can be grouped into (n – m) independent dimensionless π-terms.
Procedure (6 steps):

1. **List all variables** – dependent + independent.
2. **Write dimensions** of each variable (M, L, T).
3. **Choose m repeating variables** that:
    - Include all m dimensions.
    - Do **not** form a dimensionless group among themselves.
    - Typically: a geometric variable (L), a kinematic variable (V), and a dynamic variable (ρ or μ).
4. **Form each π-term** by combining the repeating variables with one remaining variable:
   π_i = (repeating variables)^(exponents) × (remaining variable)
   Solve exponents to make the product dimensionless.
5. **Repeat** for all remaining variables.
6. **Write the functional relationship:** π_1 = φ(π_2, π_3, …).

Example (drag force on sphere):
Variables: F, ρ, V, D (n=4). m=3 → one π-term.
Choose ρ, V, D as repeating variables.
π = F ρ^a V^b D^c. Solve to get a=-1, b=-2, c=-2 → π = F/(ρ V^2 D^2).
Thus F/(ρ V^2 D^2) = constant → same as Rayleigh.
Real-world application:

- Model testing (ships, aircraft) – scaling laws.
- Heat transfer correlations (Nusselt = f(Re, Pr)).
- Pump/fan performance curves.

Other Important Questions
OQ1 – Efficiency η of a fan depends on ρ, μ, ω, D, Q. Express in dimensionless parameters. (7 marks)
Appeared in: S24 (Q1c, 07 marks)
Ans:
Given: η = f(ρ, μ, ω, D, Q)

- η = efficiency (dimensionless)
- n = 6 variables (including η), m = 3 (M, L, T) → number of π-terms = 6 – 3 = 3.

Dimensions:

Variable	Symbol	Dimension
Efficiency	η	M^0 L^0 T^0
Density	ρ	M L^-3

Variable	Symbol	Dimension
Dynamic viscosity	μ	$M L^{-1} T^{-1}$
Rotational speed	ω	T^{-1}
Diameter	D	L
Volumetric flow rate	Q	$L^3 T^{-1}$

Choose repeating variables (m=3): ρ, ω, D (they contain M, L, T and are independent).

$\pi_1 = \eta$ (already dimensionless). So $\pi_1 = \eta$.

$\pi_2 = \mu \times \rho^a \omega^b D^c$

Write dimensions:

$[\mu] = M L^{-1} T^{-1}$

$\rho^a = M^a L^{-3a}$

$\omega^b = T^{-b}$

$D^c = L^c$

Product: $M^{(1+a)} L^{(-1 -3a + c)} T^{(-1 - b)} = M^0 L^0 T^0$

Equate exponents:

M: $1 + a = 0 \rightarrow a = -1$

T: $-1 - b = 0 \rightarrow b = -1$

L: $-1 -3(-1) + c = -1 + 3 + c = 2 + c = 0 \rightarrow c = -2$

Thus $\pi_2 = \mu \times \rho^{-1} \omega^{-1} D^{-2} = \mu/(\rho \omega D^2)$

It is common to use the reciprocal to obtain a Reynolds-like form:

$\rho \omega D^2 / \mu$ (dimensionless). So we can write $\pi_2 = \rho \omega D^2 / \mu$.

$\pi_3 = Q \times \rho^a \omega^b D^c$

$[Q] = L^3 T^{-1}$

$\rho^a = M^a L^{-3a}$

$\omega^b = T^{-b}$

$D^c = L^c$

Product: $M^a L^{(3 -3a + c)} T^{(-1 - b)} = M^0 L^0 T^0$

M: $a = 0$

T: $-1 - b = 0 \rightarrow b = -1$

L: $3 -3(0) + c = 3 + c = 0 \rightarrow c = -3$

Thus $\pi_3 = Q \times \rho^0 \omega^{-1} D^{-3} = Q/(\omega D^3)$ (flow coefficient).

Final dimensionless relation:

$\eta = \varphi(\rho \omega D^2 / \mu , Q/(\omega D^3))$

Where:

- $\rho \omega D^2 / \mu$ = Reynolds number (based on tip speed).
- $Q/(\omega D^3)$ = flow coefficient.

Real-world application:

- Fan performance curves – efficiency plotted against flow coefficient at constant Reynolds number.
- Scaling fan size for similar operation.

OQ2 – Power required by agitator: function of D, N, μ, ρ. Obtain relation using Buckingham Pi method. (7 marks)
Appeared in: W25 (Q3c, 07 marks)
Ans:
Given: Power $P = f(D, N, \mu, \rho)$
Variables ($n=5$): P, D, N, μ, ρ.
Dimensions ($m=3$): M, L, T → π-terms = $5 - 3 = 2$.
Dimensions:

Variable	Symbol	Dimension
Power	P	$M L^{2} T^{-3}$
Diameter	D	L
Rotational speed	N	T^{-1}
Dynamic viscosity	μ	$M L^{-1} T^{-1}$
Density	ρ	$M L^{-3}$

Choose repeating variables: ρ, N, D (contain M, L, T).

$\pi_1 = P \times \rho^{a} N^{b} D^{c}$
$[P] = M L^{2} T^{-3}$
$\rho^{a} = M^{a} L^{-3a}$
$N^{b} = T^{-b}$
$D^{c} = L^{c}$
Product: $M^{(1+a)} L^{(2 -3a + c)} T^{(-3 - b)} = M^{0} L^{0} T^{0}$
M: $1 + a = 0 \rightarrow a = -1$
T: $-3 - b = 0 \rightarrow b = -3$
L: $2 -3(-1) + c = 2 + 3 + c = 5 + c = 0 \rightarrow c = -5$
Thus $\pi_1 = P \times \rho^{-1} N^{-3} D^{-5} = P/(\rho N^{3} D^{5})$ → this is the **Power number (N_P)**.

$\pi_2 = \mu \times \rho^{a} N^{b} D^{c}$
$[\mu] = M L^{-1} T^{-1}$
$\rho^{a} = M^{a} L^{-3a}$
$N^{b} = T^{-b}$
$D^{c} = L^{c}$
Product: $M^{(1+a)} L^{(-1 -3a + c)} T^{(-1 - b)} = M^{0} L^{0} T^{0}$
M: $1 + a = 0 \rightarrow a = -1$
T: $-1 - b = 0 \rightarrow b = -1$
L: $-1 -3(-1) + c = -1 + 3 + c = 2 + c = 0 \rightarrow c = -2$
Thus $\pi_2 = \mu \times \rho^{-1} N^{-1} D^{-2} = \mu/(\rho N D^{2})$
Take reciprocal to obtain Reynolds number: $\rho N D^{2} / \mu$. So $\pi_2 = \rho N D^{2} / \mu$.

Final relation:
$P/(\rho N^{3} D^{5}) = \varphi(\rho N D^{2} / \mu)$ or $N_P = \varphi(Re)$
Real-world application:

- Scale-up of mixing tanks – same Power number and Reynolds number ensure dynamic similarity.

- Estimating agitator power for industrial processes.

Numerical example (turbulent regime):
For a Rushton turbine, N_P ≈ 5 when Re > 10^4.
Given ρ = 1000 kg/m^3, N = 2 rev/s, D = 0.5 m:
P = N_P × ρ N^3 D^5 = 5 × 1000 × 2^3 × (0.5)^5 = 5 × 1000 × 8 × 0.03125 = 1250 W.

OQ3 – Pressure drop per unit length depends on d, ν, ρ, μ. Find relation using Buckingham Pi method. (7 marks)
Appeared in: W24 (Q4c, 07 marks), W22 (Q4c, 07 marks)
Ans:
Note: The problem lists ν (kinematic viscosity) and μ (dynamic viscosity) together. Since ν = μ/ρ, using both is redundant. In standard GTU problems, the intended variables are:
Pressure drop per unit length (ΔP/L) depends on **diameter (d)**, **density (ρ)**, **viscosity (μ)**, and **average velocity (V)**.
Here we assume ν is a typo and solve with V (velocity).
Thus variables: ΔP/L, d, ρ, μ, V.
n = 5, m = 3 → π-terms = 2.
Dimensions:

Variable	Symbol	Dimension
Pressure drop per unit length	ΔP/L	M L^-2 T^-2
Diameter	d	L
Density	ρ	M L^-3
Dynamic viscosity	μ	M L^-1 T^-1
Velocity	V	L T^-1

Choose repeating variables: ρ, V, d (contain M, L, T).

π_1 = (ΔP/L) × ρ^a V^b d^c
[ΔP/L] = M L^-2 T^-2
ρ^a = M^a L^-3a
V^b = L^b T^-b
d^c = L^c
Product: M^(1+a) L^(-2 -3a + b + c) T^(-2 - b) = M^0 L^0 T^0
M: 1 + a = 0 → a = -1
T: -2 - b = 0 → b = -2
L: -2 -3(-1) + (-2) + c = -2 + 3 -2 + c = -1 + c = 0 → c = 1
Thus π_1 = (ΔP/L) × ρ^{-1} V^{-2} d^1 = (ΔP d) / (ρ V^2 L) → this is the **Euler number** (or friction factor related).

π_2 = μ × ρ^a V^b d^c
[μ] = M L^-1 T^-1
ρ^a = M^a L^-3a
V^b = L^b T^-b
d^c = L^c
Product: M^(1+a) L^(-1 -3a + b + c) T^(-1 - b) = M^0 L^0 T^0

M: 1 + a = 0 → a = -1
T: -1 - b = 0 → b = -1
L: -1 -3(-1) + (-1) + c = -1 + 3 -1 + c = 1 + c = 0 → c = -1
Thus π_2 = μ × ρ^{-1} V^{-1} d^{-1} = μ/(ρ V d)
Take reciprocal to get Reynolds number: ρ V d / μ. So π_2 = ρ V d / μ.

Final relation:
(ΔP d) / (ρ V^2 L) = φ(ρ V d / μ)
Or equivalently:
ΔP/L = φ(Re) × (ρ V^2 / d).
Verification for laminar flow: For Re < 2000, φ(Re) = 64/Re. Then
ΔP/L = (64/Re) × (ρ V^2 / d) = (64 μ/(ρ V d)) × (ρ V^2 / d) = 64 μ V / d^2
This matches the Hagen-Poiseuille equation: ΔP = 32 μ L V / d^2 (since ΔP/L = 32 μ V / d^2? Wait: Hagen-Poiseuille: ΔP = 32 μ L V / d^2 → ΔP/L = 32 μ V / d^2. Our derived 64 μ V / d^2 is twice that. The standard friction factor for laminar flow is f = 64/Re, and Darcy-Weisbach gives ΔP/L = f (1/d) (½ ρ V^2) = (64/Re)(1/d)(½ ρ V^2) = (64 μ/(ρ V d))(1/d)(½ ρ V^2) = 32 μ V / d^2. So the correct φ(Re) should be 32/Re? Let me re-check carefully.
Actually from π_1 = (ΔP d)/(ρ V^2 L) and π_2 = Re = ρ V d / μ, the functional relation is (ΔP d)/(ρ V^2 L) = φ(Re). For laminar flow, the analytical solution gives ΔP/L = 32 μ V / d^2.
Divide both sides by (ρ V^2 / d):
(ΔP/L) / (ρ V^2 / d) = (32 μ V / d^2) × (d/(ρ V^2)) = 32 μ/(ρ V d) = 32/Re. Therefore φ(Re) = 32/Re.
So the correct functional relation is:
(ΔP d)/(ρ V^2 L) = 32/Re for laminar flow, but in general (ΔP d)/(ρ V^2 L) = φ(Re).
Real-world application:
- Pipe flow friction factor (Moody chart).
- Design of pumping systems – pressure drop calculation.

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