GUJARAT TECHNOLOGICAL UNIVERSITY

BE-2 SEMESTER – OLD PAPER – S22 TO W24 – QUESTION BANK

Subject Name & Code:

Mathematics - 2- 3110015

Unit 1: Vector Calculus (20% Weightage)

Repeated Questions:

1. **Verification of Green's Theorem:** Appears in almost every paper.
    - **Q.1 (c) - SUM 23 (07 Marks):** Verify Greens theorem for $F = x^2 i + xy^2 j$, along the square bounded by $x = 0, x = 1, y = 0, y = 1$.
    - **Q-1 (c) - WIN 22 (Marks NA):** Evaluate $\int_C [(2xy - x^2)dx + (x + y^2)dy]$ using Green's theorem where C is closed curve formed by $y = x^2$ and $x = y^2$.
    - **Q.1 (c) - WIN 23 (07 Marks):** Verify Green's theorem in the plane for $\oint_C (xy^2 - 2xy)dx + (x^2 y + 3)dy$, where C is the rectangle with vertices $(-1, 0), (1, 0), (1, 1), (-1, 1)$.
    - **Q.1 (c) - SUM 24 (07 Marks):** Evaluate $\iint\limits_C (2x^2 - y^2)dx + (x^2 + y^2)dy$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.
    - **Q.1 (c) - WIN 24 (07 Marks):** Verify Green's theorem for $\iint\limits_C (x - 2y)dx + xdy$ around the circle $x^2 + y^2 = 4$.
2. **Finding Work Done:**
    - **Q.1 (b) - SUM 24 (04 Marks):** If a force $F = 2x^2 y\hat{i} + 3xy\hat{j}$ displaces a particle from (0,0) to (1,4) along $y = 4x^2$. Find work done.
    - **Q-2 (c)(ii) - WIN 22 (Marks NA):** Find the work done by $\overline{F} = 2xyz\hat{i} + (x^2 z + 2y)\hat{j} + x^2 y\hat{k}$ from $(0, 1, 1)$ to $(1, 2, 0)$.

3. **Conservative Field & Scalar Potential:**
   - **Q.1 (b) - SUM 23 (Marks NA):** For $F = 3x^2yzi + x^3zj + x^3yk$, show irrotational and find scalar ϕ.
   - **Q.1 (b) - WIN 23 (Marks NA):** For $\vec{F} = 3x^2y\hat{i} + (x^3 - 2yz^2)\hat{j} + (3z^2 - 2y^2z)\hat{k}$, find (i) scalar potential ϕ, (ii) work done from (2,1,1) to (3,0,1).
   - **Q-2 (c)(i) - WIN 22 (Marks NA):** If $\overline{F} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + x^2y\hat{k}$ is conservative, find its scalar potential ϕ.

Other Important Questions:

- **Q.1 (a) - SUM 23 (Marks NA):** Directional derivative of $f(x, y, z) = x^2 + 5y^2 + 3z^2$ at (1,1,1) in direction 3i+4j+5k.
- **Q-1 (a) - WIN 22 (Marks NA):** Find $curl\overline{F}$ for $\overline{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$.
- **Q.3 (a) - WIN 22 (Marks NA):** Find constant b such that $\overline{A} = (bx + 4y^2z)\hat{i} + (x^3sinz - 3y)\hat{j} - (e^x + 4cosx^2y)\hat{k}$ is solenoidal.
- **Q.1 (a) - WIN 23 (03 Marks):** Find constant a for $(x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ to be solenoidal.
- **Q.2 (a) - SUM 22 (03 Marks):** Find divergence of $F = (x^2 - y)i + (xy - y^2)j$.
- **Q.1 (a) - SUM 24 (03 Marks):** Find curl of $\vec{v} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at (2, -1, 1).
- **Q.1 (a) - WIN 24 (03 Marks):** Find a such that $(x + 3y)^2 + (y - 2z)^2 + (x + az)^2$ is solenoidal.
- **Q.3 (a) - WIN 24 (03 Marks):** Find arc length of curve $\bar{r}(t) = t^2\hat{i} + t^3\hat{j}$ between (1,1) and (4,8).
- **Q.2 (c) - SUM 22 (07 Marks):** Integrate $f(x, y, z) = 3x^2 - 2y + z$ over the line segment from origin to (2,2,2).

Unit 2: Laplace Transforms (20% Weightage)

Repeated Questions:

1. **Solving IVP using Laplace Transform:**
   - **Q.3 (c) - SUM 23 (Marks NA):** Solve $y'' - 5y' - 6y = e^{3t}$ with $y(0) = 3, y'(0) = 2$.
   - **Q.3 (c) - WIN 22 (Marks NA):** Solve $\frac{dy}{dt} + y = cos2t, y(0) = 1$.
   - **Q.2 (c) - WIN 23 (Marks NA):** Solve $y'' - y = t, y(0) = 1, y'(0) = -1$.
   - **Q.3 (c) - SUM 22 (07 Marks):** Solve $y'' - y' - 2y = 0, y(0) = 1, y'(0) = 0$.
   - **Q.2 (c) OR - SUM 24 (07 Marks):** Solve $y'' + 4y' + 8y = 1, y(0) = 0, y'(0) = 1$.
   - **Q.3 (c) - WIN 24 (07 Marks):** Solve $y'' - 3y' + 2y = 4t, y(0) = 1, y'(0) = -1$.
2. **Inverse Laplace using Convolution Theorem:**
   - **Q.3 (b) - SUM 23 (Marks NA):** Find Inverse Laplace of $\frac{1}{(s-1)(s-2)(s-3)}, \frac{1}{s^4-9s^2}, \frac{s+1}{s^2+2s+10}, tan^{-1}(\frac{s}{4})$.
   - **Q-2 (c) - WIN 22 (Marks NA):** Find inverse Laplace of $\frac{1}{(s^2+4)^2}$ using convolution.
   - **Q.2 (c) - SUM 24 (07 Marks):** Find inverse Laplace of $\frac{1}{(s^2+a^2)^2}$ using convolution theorem.
3. **Second Shifting Theorem:**
   - **Q.3 (a) OR - WIN 22 (Marks NA):** State Second Shifting theorem. Find inverse Laplace of $\frac{e^{-as}}{s}$.
   - **Q.3 (b) - SUM 22 (04 Marks):** State second shifting theorem and find inverse Laplace of $\frac{se^{-\pi s}}{s^2+1}$.
   - **Q.4 (a) - SUM 24 OR (03 Marks):** Find Laplace transform of $t^2u(t-2)$.

Other Important Questions:

- **Q.3 (a) - SUM 23 (Marks NA):** Find Laplace of (i) e^{3tt} (ii) t cos3t (iii) $\frac{\sin^2 5t}{t}$.
- **Q-2 (a) - WIN 22 (Marks NA):** Find Laplace of $t^2 - e^{-2t} + cosh^2 3t$.
- **Q.2 (a) - WIN 23 (Marks NA):** (i) $L[e^{-t}t^5]$ (ii) $L^{-1}\left[\frac{1}{s^2+2s+2}\right]$.
- **Q.2 (b) - WIN 23 (Marks NA):** (i) $L\left[\frac{1-\cos 2t}{t}\right]$ (ii) $L^{-1}\left[\frac{54}{(s^2+9)(s^2-9)}\right]$.
- **Q.3 (a) - SUM 22 (03 Marks):** Convolution of t and e^t.
- **Q.3 (b) - SUM 22 (04 Marks):** Laplace of $\frac{\cos at-\cos bt}{t}$.
- **Q.3 (a) - SUM 22 (03 Marks):** Inverse Laplace of $\frac{s-4}{s^2-4}$.
- **Q.3 (c) - SUM 22 (07 Marks):** State convolution theorem and find inverse Laplace of $\frac{1}{s(s^2+4)}$.
- **Q.2 (a) - SUM 24 (03 Marks):** Laplace of $f(t) = \int_0^t \frac{\sin t}{t} dt$.
- **Q.3 (b) - SUM 24 (04 Marks):** Inverse Laplace of $\frac{-s}{s^2+\pi e^{-s}}$.
- **Q.3 (b) - WIN 24 (04 Marks):** Laplace of $\frac{e^{-t}\sin t}{t}$.
- **Q.3 (a) OR - WIN 24 (03 Marks):** Inverse Laplace of $\tan^{-1} s$.
- **Q.4 (a) - WIN 24 (03 Marks):** Inverse Laplace of $\frac{e^{-\pi s}}{s^2-2s+2}$.
- **Q.4 (b) - WIN 24 OR (Marks NA):** Laplace of $\sin\sqrt{t}$.
- **Q.5 (a) - WIN 24 OR (Marks NA):** Laplace of $\iint_0^t \sin at\, dt\, dt$.

Unit 3: Fourier Integrals (~5-7% Weightage - Part of 20% with Laplace)

Repeated Questions:

1. **Fourier Sine/Cosine Integral of e^{-kx}:**
   - **Q.2 (b) - WIN 22 (Marks NA):** Find Fourier cosine integral of $f(x) = \frac{\pi}{2}e^{-x}, x \geq 0$.
   - **Q.2 (b) - SUM 22 (04 Marks):** Find Fourier cosine integral of $f(x) = e^{-kx}(x > 0, k > 0)$.
   - **Q.2 (b) - SUM 24 (04 Marks):** Find Fourier cosine integral of $f(x) = e^{-kx}, x > 0, k > 0$.
   - **Q.2 (b) - WIN 24 (04 Marks):** Find Fourier sine integral of $f(x) = e^{-bx}$.

Other Important Questions:

- **Q.3 (c) - SUM 23 (Marks NA):** Find Fourier integral of $f(x) = \begin{cases} 0, & |x| < 5 \\ 5, & |x| > 5 \end{cases}$.
- **Q.2 (c) OR - WIN 23 (Marks NA):** Find Fourier sine integral of $f(x) = e^{-bx}$ and show that $\frac{\pi}{2}e^{-bx} = \int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + b^2} d\lambda$.

Unit 4: First Order Ordinary Differential Equations (14% Weightage)

Repeated Questions:

1. **Exact Differential Equations:**
    - **Q.4 (c)(ii) - SUM 23 (07 Marks):** Solve $(x^3 + y^3)dx - xy^2dy = 0$.
    - **Q.3 (c)(ii) - WIN 23 (Marks NA):** Solve $(x^3 + y^3)dx - xy^2dy = 0$.
    - **Q.5 (b) - SUM 22 (Marks NA):** Solve $(y \cos x + 2xe^y) + (sinx + x^2e^y - 1)y' = 0$.
2. **Linear Differential Equations:**
    - **Q.4 (c)(i) - SUM 23 (07 Marks):** Solve $\frac{dy}{dx} + y \tan x = \sin 2x$, $y(0) = 2$.
    - **Q.3 (a) - WIN 23 (Marks NA):** Solve $\frac{dy}{dx} + y = e^{-x}$, $y(1) = 1$.
    - **Q.4 (a) - SUM 22 (03 Marks):** Solve $\frac{dy}{dx} - 2y = 4 - x$.
3. **Equations Solvable for p (Clairaut's, etc.):**
    - **Q.4 (c)(i) - SUM 23 OR (07 Marks):** Solve $(y - px)(p^2 + 1) = \tan^{-1} p$.
    - **Q.4 (c)(ii) - SUM 23 OR (07 Marks):** Solve $p^2x^2 = x^2 + p^2$.
    - **Q.3 (c)(i) - WIN 23 (Marks NA):** Solve $p = \sin(y - px)$.
    - **Q.3 (c)(i) - SUM 24 (07 Marks):** Solve $y + px = x^4p^2$.
    - **Q.3 (c)(ii) - SUM 24 (07 Marks):** Solve $p^2 - xp + y = 0$ (Clairaut's Form).
    - **Q.4 (b) - SUM 22 (04 Marks):** Solve $p^2 + 2py \cot x = y^2$.
    - **Q.5 (b) - SUM 22 (Marks NA):** Solve $y = 2px + p^2y$.
    - **Q.3 (b) - WIN 24 (04 Marks):** Solve $x^2p^2 + 3xyp + 2y^2 = 0$.

Other Important Questions:

- **Q.4 (a) - SUM 23 (03 Marks):** Solve $e^x \cos y dx - e^x \sin y dy = 0$.
- **Q.4 (a) - SUM 23 OR (03 Marks):** Solve $(x^2 - y^2)dx + xydy = 0$ (Homogeneous).
- **Q.1 (b) - WIN 24 (04 Marks):** Solve $ye^x dx + (2y + e^x)dy = 0$.
- **Q.3 (a) OR - SUM 24 (03 Marks):** Solve $\left(y^2 - x^2\right) dx + 2xydy = 0$ (Homogeneous).

- **Q.4 (a) - SUM 24 (03 Marks):** Solve $9yy' + 4x = 0$ (Variable Separable).
- **Q.3 (c)(i) - WIN 23 (Marks NA):** Solve $(x + y - 1)dx + (2x + 2y - 3)dy = 0$.
- **Q.3 (a) - WIN 23 (Marks NA):** Solve $y^2 \frac{dx}{dy} + yx = y^3 x^2$ (Bernoulli's).
- **Q-1 (b) - WIN 22 (Marks NA):** Solve $x \frac{dy}{dx} + y + 1 = 0$.

Unit 5: Higher Order Linear ODEs (26% Weightage)

Repeated Questions:

1. **Method of Undetermined Coefficients:**
   - **Q.5 (a) - SUM 23 (03 Marks):** Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = e^{-4x} - 12x$.
   - **Q.5 (b) - SUM 23 (04 Marks):** Solve $\frac{d^2y}{dx^2} + 4y = \sin x$.
   - **Q-4 (c) - WIN 22 (Marks NA):** Solve $y'' + 4y = 2\sin 3x$.
   - **Q.4 (c) - WIN 23 (Marks NA):** Solve $y'' + 9y = 2x^2$.
   - **Q.5 (b) - SUM 23 OR (04 Marks):** Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x^2$.
   - **Q.2 (c) OR - WIN 24 (07 Marks):** Solve $(D^2 - 9)y = x + e^{2x} - \sin 2x$.
2. **Method of Variation of Parameters:**
   - **Q.4 (c) - SUM 24 (07 Marks):** Solve $\frac{d^2y}{dx^2} + y = \sin x$.
   - **Q.4 (c) - WIN 23 OR (07 Marks):** Solve $y'' - 2y' + y = xe^x \sin x$.
   - **Q.4 (c) - SUM 22 (07 Marks):** Solve $y'' + 4y = 4\tan 2x$.
   - **Q.5 (c)(i) - SUM 23 OR (07 Marks):** Solve $\frac{d^2y}{dx^2} + 16y = \cot 4x$.
   - **Q.4 (c) - WIN 24 (07 Marks):** Solve $(D^2 - 2D + 2)y = e^x \tan x$.
3. **Euler-Cauchy Equations:**
   - **Q.4 (b) - SUM 23 (04 Marks):** Solve $(x^2D^2 - 7xD + 12)y = x^2$.
   - **Q.5 (c)(ii) - SUM 23 OR (07 Marks):** Solve $\frac{x^2d^2y}{dx^2} - 6x\frac{dy}{dx} + 6y = x^2 + \frac{1}{x^2}$.
   - **Q.4 (b) - SUM 22 (04 Marks):** Solve $x^2y'' - 3xy' + 4y = 0$.
   - **Q.5 (b) - SUM 24 (Marks NA):** Solve $(x^2D^2 - xD + 2)y = 6$.

Other Important Questions:

- **Q.4 (b) - SUM 23 (04 Marks):** Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0, y(0) = 1, y'(0) = 2.$
- **Q.4 (b) - SUM 23 OR (04 Marks):** Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 14y = 0, y(0) = 3, y'(0) = 1.$
- **Q-4 (a) - WIN 22 (Marks NA):** Find second solution of $y'' + 2y' + y = 0$ given $y_1 = e^{-x}$.
- **Q-4 (a) - WIN 22 OR (Marks NA):** Solve $2D^2y + Dy - 6y = 0.$
- **Q-4 (b) - WIN 22 (Marks NA):** Solve $(D^2 - 2D + 1)y = 10e^x.$
- **Q.3 (b) - WIN 23 (Marks NA):** Solve (i) $(D^2 + 7D - 18)y = 0$, (ii) $y'' + y' + 2y = 0.$
- **Q.4 (a) - WIN 23 (Marks NA):** Solve $(D^2 + 3D + 2)y = \sin 2x.$
- **Q.4 (a) - WIN 23 OR (Marks NA):** Solve $(D^2 + 4D)y = x + x^2.$
- **Q.4 (b) - WIN 23 (Marks NA):** Solve $x^3y''' + x^2y'' = x^3.$
- **Q.4 (b) - WIN 23 OR (Marks NA):** Solve $(D^2 - 6D + 9)y = x^2e^{3x}.$
- **Q.3 (a) - SUM 24 (03 Marks):** Solve $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0.$
- **Q.4 (b) - SUM 24 (Marks NA):** Solve $(D^2 + 9)y = 2\sin 3x + \cos 3x.$
- **Q.4 (c) - SUM 24 OR (Marks NA):** Solve $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x.$
- **Q.4 (b) - WIN 24 (04 Marks):** Solve $(D^2 + 1)y = e^{-x}.$
- **Q.5 (b) - SUM 24 (Marks NA):** Solve $(D^2 - 1)y = xe^x.$