GUJARAT TECHNOLOGICAL UNIVERSITY

BE-1 SEMESTER – OLD PAPER – W24 & S25 – QUESTION BANK

Subject Name & Code:

Mathematics -1- BE01000041

Unit 1: Basic Calculus

Repeated Questions:

1. **Evaluate limits using L'Hôpital's Rule.**
   - (Summer 2025 - Q2a - 03 Marks): Evaluate $\lim_{x\to0} (3x - \sin x) / x$
   - (Winter 2024 - Q1a - 03 Marks): Evaluate $\lim_{x\to0} (1/x^2 - 1/\sin^2 x)$
2. **Find volumes of solids of revolution.**
   - (Summer 2025 - Q1c - 07 Marks): Find volume generated by revolving $y = \sqrt{x}$, $0 \le x \le 4$ about the x-axis.
   - (Winter 2024 - Q2c - 07 Marks): Find volume of a sphere generated by rotating $x^2 + y^2 = a^2$ about the x-axis.

Non-Repeated Questions:

- (Summer 2025 - Q1a - 03 Marks): Investigate convergence of $\int_0^1 1/(1-x)\, dx$
- (Winter 2024 - Q2b - 04 Marks): Discuss convergence of $\int_{-\infty}^{\infty} 1/(1+x^2)\, dx$

Unit 2: Single-Variable Calculus (Differentiation)

Repeated Questions:

1. **Find Taylor/Maclaurin series expansion of functions.**
   - (Summer 2025 - Q2b - 04 Marks): Find Taylor series for `f(x) = e^x` at `x=2`.
   - (Summer 2025 - Q2c - 07 Marks): Find Maclaurin series for `f(x) = (1+x)^k` and hence for `1/(1-x)`.
   - (Winter 2024 - Q1c - 07 Marks): Expand `e^x` into a power series and find terms needed to calculate `e` with error < `10^{-4}`.
2. **Find local extreme values of a function.**
   - (Summer 2025 - Q2c (OR) - 07 Marks): Find local extrema of $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$.
   - (Winter 2024 - Q1b - 04 Marks): Use the second derivative test to find extrema of $f(x) = x^4 - 4x^3 + 10$.

Non-Repeated Questions:

- (Winter 2024 - Q1b): The question itself includes the application of the second derivative test.

(b) Write second derivative test for local extrema and find the extrema of $f(x) = x^4 - 4x^3 + 10$ **04**

Unit 3: Sequences and Series

Repeated Questions:

1. **Test the convergence of a given series.**
   - (Summer 2025 - Q3a - 03 Marks): Test convergence of $\sum_{n=1}^{\infty} 1/(n^2+1)$.
   - (Summer 2025 - Q3b - 04 Marks): Test convergence of $\sum_{n=1}^{\infty} (-1)^{n+1}/n$.
   - (Winter 2024 - Q4a - 03 Marks): Test convergence of $\sum_{n=0}^{\infty} 4^n n! / (2n)!$.
   - (Winter 2024 - Q4b - 04 Marks): Test convergence and find sum of $\sum_{n=0}^{\infty} 1/(n(n+1))$.
   - (Winter 2024 - Q4a (OR) - 03 Marks): Test convergence of $\sum_{n=0}^{\infty} (4^n + 3)/5^n$.
2. **Find the interval and radius of convergence of a power series.**
   - (Winter 2024 - Q4c - 07 Marks): For $\sum_{n=0}^{\infty} (x-5)^n / n^2$.
   - (Winter 2024 - Q4c (OR) - 07 Marks): For $\sum_{n=0}^{\infty} (-1)^n x^n / [3^n (n+1)]$.

Non-Repeated Questions:

- (Summer 2025 - Q3c - 07 Marks): Define absolute & conditional convergence. Investigate $\sum_{n=1}^{\infty} \sin n / n^2$.
- (Summer 2025 - Q3c (OR) - 07 Marks): Find x for which $\sum_{n=1}^{\infty} (x-3)^n / n$ converges.
- (Winter 2024 - Q4b (OR) - 04 Marks): This question also asks to find the sum of the convergent series.

Unit 4: Multivariable Calculus – Differentiation

Repeated Questions:

1. **Evaluate limits in two variables / Check existence.**
   - (Summer 2025 - Q4a - 03 Marks): Does $\lim_{(x,y)\to(0,0)} (x y^2)/(x^2 + y^4)$ exist?
   - (Winter 2024 - Q3a - 03 Marks): Determine if $\lim_{(x,y)\to(0,0)} (x^6 - y^2)/(x^3 y)$ exists, find it if it does.
2. **Use the Chain Rule for partial derivatives.**
   - (Summer 2025 - Q4b - 04 Marks): Find dz/dt if $z = x^2y + 3xy^4$, $x = \sin 2t$, $y = \cos t$, at $t=0$.
   - (Winter 2024 - Q3b - 04 Marks): Find dw/dt if $w = x^2y - y^2$, $x = \sin t$, $y = e^t$, at $t=0$.
3. **Find extreme values of functions of two variables (with/without constraints).**
   - (Summer 2025 - Q4c - 07 Marks): Find extrema of $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$ (Constraint).
   - (Winter 2024 - Q3c - 07 Marks): Find extrema of $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$.
   - (Winter 2024 - Q3c (OR) - 07 Marks): Use Lagrange multipliers to find dimensions of an open-top rectangular box of volume 32 ft^3 that uses the least material.

Non-Repeated Questions:

- (Winter 2024 - Q3a (OR) - 03 Marks): If $u = \tan^{-1}(y/x)$, prove $\partial^2u/\partial x^2 + \partial^2u/\partial y^2 = 0$.
- (Winter 2024 - Q3b (OR) - 04 Marks): Find tangent plane & normal line to $x^2 + y^2 + z - 9 = 0$ at $(1, 2, 4)$.

Unit 5: Multivariable Calculus – Integration

Repeated Questions:

1. **Evaluate double integrals over rectangular/general regions.**
   - (Summer 2025 - Q5a - 03 Marks): Evaluate $\int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$
   - (Summer 2025 - Q5b - 04 Marks): Evaluate $\iint_R (3x + 4y^2)\, dA$ where R is between circles $x^2+y^2=1$ and $x^2+y^2=4$ in the upper half-plane.
   - (Winter 2024 - Q5a - 03 Marks): Evaluate $\iint_R y^2 x\, dA$ over $R = \{(x,y): -3 \le x \le 2,\ 0 \le y \le 1\}$.
2. **Change the order of integration.**
   - (Summer 2025 - Q5c - 07 Marks): Sketch region, reverse order, and evaluate $\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$.
   - (Winter 2024 - Q5c - 07 Marks): Change order and evaluate $\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$.
3. **Use double integrals to find areas.**
   - (Summer 2025 - Q5b - 04 Marks): Implied in evaluating the integral over a specific region.
   - (Winter 2024 - Q5b - 04 Marks): Find area between $y = (1/2)x^2$ and $y = 2x$.

Non-Repeated Questions:

- (Summer 2025 - Q5c (OR) - 07 Marks): Evaluate the triple integral $\iiint_B x y z^2\, dV$ for $B = [0,1]\times[-1,2]\times[0,3]$.
- (Summer 2025 - Q5c (OR) - 07 Marks): Find area bounded by $y = 2x^2$ and $y^2 = 4x$.
- (Winter 2024 - Q5b (OR) - 04 Marks): Evaluate $\iint_R e^{x^2+y^2}\, dA$ over semi-circle $y = \sqrt{1-x^2}$ using polar coordinates.
- (Winter 2024 - Q5c (OR) - 07 Marks): Find mass, first moments, and center of mass of a triangular plate with density $\delta(x,y) = 6x + 6y + 6$.