GUJARAT TECHNOLOGICAL UNIVERSITY

BE-4 SEMESTER – OLD PAPER – S22 TO W25 – QUESTION BANK ANSWER

Subject Name & Code:

ANALOG CIRCUIT DESIGN (3141002)

Unit 1: Transistor at High Frequencies

Q.1 – Derive the expression for the CE short-circuit current gain A_i as a function of frequency.
(7 marks)
Appeared in: S23 (Q2c), W23 (Q2c), S22 (Q2c)
Ans:
Given: Common-emitter (CE) amplifier with output short-circuited. The high-frequency hybrid-π model of BJT includes:

- r_π – input resistance, $C_\pi = C_{be}$ (base-emitter capacitance)
- $C_\mu = C_{bc}$ (base-collector capacitance), $g_m v_\pi$ – dependent current source
- r_o neglected for short-circuit output condition.

To find: Short-circuit current gain $A_i(s) = \frac{I_c(s)}{I_b(s)}$ as function of frequency.
Formula: Using current divider and Miller effect simplification (for short-circuit output, C_μ appears directly from base to ground).
Solution:

1. **Input circuit:** Base current I_b splits into I_{C_π} and I_{r_π}.
$$I_b = I_{r_\pi} + I_{C_\pi} = \frac{V_\pi}{r_\pi} + sC_\pi V_\pi = V_\pi\left(\frac{1}{r_\pi} + sC_\pi\right)$$
2. **Collector current:** $I_c = g_m V_\pi$ (since output is shorted, no current flows through C_μ or r_o).
3. **Current gain:**
$$A_i(s) = \frac{I_c}{I_b} = \frac{g_m V_\pi}{V_\pi\left(\frac{1}{r_\pi} + sC_\pi\right)} = \frac{g_m}{\frac{1}{r_\pi} + sC_\pi}$$
4. **Low-frequency current gain** $\beta_0 = g_m r_\pi$. Substitute $g_m = \frac{\beta_0}{r_\pi}$:
$$A_i(s) = \frac{\beta_0/r_\pi}{\frac{1}{r_\pi} + sC_\pi} = \frac{\beta_0}{1 + sr_\pi C_\pi}$$
5. **Cut-off frequency** $\omega_\beta = \frac{1}{r_\pi C_\pi}$ (or $f_\beta = \frac{1}{2\pi r_\pi C_\pi}$).
$$A_i(s) = \frac{\beta_0}{1 + s/\omega_\beta}$$

6. **Magnitude response:** $|A_i| = \frac{\beta_0}{\sqrt{1+(f/f_\beta)^2}}$

Final Answer:

$$\boxed{A_i(j\omega) = \frac{\beta_0}{1 + j\omega r_\pi C_\pi}}$$

where $\beta_0 = g_m r_\pi$ is low-frequency current gain, and $f_\beta = \frac{1}{2\pi r_\pi C_\pi}$ is the –3 dB frequency.
Real-world application: Used to determine bandwidth of analog amplifiers (e.g., in RF front-ends, op-amp internal stages). GTU numerical example: If $\beta_0 = 100$, $r_\pi = 2.5\ k\Omega$, $C_\pi = 20\ pF$, find f_β.
Solution: $f_\beta = 1/(2\pi \times 2.5k \times 20p) \approx 3.18\ MHz$.

Q.2 – Derive the relationship between hybrid-π and h-parameters. (7 marks)
Appeared in: S25 (Q5b, 4 marks), W25 (Q4c, 7 marks), S22 (Q4b, 4 marks) → Highest 7 marks
Ans:
Given: Low-frequency h-parameters ($h_{ie}, h_{fe}, h_{re}, h_{oe}$) and high-frequency hybrid-π parameters ($r_\pi, g_m, r_o, C_\pi, C_\mu$ – but for DC/low-frequency, capacitances are open).
To find: Relationship between h-parameters and hybrid-π parameters.
Assumptions: Low-frequency (capacitances neglected), common-emitter configuration.
Formulas:

- $h_{ie} = \frac{v_{be}}{i_b}|_{v_{ce}=0}$ (input impedance with output shorted)
- $h_{fe} = \frac{i_c}{i_b}|_{v_{ce}=0}$ (forward current gain)
- $h_{re} = \frac{v_{be}}{v_{ce}}|_{i_b=0}$ (reverse voltage gain)
- $h_{oe} = \frac{i_c}{v_{ce}}|_{i_b=0}$ (output admittance)

Solution:

1. **Hybrid-π model (low-frequency):**
   - r_π – base-emitter resistance
   - $g_m v_\pi$ – dependent source, $g_m = I_C/V_T$
   - r_o – output resistance (Early effect)
2. **Relation for h_{ie}:** With $v_{ce} = 0$, $v_{be} = i_b \cdot r_\pi$
$$h_{ie} = r_\pi$$
3. **Relation for h_{fe}:** With $v_{ce} = 0$, $i_c = g_m v_\pi = g_m(i_b r_\pi) = g_m r_\pi i_b$
$$h_{fe} = g_m r_\pi = \beta_0$$
4. **Relation for h_{re}:** Open input ($i_b = 0 \rightarrow v_\pi = 0$), then use voltage divider from v_{ce} to v_{be} through r_π and r_μ (where $r_\mu = 1/(sC_\mu)$ at DC is infinite? Actually at DC, C_μ is open, so $h_{re} = 0$ in ideal model. Including r_μ (feedback resistance) gives:
$$v_{be} = v_{ce} \cdot \frac{r_\pi}{r_\pi + r_\mu} \Rightarrow h_{re} \approx \frac{r_\pi}{r_\mu}$$

Typically $h_{re} \approx 10^{-4}$ to 10^{-3}.

5. **Relation for h_{oe}:** With $i_b = 0$ $(v_\pi = 0)$, $i_c = v_{ce}/r_o$
$$h_{oe} = \frac{1}{r_o}$$

Final Answer (summary table):

h-parameter	Hybrid-π equivalent
h_{ie}	r_π
h_{fe}	$g_m r_\pi = \beta_0$
h_{re}	$\frac{r_\pi}{r_\mu}$ (≈ 0 for ideal)
h_{oe}	$1/r_o$

Real-world application: Used to convert low-frequency h-parameter datasheet values into hybrid-π model for high-frequency analysis (e.g., designing RF amplifiers, oscillator circuits).

Q.3 – Derive the high-frequency transconductance equation for g_m for CE amplifier. (7 marks)
Appeared in: S25 (Q5c), S22 (Q5c)
Ans:
Given: Bipolar junction transistor in common-emitter configuration. Transconductance $g_m = \frac{\partial I_C}{\partial V_{BE}}$ at a given bias point.
To find: Expression for g_m including high-frequency effects (minor influence – g_m is essentially a DC/low-frequency parameter, but its magnitude depends on bias current which may be limited at high frequencies by device capacitances). The derivation below is the standard low-frequency g_m used in all high-frequency models.
Formula: $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$ where $V_T = kT/q$.
Solution:

1. **Definition:** Transconductance is the slope of the transfer characteristic:
$$g_m = \frac{dI_C}{dV_{BE}}$$

2. **Differentiate Ebers-Moll equation:**
$$g_m = \frac{d}{dV_{BE}}\left[I_S e^{V_{BE}/V_T}\right] = I_S e^{V_{BE}/V_T} \cdot \frac{1}{V_T} = \frac{I_C}{V_T}$$

3. **At room temperature** $(T = 300K)$, $V_T \approx 26\, mV$:
$$g_m = \frac{I_C}{26\, mV} \text{(in Siemens)}$$

4. **High-frequency consideration:** The above equation holds even at high frequencies because g_m is a bias-dependent small-signal parameter. However, at very high frequencies, parasitic capacitances cause the *effective* g_m to roll off due to phase shift and current division. The **unity-gain frequency** f_T relates to g_m as:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

Final Answer:

$$\boxed{g_m = \frac{I_C}{V_T} = \frac{I_C}{kT/q}}$$

where I_C = collector bias current (mA), V_T = thermal voltage ≈ 26 mV at 300 K.
Numerical example (GTU style):
Given: $I_C = 2\ mA, T = 300K$. Find g_m.
$g_m = 2\ mA/26\ mV = 0.0769\ S = 76.9\ mS$.

Q.4 – List the parameters affecting the transistor at high frequencies.
(4 marks)
Appeared in: S23 (Q2b)
Ans:
The key parameters (internal capacitances and resistances) that affect transistor behavior at high frequencies are:

- **Junction capacitances:**
  - C_{be} (base-emitter capacitance) – combination of diffusion capacitance (C_d) and depletion capacitance (C_{je})
  - C_{bc} (base-collector capacitance) – also called Miller capacitance C_μ
- **Diffusion capacitance** C_d – dominant at forward bias, proportional to I_C
- **Base-spreading resistance** $r_{bb'}$ (or r_x) – limits charging of C_{be}
- **Transit time** τ_f – related to $C_d = g_m\tau_f$
- **Current gain cut-off frequency** f_T and f_β
- **Parasitic capacitances** – package and lead inductances (at very high frequencies)
- **Early effect** (output resistance r_o) – becomes less significant at high frequencies but affects gain

Real-world application: These parameters determine the bandwidth of RF amplifiers, oscillators, and high-speed switching circuits.

Q.5 – Derive the expression for gain-bandwidth product.
(3 marks)
Appeared in: S25 (Q5a)
Ans:
Definition: Gain-bandwidth product (GBW or f_T) is the frequency at which the short-circuit common-emitter current gain $|A_i|$ drops to unity (0 dB).
Given: Current gain $A_i(s) = \frac{\beta_0}{1+s/\omega_\beta}$
To find: Gain-bandwidth product f_T.
Solution:

1. At high frequencies ($f \gg f_\beta$), $A_i(jf) \approx \frac{\beta_0}{j(f/f_\beta)}$
2. Magnitude: $|A_i| = \frac{\beta_0}{f/f_\beta}$
3. Set $|A_i| = 1$ to find f_T:

$$1 = \frac{\beta_0}{f_T/f_\beta} \Rightarrow f_T = \beta_0 \cdot f_\beta$$

4. Substitute $f_\beta = \frac{1}{2\pi r_\pi C_\pi}$ and $\beta_0 = g_m r_\pi$:

$$f_T = g_m r_\pi \cdot \frac{1}{2\pi r_\pi C_\pi} = \frac{g_m}{2\pi C_\pi}$$

More accurately including C_μ: $f_T \approx \frac{g_m}{2\pi(C_\pi + C_\mu)}$

Final Answer:

$$\boxed{f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}}$$

Q.6 – Derive the high-frequency current gain for CE amplifier with R_s.
(7 marks)
Appeared in: S25 (Q5c)
Ans:
Given: CE amplifier with source resistance R_s in series with base. Hybrid-π model includes r_π, C_π, C_μ, $g_m v_\pi$, and r_o (neglected for current gain).
To find: Current gain $A_i(s) = I_c/I_s$ where I_s is source current.
Assumptions: Output short-circuited (to simplify). Miller effect applied to C_μ.
Solution:

1. **Input circuit** with R_s and Miller capacitance:
   Miller capacitance seen at input: $C_M = C_\mu(1 + g_m R_L)$ but for short-circuit output $R_L = 0$, so $C_M = C_\mu$.
   Total input capacitance $C_{in} = C_\pi + C_\mu$.
   Input impedance $Z_{in} = r_\pi \parallel \frac{1}{sC_{in}}$.
2. **Source current** I_s splits between R_s and Z_{in}:
   $V_\pi = I_s \cdot (Z_{in} \parallel R_s)$? Better: voltage divider.

Actually, $V_\pi = I_s \cdot (R_s \parallel Z_{in})$? Let's derive properly:
Total resistance from base to ground = Z_{in}. Source current I_s flows through parallel combination of R_s and Z_{in}? No – I_s is the current from source; the base node voltage $V_\pi = I_s \cdot (R_s \parallel Z_{in})$ **only if** the source is ideal current source. For a practical voltage source with series R_s, it's easier to use voltage division.
Better approach: The source is a current source I_s with parallel R_s (Norton equivalent). Then:

$$V_\pi = I_s \cdot (R_s \parallel Z_{in})$$

where $Z_{in} = r_\pi \parallel \frac{1}{s(C_\pi + C_\mu)}$.

3. **Short-circuit collector current:** $I_c = g_m V_\pi$.
4. **Current gain:**

$$A_i(s) = \frac{I_c}{I_s} = g_m \cdot (R_s \parallel Z_{in})$$

Let $C_T = C_\pi + C_\mu$. Then

$$Z_{in} = \frac{r_\pi}{1 + s r_\pi C_T}$$

Parallel combination:

$$R_s \parallel Z_{in} = \frac{R_s \cdot \frac{r_\pi}{1 + s r_\pi C_T}}{R_s + \frac{r_\pi}{1 + s r_\pi C_T}} = \frac{R_s r_\pi}{R_s(1 + s r_\pi C_T) + r_\pi}$$

$$= \frac{R_s r_\pi}{R_s + r_\pi + s R_s r_\pi C_T}$$

5. **Thus:**

$$A_i(s) = g_m \cdot \frac{R_s r_\pi}{R_s + r_\pi} \cdot \frac{1}{1 + s \frac{R_s r_\pi}{R_s + r_\pi} C_T}$$

Define $\beta_0' = g_m \cdot \frac{R_s r_\pi}{R_s + r_\pi}$ and time constant $\tau = \frac{R_s r_\pi}{R_s + r_\pi} C_T$.

6. **Final expression:**

$$A_i(s) = \frac{\beta_0'}{1 + s\tau}$$

Final Answer:

$$\boxed{A_i(j\omega) = \frac{g_m(R_s \parallel r_\pi)}{1 + j\omega(R_s \parallel r_\pi)(C_\pi + C_\mu)}}$$

where $R_s \parallel r_\pi = \frac{R_s r_\pi}{R_s + r_\pi}$.

Q.7 – Explain why junction capacitances exist in the hybrid-π model but not in the h-parameter model.
(3 marks)
Appeared in: W24 (Q1a)
Ans:

- **Hybrid-π model** is a **high-frequency model** that includes parasitic junction capacitances (C_{be}, C_{bc}) because at high frequencies, these capacitances provide significant reactive paths and cannot be ignored.
- **h-parameter model** is a **low-frequency (or mid-band) model** that treats the transistor as a linear two-port network using incremental resistances and dependent sources. It assumes **capacitive reactances are infinite** (open circuit) at low frequencies, hence capacitances are omitted.
- **Validity:** h-parameters are valid only for frequencies where the effect of junction capacitances is negligible (typically below $f_\beta/10$). Hybrid-π model is valid up to f_T.

Real-world application: In audio amplifiers (20 Hz–20 kHz), h-parameters suffice. In RF circuits (MHz–GHz), hybrid-π model with capacitances is mandatory.

Q.8 – Derive the input conductance for the hybrid-π model in CE configuration.
(7 marks)
Appeared in: W24 (Q2c)
Ans:
Given: Hybrid-π model of BJT in CE configuration including r_π, C_π, C_μ, g_m, and load R_L.
To find: Input conductance $G_{in} = \frac{1}{Z_{in}} = \frac{I_b}{V_{be}}$ (real part) at high frequency.
Assumptions: Miller effect applied to C_μ because $V_{ce} = -g_m V_\pi R_L$ (for CE with load).

Solution:

1. **Miller capacitance:** The feedback capacitance C_μ appears at input as
$$C_M = C_\mu(1 + g_m R_L)$$

(since voltage gain $A_v = -g_m R_L$).

2. **Total input capacitance:**
$$C_{in} = C_\pi + C_\mu(1 + g_m R_L)$$

3. **Input admittance:**
$$Y_{in} = \frac{1}{r_\pi} + j\omega C_{in}$$

4. **Input conductance** is the real part:
$$G_{in} = \frac{1}{r_\pi}$$

(Independent of frequency in this ideal model – r_π remains constant. However, at very high frequencies, C_μ causes an additional resistive component due to feedback, but typically negligible.)

5. **More accurate expression** including r_o and C_μ feedback resistance:
   The real part can be affected by r_μ (feedback resistor), but in standard hybrid-π,
$$G_{in} = \frac{1}{r_\pi} + \frac{1}{r_\mu}$$

where $r_\mu \approx \beta_0 r_o$ (large). So $G_{in} \approx 1/r_\pi$.

Final Answer:
$$\boxed{G_{in} = \frac{1}{r_\pi}}$$

Input conductance of the hybrid-π model in CE is simply the reciprocal of base-emitter resistance, independent of frequency up to f_T.

Diagram Prompt for Hybrid-π Model (CE):

[DG PROMPT]

Title: High-frequency hybrid-π model of BJT in CE configuration

Description: Draw a vertical line for the base node (left), collector node (right), and emitter node (bottom). From base to emitter, place a resistor labeled r_π in parallel with a capacitor labeled C_π. From base to collector, place a capacitor labeled C_μ. From collector to emitter, place a current source $g_m v_\pi$ (arrow pointing from collector to emitter) and a resistor r_o in parallel. Label the base terminal B, collector C, emitter E. Add an external load resistor R_L from collector to emitter. Show input voltage V_{be} across r_π and C_π.

Proportions: r_π and C_π side by side; C_μ angled from base to collector. Use arrows for I_b into base, I_c into collector.

Q.9 – Derive the base-spreading resistance for the hybrid-π model in CE configuration. (7 marks)

Appeared in: W24 (Q2c OR)

Ans:

Given: Base-spreading resistance $r_{bb'}$ (or r_x) is the ohmic resistance of the neutral base

region between the base terminal and the intrinsic base region where r_π and C_π are located.
To find: Expression for $r_{bb'}$ in terms of physical parameters and its placement in the hybrid-π model.
Solution:

1. **Physical origin:** In a BJT, the base region has a finite resistivity. The extrinsic base (contact to active region) introduces a series resistance $r_{bb'}$. It is **not** a function of bias (ohmic).
2. **Hybrid-π model inclusion:** $r_{bb'}$ is placed in series with the base terminal, before the internal node B' where r_π and C_π connect.
3. **Derivation from measurement:** $r_{bb'}$ can be extracted from:
    - **Input impedance at very high frequency** – At frequencies where C_π shorts r_π, the input impedance approaches $r_{bb'}$.
    - **Noise figure measurements** – $r_{bb'}$ contributes thermal noise.
    - **Cut-off frequency f_T roll-off** – $r_{bb'}$ limits charging of C_π and reduces effective f_T.
4. **Mathematical expression** (approximate from device physics):

$$r_{bb'} = \frac{\rho_B \cdot W_B}{3 \cdot L_E \cdot Z_E}$$

where ρ_B = base resistivity, W_B = base width, L_E = emitter length, Z_E = emitter width.
Typical values: 10 Ω to 500 Ω.

5. **Effect on gain:** $r_{bb'}$ forms a voltage divider with r_π, reducing V_π:

$$V_\pi = V_{be} \cdot \frac{r_\pi}{r_{bb'} + r_\pi}$$

Hence the effective transconductance becomes $g'_m = g_m \cdot \frac{r_\pi}{r_{bb'}+r_\pi}$.
Final Answer:
Base-spreading resistance $r_{bb'}$ is a lumped ohmic resistance (typically 10–500 Ω) placed between the base terminal B and the internal base node B' in the hybrid-π model. It reduces input impedance and high-frequency gain.
Diagram Prompt (Hybrid-π with $r_{bb'}$):
[DG PROMPT]
Title: Hybrid-π model including base-spreading resistance
Description: Draw the standard hybrid-π model. Add a resistor $r_{bb'}$ in series with the base terminal before the parallel combination of r_π and C_π. Label external base as B, internal node as B'. Connect $r_{bb'}$ from B to B'. From B' to emitter E, place r_π and C_π in parallel.
Keep C_μ from B' to collector. All other components same. Annotate: $r_{bb'}$ = base spreading resistance.

Q.10 – Explain how and why transconductance varies with $| I_C |$, $| V_{CE} |$, and temperature at high frequency.
(4 marks)
Appeared in: W23 (Q1b)
Ans:
Transconductance $g_m = I_C/V_T$ depends on bias and temperature. At high frequencies, the **same DC relationship holds**, but the **effective** g_m may be reduced due to parasitic elements.

Parameter	Variation	Why?
(I_C) (collector current)	$g_m \uparrow$ linearly with I_C	$g_m = I_C/V_T$ – direct proportionality
(V_{CE}) (collector-emitter voltage)	$g_m \uparrow$ slightly with V_{CE} (Early effect)	Higher V_{CE} increases I_C (Early effect) $\Rightarrow g_m$ increases marginally. At high frequencies, larger V_{CE} reduces C_{bc} (depletion capacitance), improving bandwidth.
Temperature	$g_m \downarrow$ as $T \uparrow$	$V_T = kT/q$ increases with temperature, so $g_m = I_C/V_T$ decreases for fixed I_C.

High-frequency consideration: Although g_m itself is frequency-independent up to f_T, the **gain** rolls off due to C_π and C_μ. At very high frequencies, the **effective** transconductance seen at output is reduced by phase shift and current division through C_π.
Real-world application: Biasing circuits for RF amplifiers must stabilize g_m against temperature (using current mirrors) and choose I_C to optimize bandwidth vs. power consumption.

Q.11 – Derive the emitter diffusion capacitance in terms of base width, diffusion constant, and transconductance.
(7 marks)
Appeared in: W23 (Q2c OR)
Ans:
Given: Emitter diffusion capacitance C_d (part of C_π) arises from minority carrier storage in the base region under forward bias.
To find: Expression for C_d in terms of base width W_B, diffusion constant D_n (for npn), and transconductance g_m.
Assumptions: One-dimensional uniform base, no recombination, low-level injection.
Solution:

1. **Physical origin:** Stored minority carrier charge Q_n in the base is proportional to collector current:

$$Q_n = \tau_f \cdot I_C$$

where τ_f is the forward transit time.

2. **Transit time** τ_f for a uniform base is:

$$\tau_f = \frac{W_B^2}{2D_n}$$

(from diffusion equation: time to diffuse across base width).

3. **Diffusion capacitance** is defined as the derivative of stored charge with respect to V_{BE}:

$$C_d = \frac{dQ_n}{dV_{BE}} = \tau_f \cdot \frac{dI_C}{dV_{BE}} = \tau_f \cdot g_m$$

4. **Substitute** τ_f:

$$C_d = \frac{W_B^2}{2D_n} \cdot g_m$$

5. **Alternative form** using $g_m = I_C/V_T$:

$$C_d = \frac{W_B^2}{2D_n} \cdot \frac{I_C}{V_T}$$

Final Answer:

$$\boxed{C_d = g_m \cdot \frac{W_B^2}{2D_n}}$$

where W_B = base width (m), D_n = diffusion constant for electrons in base (m²/s), g_m = transconductance (S).
Numerical example: If $W_B = 0.5\ \mu m$, $D_n = 20\ cm^2/s = 0.002\ m^2/s$, $g_m = 0.1\ S$, then $C_d = 0.1 \times (0.5 \times 10^{-6})^2/(2 \times 0.002) = 0.1 \times (0.25 \times 10^{-12})/0.004 = 0.1 \times 6.25 \times 10^{-1} \ \ = 6.25\ pF$.
Real-world application: Determines the upper frequency limit of bipolar transistors; small W_B and high D_n reduce C_d, increasing f_T.

Q.12 – Explain the validity of the hybrid-π model.
(3 marks)
Appeared in: S23 (Q2a), W24 (Q1a)
Ans:
The **hybrid-π model** is valid under the following conditions:

- **High-frequency operation** – It accurately represents transistor behavior up to the unity-gain frequency f_T (typically MHz to GHz) because it includes junction capacitances C_π and C_μ.
- **Small-signal condition** – The model is linearized around the DC operating point, valid for signal amplitudes much smaller than the thermal voltage V_T (~26 mV).
- **Forward active region** – The transistor must be biased in the active region (V_{BE} forward, V_{CB} reverse or small forward).
- **Lumped approximation** – Valid when the physical dimensions of the transistor are much smaller than the wavelength of the signal (i.e., distributed effects negligible).
- **Limitations** – Fails at frequencies approaching f_T or above due to parasitic inductances and transmission line effects. Also inaccurate for large-signal switching.

Real-world application: Used extensively in RF amplifier design, oscillator analysis, and high-speed digital circuit simulation (e.g., in SPICE models like Gummel-Poon).

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