GUJARAT TECHNOLOGICAL UNIVERSITY

BE-1 SEMESTER – OLD PAPER – S22 TO W24 – QUESTION BANK

Subject Name & Code:

Mathematics -1- 3110014

Unit 1: Calculus - I (L'Hospital's Rule, Improper Integrals, Beta/Gamma, Applications)

Repeated Questions:

1. **Evaluating Limits using L'Hospital's Rule:**
   - **Q.1 (a) - SUM 24 (03 Marks):** Evaluate $\lim_{x\to 1} \frac{x^x - x}{1-x+\log x}$.
   - **Q.1 (b) - WIN 24 (04 Marks):** Evaluate $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.
   - **Q.1 (a) - WIN 23 (03 Marks):** Evaluate $\lim_{x\to 0^+} x \ln x$.
2. **Beta and Gamma Functions:**
   - **Q.1 (b) - SUM 24 (04 Marks):** Define Beta function and evaluate $\int_0^1 x^{1/5}(1 - x^{1/3})^5 dx$.
   - **Q.1 (b) - WIN 23 (04 Marks):** Define beta and gamma functions and state their relationship.
   - **Q.3 (a) - WIN 22 OR (03 Marks):** Define Beta function. Evaluate $\int_0^1 x^4(1 - \sqrt{x})^5 dx$.

Non-Repeated Questions:

- **Q.1 (a) - WIN 22 (03 Marks):** Evaluate $\lim_{x\to 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$.
- **Q.1 (a) - SUM 23 (03 Marks):** Evaluate $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.
- **Q.1 (b)(1) - SUM 22 (02 Marks):** Find $\lim_{x\to a} \frac{\sin x - \sin a}{(x-a)^2}$.
- **Q.1 (b)(2) - SUM 22 (02 Marks):** Check convergence of $\int_0^\infty \frac{\sin^2 x}{x^2} dx$.
- **Q.3 (a) - SUM 24 OR (03 Marks):** Check convergence of $\int_0^5 \frac{1}{\sqrt{x}} dx$.

- **Q.3 (b) - SUM 24 OR (04 Marks):** Evaluate $\int_0^\infty 3x^4 e^{-x} dx$ (Gamma function).
- **Q.3 (a) - WIN 24 OR (03 Marks):** Evaluate $\int_0^1 (\ln x)^5 dx$ (Gamma function).
- **Q.1 (c)(2) - SUM 22 (03 Marks):** Prove $\Gamma(n) = (n-1)\Gamma(n-1)$.
- **Q.1 (c)(1) - SUM 22 (04 Marks):** Find arc length of $f(x) = \frac{x^3}{12} + \frac{1}{x}, 1 \leq x \leq 4$.
- **Q.3 (b) - SUM 23 (04 Marks):** Find arc length of $f(x) = \ln(\cos x), 0 \leq x \leq \pi/4$.
- **Q.2 (c) - SUM 24 (07 Marks):** Find area of surface of revolution of a quadrant of a circular arc about a tangent.
- **Q.2 (c) OR - SUM 24 (07 Marks):** Find length of the loop of the curve $9ay^2 = (x-2a)^2(x-5a)$.
- **Q.4 (c) - WIN 24 OR (07 Marks):** Find length of parabola $x^2 = 4y$ inside circle $x^2 + y^2 = 6y$.

Unit 2: Sequences and Series

Repeated Questions:

1. **Testing Convergence of Series (Various Tests):**
   - **Q.3 (a) - WIN 22 (03 Marks):** Test convergence of $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n+1}$ (Divergence Test).
   - **Q.4 (a) - SUM 23 OR (03 Marks):** Discuss convergence of $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$ (Divergence Test).
   - **Q.4 (a) - WIN 24 (03 Marks):** Test convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ (Comparison Test).
   - **Q.5 (a) - WIN 24 (03 Marks):** Test convergence of $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ (Integral Test).
   - **Q.2 (a) - SUM 22 (03 Marks):** Investigate convergence of $\sum_{n=1}^{\infty} \frac{n^2}{7^n}$ (Ratio Test).
   - **Q.3 (b) - SUM 24 (04 Marks):** Test convergence of $\sum_{n=1}^{\infty} \frac{(n^2+1)^{1/2}}{n^2}$ (Limit Comparison Test).
   - **Q.3 (c)(ii) - SUM 23 (04 Marks):** Use Ratio test for $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$.
2. **Telescoping Series:**
   - **Q.1 (b) - SUM 23 (04 Marks):** Show $\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is convergent and find its sum.
   - **Q.2 (b) - WIN 22 (04 Marks):** Show $\sum_{n=0}^{\infty} \left(\frac{e}{n}\right)^n$ is convergent and find its sum. *(Note: This is not telescoping, but appears in the same slot).*
3. **Power Series & Taylor/Maclaurin Series:**
   - **Q.5 (a) - WIN 23 (03 Marks):** Find Maclaurin series for $\cos x$.
   - **Q.5 (a) - SUM 22 (Marks NA):** Find Maclaurin series for $f(x) = e^{2x} \sinh x$ up to x^4.
   - **Q.4 (a) - SUM 24 (03 Marks):** Prove $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$.
   - **Q.2 (b) - WIN 24 (04 Marks):** Expand $x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x-3)$ (Taylor series).
   - **Q.2 (b) - SUM 24 (04 Marks):** Express $f(x) = \frac{2}{x^2+3x-8}$ in terms of $(x-2)$.

Non-Repeated Questions:

- **Q.3 (a) - WIN 22 OR (03 Marks):** Test convergence of $\sum_{n=1}^{\infty} \frac{(n+1)\sin((2n-1)\pi/2)}{n\log(n+1)}$.
- **Q.2 (b) - SUM 22 (04 Marks):** Investigate convergence of $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$ (Ratio Test).
- **Q.3 (b) - WIN 23 (04 Marks):** Determine convergence of $\sum_{n=1}^{\infty} ne^{-n^2}$ (Integral Test).
- **Q.3 (c) - WIN 23 (Marks NA):** Test convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ (Alternating Series Test).
- **Q.5 (b) - SUM 22 (Marks NA):** Show $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges (Alternating Series Test).
- **Q.5 (c) - WIN 23 (Marks NA):** Find interval of convergence of $x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$.
- **Q.4 (b) - WIN 24 OR (04 Marks):** Test convergence of $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$ (Root Test).
- **Q.3 (c)(i) - SUM 23 (03 Marks):** Using Sandwich theorem, find limit of sequence $a_n = \frac{\cos n}{n}$.
- **Q.3 (c)(ii) - SUM 23 (04 Marks):** Define Radius of Convergence. Find it for $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}$.

- **Q.2 (a) - SUM 24 (03 Marks):** Show sequence $u_n = \frac{\sin n}{n}$ converges to 0.
- **Q.3 (a) - WIN 24 (03 Marks):** Show sequence $u_n = \frac{\sin n}{n}$ converges to 0.
- **Q.5 (a) - WIN 23 (Marks NA):** Define monotonic sequence. Is $\left(\frac{1}{n^2}\right)$ monotonic?
- **Q.4 (a) - SUM 23 (03 Marks):** Find sum of series $\sum_{n=0}^{\infty} \frac{2^n - 1}{3^n}$ (Geometric Series).
- **Q.5 (b) - WIN 23 (Marks NA):** Investigate convergence of $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$ (Geometric Series).
- **Q.3 (b) - WIN 22 (04 Marks):** Find approximate value of $\sqrt{25.15}$ using Taylor's theorem.
- **Q.3 (c)(ii) - SUM 23 (04 Marks):** Represent $f(x) = 2x^3 + x^2 + 3x - 8$ in powers of $(x-1)$ (Taylor series).

Unit 3: Fourier Series

Repeated Questions:

1. **Fourier Series of Standard Functions (x, x^2, |x|):**
   - **Q.2 (c) - SUM 22 (07 Marks):** Find Fourier series of $f(x) = x^2, -\pi < x < \pi$.
   - **Q.2 (c) OR - SUM 22 (07 Marks):** Find Fourier series of $f(x) = x, -1 < x < 0$.
   - **Q.2 (c) - WIN 22 (07 Marks):** Find Fourier series of $f(x) = x + |x|, -\pi < x \leq \pi$.
   - **Q.2 (c) OR - WIN 24 (07 Marks):** Find Fourier series of $f(x) = x + |x|, -\pi < x < \pi$.
   - **Q.2 (c) - WIN 23 OR (07 Marks):** Find Fourier series of $f(x) = x + \pi, -\pi < x < \pi$.
2. **Half-Range Expansions:**
   - **Q.2 (c) OR - WIN 22 (07 Marks):** Find half-range sine series of $f(x) = x - x^2$ in (0,1).
   - **Q.4 (b) - SUM 24 (04 Marks):** Find Fourier sine series of $f(x) = e^x$ in $0 < x < \pi$.
   - **Q.3 (b) - WIN 24 OR (04 Marks):** Find cosine series for $f(x) = \pi - x$ in $0 < x < \pi$.

Non-Repeated Questions:

- **Q.2 (c) - SUM 23 (07 Marks):** Find Fourier series of $f(x) = x + x^2; -\pi < x < \pi$.
- **Q.2 (c) OR - SUM 23 (07 Marks):** Find Fourier series for $f(x) = \begin{cases} \sin x, & 0 \leq x \leq \pi \\ 0, & \pi \leq x \leq 2\pi \end{cases}$.
- **Q.1 (c) - SUM 24 (07 Marks):** Find Fourier series of $f(x) = \begin{cases} x + \pi, & -\pi < x < 0 \\ -x + \pi, & 0 < x < \pi \end{cases}$.
- **Q.3 (c) - SUM 24 OR (07 Marks):** Find Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$ and deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$.
- **Q.2 (c) - WIN 24 (07 Marks):** Find Fourier series of $f(x) = \begin{cases} x^2, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$.

Unit 4: Multivariable Calculus - I (Limits, Continuity, Differentiation)

Repeated Questions:

1. **Directional Derivative:**
   - **Q.3 (b) - WIN 23 (04 Marks):** Find directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at (2,1,3) in direction of $\vec{i} - 2\vec{k}$.
   - **Q.2 (a) - WIN 24 (03 Marks):** Find directional derivative of $f(x,y,z) = xy^2 + yz^3$ at (2,-1,1) in direction of $\hat{i} + 2\hat{j} + 2\hat{k}$.
   - **Q.4 (a) - SUM 24 OR (03 Marks):** Find directional derivative of $f(x,y,z) = x^2yz + 4xz^2$ at (1,-2,-1) in direction of $2\hat{i} - \hat{j} - 2\hat{k}$.
2. **Tangent Plane and Normal Line:**
   - **Q.2 (b) - SUM 23 (04 Marks):** Find equations of tangent plane and normal line to surface $\frac{x^2}{2} - \frac{y^2}{3} - z = 0$ at (2,3,-1).
   - **Q.4 (b) - WIN 22 (04 Marks):** Find equations of tangent plane and normal line to surface $x^2yz + 3y^2 = 2xz^2 - 8z$ at (1,2,-1).
   - **Q.1 (a) - WIN 24 (03 Marks):** Find equation of tangent plane and normal line to surface $xyz = 6$ at (1,2,3).
   - **Q.3 (b) - WIN 23 OR (04 Marks):** Find equation of tangent plane to $z = 3x^2 - xy$ at (1,2,1).
3. **Finding Local Extreme Values:**
   - **Q.4 (c) - SUM 23 (07 Marks):** Find local max/min of $f(x,y) = x^3 + 3xy^2 - 15x + y^3 - 15y$.
   - **Q.3 (c) - WIN 23 (07 Marks):** Find local extreme values of $f(x,y) = 4x^2 + 9y^2 + 8x - 36y + 24$.
   - **Q.4 (c) - SUM 24 (07 Marks):** Find extreme values of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.
   - **Q.3 (c) - WIN 24 OR (07 Marks):** Find extreme values of $f(x,y) = x^3 + xy^2 + 21x - 12x^2 - 2y^2$.

Non-Repeated Questions:

- **Q.4 (a) - WIN 22 (03 Marks):** Show $f(x,y) = \frac{x^3 y}{x^4+y^4}$ is not continuous at (0,0).
- **Q.3 (b) - SUM 22 (Marks NA):** Find derivative of $f(x,y) = x^2 + xy + y^2$ in direction $\hat{i} + \hat{j}$ at (1,1).
- **Q.3 (b) - SUM 22 (04 Marks):** Find tangent plane of $z = e^x \cos y$ at (0,0,0).
- **Q.3 (c) - SUM 22 (07 Marks):** Find local extreme values of $f(x,y) = xy - x^2 - y^2 - x$.
- **Q.1 (b) - WIN 22 (04 Marks):** If $z = \tan^{-1}(x/y), x = u\cos v, y = u \sin v$, find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ (Chain Rule).
- **Q.3 (a) - WIN 23 (03 Marks):** If $w = x^2 + y^2, x = r - s, y = r + s$, prove $\frac{\partial w}{\partial s} = 4s$ (Chain Rule).
- **Q.3 (a) - WIN 23 OR (03 Marks):** Determine if $\lim_{(x,y)\to(0,0)} \frac{2x^2 y}{x^3+y^3}$ exists.
- **Q.4 (b) - SUM 23 OR (04 Marks):** Show $\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$ does not exist.
- **Q.3 (c) - WIN 23 OR (07 Marks):** Find max/min of $f(x,y) = 3x + 4y$ on circle $x^2 + y^2 = 1$ (Lagrange Multipliers).
- **Q.3 (c) - SUM 22 OR (07 Marks):** Find min value of $x^2 y z^2$ subject to $x + y + 2z = 5$ (Lagrange Multipliers).
- **Q.4 (c) - SUM 23 OR (07 Marks):** Find max volume of an open box made from 12 m^2 cardboard (Application of Extrema).
- **Q.3 (c) - WIN 24 (07 Marks):** Find max volume of an open box made from 12 m^2 cardboard (Application of Extrema).

Unit 5: Multivariable Calculus - II (Multiple Integrals)

Repeated Questions:

1. **Changing Order of Integration:**
   - **Q.4 (b) - SUM 22 (Marks NA):** Evaluate $\int_0^2 \int_{x/2}^1 \frac{1}{3} e^{y^2} dydx$ by changing order.
   - **Q.5 (c) - WIN 22 (07 Marks):** Evaluate $\int_0^1 \int_{1-\sqrt{1-y^2}}^{1+\sqrt{1-y^2}} dxdy$ by changing order.
   - **Q.4 (b) - WIN 23 (04 Marks):** Sketch region and reverse order for $\int_0^2 \int_{x^2}^{2x} (4x + 2)dydx$.
   - **Q.4 (c) - SUM 24 OR (07 Marks):** Change order and evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{\cos^{-1} x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}} dxdy$.
   - **Q.4 (c) - WIN 24 OR (07 Marks):** Change order and evaluate $\int_0^1 \int_x^\infty e^{-y} dydx$.
2. **Polar Coordinates:**
   - **Q.4 (c)(1) - SUM 22 OR (Marks NA):** Change to polar and solve $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)} dydx$.
   - **Q.4 (c) - WIN 23 OR (07 Marks):** Evaluate $\iint_R xy\sqrt{x^2 + y^2} dxdy$ over $1 \leq x^2 + y^2 \leq 4, x, y \geq 0$ using polar coordinates.
   - **Q.5 (c) - SUM 24 (07 Marks):** Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} dydx$ using polar coordinates.
3. **Triple Integrals:**
   - **Q.5 (a) - WIN 24 OR (03 Marks):** Evaluate $\int_0^1 \int_0^x \int_0^{xyz} dzdydx$.
   - **Q.5 (a) - SUM 24 OR (03 Marks):** Evaluate $\int_0^2 \int_0^y \int_0^{xyz} dzdxdy$.

Non-Repeated Questions:

- **Q.2 (a) - SUM 23 (03 Marks):** Evaluate $\iint_0^1 \int_0^{2\pi} 2xyzdzdydx$. *(Note: Limits suggest double integral)*
- **Q.4 (a) - WIN 23 (03 Marks):** Calculate $\iint_R (100 - 6x^2y)dA$ for $R : 0 \leq x \leq 2, -1 \leq y \leq 1$.
- **Q.4 (c) - WIN 23 (07 Marks):** Calculate $\iint_R \frac{\sin x}{x} dA$ over triangle bounded by y=x, x=1, x-axis.

- **Q.4 (a) - WIN 23 OR (03 Marks):** Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dxdy$.
- **Q.4 (b) - WIN 23 OR (04 Marks):** Find area of region bounded by y=x and y=x^2 in first quadrant.
- **Q.4 (c) - WIN 23 OR (07 Marks):** Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)dydx$ using polar coordinates.
- **Q.5 (a) - SUM 24 (03 Marks):** Evaluate $\int_0^3 \int_0^1 (x^2+3y^2)dydx$.
- **Q.4 (b) - WIN 24 (04 Marks):** Evaluate $\int_0^{\pi/2} \int_0^1 r^2 \cos\theta dr d\theta$.
- **Q.4 (c)(1) - SUM 22 (Marks NA):** Find area of region covered by x=1, x=4, y=0, y=√x.
- **Q.4 (c)(2) - SUM 22 (Marks NA):** Evaluate $\int_0^1 \int_0^{x^{1/4}} \int_0^{y^2} \sqrt{z} dzdydx$.
- **Q.4 (a) - SUM 22 OR (03 Marks):** Evaluate $\iint_R xydA$ over region bounded by x-axis, x=2a, x^2=4ay.
- **Q.4 (b) - SUM 22 OR (Marks NA):** Evaluate $\int_0^1 \int_0^{\cos^{-1} y} e^{\sin x} dxdy$ by changing order.
- **Q.4 (c)(2) - SUM 22 OR (Marks NA):** Find Jacobian for transformation x+y=u, y=uv.
- **Q.5 (a) - WIN 22 (03 Marks):** Evaluate $\int_0^1 \int_x^1 \int_0^{y-x} dzdydx$.
- **Q.5 (b) - WIN 22 (Marks NA):** Evaluate $\iint_R y(x-y^2)dydx$ over region bounded by y=√x, y=x^3.

Unit 6: Linear Algebra (Matrices, Eigenvalues, Diagonalization)

Repeated Questions:

1. **Gauss Elimination / Gauss-Jordan Method:**
   - **Q.1 (c) - WIN 23 (07 Marks):** Solve system using Gauss-Jordan: $x_3 + x_4 + x_5 = 0, -x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0, x_1 + x_2 - 2x_3 - x_5 = 0, 2x_1 + 2x_2 - x_3 + x_5 = 0.$
   - **Q.3 (c) - WIN 22 (07 Marks):** Solve system using Gauss-Jordan: $x_1 + x_2 + 2x_3 = 8, -x_1 - 2x_2 + 3x_3 = 1, 3x_1 - 7x_2 + 4x_3 = 10.$
   - **Q.1 (c) - WIN 24 (07 Marks):** Solve system using Gauss Elimination: $x + y + 2z = 9, 2x + 4y - 3z = 1, 3x + 6y - 5z = 0.$
   - **Q.4 (b) - SUM 24 OR (04 Marks):** Solve system using Gauss-Jordan: $-2y + 3z = 1, 3x + 6y - 3z = -2, 6x + 6y + 3z = 5.$
2. **Finding Eigenvalues and Eigenvectors:**
   - **Q.1 (c) - SUM 23 (07 Marks):** Find eigenvalues/eigenvectors of $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}.$
   - **Q.2 (c) - WIN 23 (07 Marks):** Find eigenvalues/eigenvectors of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}.$
   - **Q.3 (c) - WIN 22 OR (07 Marks):** Find eigenvalues/eigenvectors of $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}.$
   - **Q.5 (c) - SUM 24 OR (07 Marks):** Find eigenvalues/eigenvectors of $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}.$
3. **Cayley-Hamilton Theorem:**
   - **Q.4 (a) - WIN 22 (03 Marks):** Use Cayley-Hamilton to find A^3 for $A = \begin{bmatrix} -3 & -1 & 1 \\ 0 & 2 & 3 \\ 1 & -2 & 1 \end{bmatrix}.$
   - **Q.5 (b) - SUM 24 (04 Marks):** Apply Cayley-Hamilton to $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, deduce $A^8 = 625I.$
   - **Q.5 (b) - WIN 23 (04 Marks):** State and verify Cayley-Hamilton for $A = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}.$

Non-Repeated Questions:

- **Q.2 (a) - WIN 23 (03 Marks):** Define rank. Find rank of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.
- **Q.3 (b) - WIN 22 (04 Marks):** Find rank of $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$.
- **Q.4 (b) - WIN 22 (04 Marks):** Find inverse of $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ using Gauss-Jordan.
- **Q.5 (b) - WIN 24 OR (Marks NA):** Find inverse of $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ using Gauss-Jordan.
- **Q.5 (a) - SUM 22 (Marks NA):** Find characteristic equation of $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$.
- **Q.5 (a) - SUM 22 OR (Marks NA):** Show $A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$ satisfies its characteristic equation.
- **Q.5 (c) - SUM 22 (Marks NA):** Solve system $x + y + w = 1, 2x + z + w = 3, 2y + z + 2w = 2$.
- **Q.5 (c) - SUM 22 OR (Marks NA):** Show $A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 2 \\ -4 & -8 & 7 \end{bmatrix}$ is diagonalizable. Find eigenvector matrix and diagonal matrix.
- **Q.5 (b) - WIN 24 (Marks NA):** Find a and b such that $A = \begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has eigenvalues 3 and -2.
- **Q.5 (c) - WIN 24 (Marks NA):** Show $\begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ is not diagonalizable.
- **Q.5 (c) - WIN 24 (Marks NA):** If $u = f(\frac{y-x}{xy}, \frac{z-x}{xz})$, show $x^2u_x + y^2u_y + z^2u_z = 0$.