GUJARAT TECHNOLOGICAL UNIVERSITY
BE-3 SEMESTER – PAPER SOLUTION – WINTER 2024

Subject Name & Code:
Mechanics of Solids- 3130608

MARKS

Q.1 **(a)** Define following terms; **03**
i) Rigid body ii) Mass iii) Equilibrant

Answer:

(i) Rigid Body:
A rigid body is an idealization of a solid body in which deformation is neglected. It is assumed that the distance between any two particles of the body remains constant even when external forces are applied.
(ii) Mass:
Mass is the quantity of matter possessed by a body. It is a scalar quantity and is a measure of the body's inertia. Its SI unit is kilogram (kg).
(iii) Equilibrant:
An equilibrant is a force that brings a system of forces into equilibrium. It is equal in magnitude but opposite in direction to the resultant of all the acting forces.

(b) State any two assumptions made in analysis of plane trusses. Plot neat sketch of perfect truss, deficient truss and redundant truss. **04**

Answer:

Assumptions for Truss Analysis:
1. All joints are pin-jointed and allow free rotation.
2. Loads are applied only at joints.
3. The weight of members is negligible compared to the applied loads (or is lumped at joints).
4. Members are connected by frictionless pins.

(Any two of the above can be written.)
Types of Trusses:
1. **Perfect Truss:**
   When the number of members (m) = 2j − 3
   Where j = number of joints
2. **Deficient Truss:**
   When **m < 2j − 3**, truss lacks enough members and is unstable.
3. **Redundant Truss:**
   When **m > 2j − 3**, truss has extra members and is statically indeterminate.

Sketches:

(c) Obtain the magnitude and direction of equilibrant force for the force system shown in Figure-1. **07**

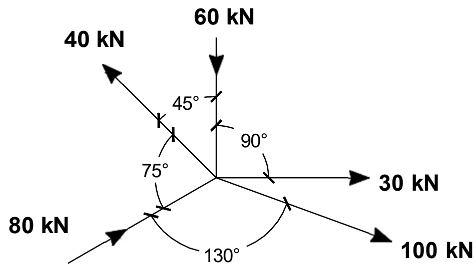

Figure-1

Answer:

We will:

1. Resolve each force into **horizontal (Fx)** and **vertical (Fy)** components.
2. Sum all components:

- $R_x = \sum F_x$
- $R_y = \sum F_y$

3. Find magnitude of resultant:

- $R = \sqrt{R_x^2 + R_y^2}$

4. Direction of resultant:

- $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

5. **Equilibrant** is equal in magnitude but opposite in direction to the resultant.

Let's Solve (From Figure-1):
Refer to given forces (e.g., 60 kN @ 45°, 100 kN @ 90°, etc.)

Force	Magnitude (kN)	Angle (°)	Fx (kN)	Fy (kN)
F1	60	45°	$60\cos 45^\circ = 42.43$	$60\sin 45^\circ = 42.43$
F2	100	90°	$100\cos 90^\circ = 0$	$100\sin 90^\circ = 100$
F3	120	130°	$120\cos 130^\circ = -77.16$	$120\sin 130^\circ = 91.$
F4	80	75°	$80\cos 75^\circ = 20.71$	$80\sin 75^\circ = 77.28$
F5	30	135°	$30\cos 135^\circ = -21.21$	$30\sin 135^\circ = 21.21$
F6	40	0°	$40\cos 0^\circ = 40$	$40\sin 0^\circ = 0$

Step 1: Summing components

$$R_x = 42.43 + 0 - 77.16 + 20.71 - 21.21 + 40 = 4.77 \text{ kN}$$

$$R_y = 42.43 + 100 + 91.93 + 77.28 + 21.21 + 0 = 332.85 \text{ kN}$$

Step 2: Resultant force

$$R = \sqrt{(4.77)^2 + (332.85)^2} = \sqrt{22.76 + 110791.12} = \sqrt{110813.88} \approx 333 \text{ kN}$$

Step 3: Direction of Resultant

$$\theta = \tan^{-1}\left(\frac{332.85}{4.77}\right) \approx \tan^{-1}(69.8) \approx 89.18^\circ$$

Final Answer:

- **Resultant = 333 kN at 89.18° (approx)**
- **Equilibrant = 333 kN at (89.18° + 180°) = 269.18°**

Q.2 (a) State and explain the Law of polygon of forces. **03**

Answer:

Law of Polygon of Forces:
This law states that **if a number of forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order**, then their **resultant is represented by the closing side of the polygon taken in the opposite order**.
Explanation:

- Consider forces $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$, **...**, $\mathbf{F_n}$ acting at a point.
- If we draw these forces **head-to-tail in sequence**, forming a polygon, and the closing side (from last point to the first) represents the **resultant**, then the system is in equilibrium.
- If the polygon is closed (i.e., the resultant = 0), the body is in **equilibrium**.

Diagram (to be drawn by you in exam):

(b) Derive the relation, with usual notations, among loading, shear force and bending moment at a section for a determinate beam. **04**

Answer:

Let:

- w(x) = intensity of distributed load at section x (N/m)
- V(x) = shear force at section x
- M(x) = bending moment at section x

Derivation:

1. Consider a small element of length **dx** cut from a loaded beam.
2. At the left face:
   - Shear force = V
   - Bending moment = M
3. At the right face:

- Shear force = $V + \frac{dV}{dx}dx$
- Bending moment = $M + \frac{dM}{dx}dx$

Applying Equilibrium:
1. Vertical Force Equilibrium:

$$V + w(x)dx - [V + \frac{dV}{dx}dx] = 0 \Rightarrow \frac{dV}{dx} = -w(x)$$

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2. Moment Equilibrium:
Taking moment about left face:

$$M + Vdx + \frac{1}{2}w(x)dx^2 - [M + \frac{dM}{dx}dx] = 0 \Rightarrow \frac{dM}{dx} = V$$

Final Relations:

$$\boxed{\frac{dV}{dx} = -w(x)} \quad \text{and} \quad \boxed{\frac{dM}{dx} = V}$$

(c) Calculate support reactions for the beam shown in Figure-2. Also plot Bending Moment and Shear Force diagrams showing values at important locations. **07**

Answer:

📌 **Beam Description:**

- Simply supported beam AB
- Span L=6 m (from A to B)
- Point load = **120 kN at C (1 m from A)**
- UDL = **60 kN/m from D (2 m from A) to B (total 4 m)**

Step 1: Convert UDL to Point Load

- UDL = 60×4=240 kN
- Acts at center of UDL → 2 m from D → 4 m from A

Step 2: Reactions at Supports A and B

Let:

- RA = Reaction at A
- RB = Reaction at B

Taking **moments about A:**

$$\sum M_A = 0 \Rightarrow -120(1) - 240(4) + R_B(6) = 0$$

$$-120 - 960 + 6R_B = 0 \Rightarrow 6R_B = 1080 \Rightarrow \boxed{R_B = 180 \text{ kN}}$$

Now, vertical force equilibrium:

$$R_A + R_B = 120 + 240 = 360 \Rightarrow R_A = 360 - 180 = \boxed{180 \text{ kN}}$$

✅ **Final Reactions:**

- $\boxed{R_A = 180 \text{ kN}, \; R_B = 180 \text{ kN}}$
-

Step 3: Shear Force Diagram (SFD)

Point	x (m)	SF (kN)
A	0	+180

Just left of C	1	+180
Just right of C	1	180−120=+60
D	2	+60
Just before B	6	60−240=−180
B	6	−180+180=0

Step 4: Bending Moment Diagram (BMD)
Let's calculate bending moment at key points using:

$$M_x = R_A \cdot x - (\text{moment of loads left of section})$$

- At A: MA=0
- At C (x = 1):

$$M_C = 180(1) = \boxed{180 \text{ kNm}}$$

- At D (x = 2):

$$M_D = 180(2) - 120(1) = 360 - 120 = \boxed{240 \text{ kNm}}$$

At point just before B (x = 6):
Total UDL to the left = 240 kN acting at 4 m from A

$$M_B = 180(6) - 120(1) - 240(4) = 1080 - 120 - 960 = 0$$

✅ **SFD and BMD Summary**

🔹 **SFD Values (kN):**

- A: +180
- C: Drop to +60
- D to B: linear drop from +60 to -180
- B: 0

🔹 **BMD Values (kNm):**

- A: 0
- C: 180
- D: 240
- B: 0

OR

(c) Calculate support reactions for the beam shown in Figure-2. Also obtain the location of maximum Bending Moment, Maximum Bending Moment and Maximum Shear Force in the beam. **07**

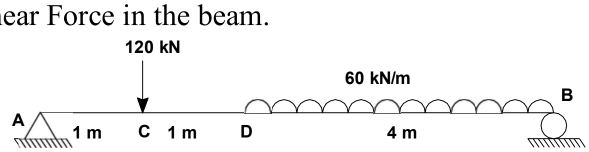

Figure-2

Answer:

Given:

- Simply supported beam AB=6 m

- Point load: 120 kN at 1 m from A (Point C)
- UDL: 60 kN/m from D (x = 2 m) to B (x = 6 m)

✅ **Step 1: Support Reactions**
Already calculated earlier:

- $\boxed{R_A = 180 \text{ kN}, \; R_B = 180 \text{ kN}}$

✅ **Step 2: Shear Force Equation**
Let's define SF from A to B.
We break into intervals:

Interval I: 0<x<1 m

$$V(x) = R_A = \boxed{180 \text{ kN}} \quad (\text{constant})$$

Interval II: 1<x<2 m
At x = 1, 120 kN load drops the shear:

$$V(x) = 180 - 120 = \boxed{60 \text{ kN}} \quad (\text{constant})$$

Interval III: 2<x<6 m
UDL starts. Shear varies linearly.
Let x be distance from A.

$$V(x) = 180 - 120 - 60(x - 2)$$

$$V(x) = 60 - 60(x - 2) = 180 - 60x$$

✅ **Step 3: Find Location of Max Bending Moment**
Maximum Bending Moment occurs where Shear Force = 0
Set:

$$180 - 60x = 0 \Rightarrow x = 3\,m$$

✅ **Step 4: Maximum Bending Moment at x=3 m**
We compute M3 using:

$$M = R_A \cdot x - \text{Moment of loads left of section}$$

- Point load: 120 kN at 1 m
- UDL from x = 2 to 3 ⇒ length = 1 m ⇒ total = 60 kN
  Acts at 0.5 m from section (i.e., at 2.5 m)

$$M_3 = 180(3) - 120(2) - 60(0.5) = 540 - 240 - 30 = \boxed{270 \text{ kNm}}$$

✅ **Step 5: Maximum Shear Force**
From earlier shear force values:

- From A to C: $+180 \text{ kN}$
- Just after point load: $+60 \text{ kN}$
- UDL reduces it linearly to -60 kN at B (before reaction)

Hence:

$$\boxed{\text{Maximum Shear Force} = +180 \text{ kN at A}}$$

✅ **Final Answers:**

- **Support Reactions:** RA=RB=180 kN
- **Location of Maximum BM:** x=3 m
- **Maximum Bending Moment:** 270 kNm
- **Maximum Shear Force:** 180 kN

Q.3 (a) State Pappus-Guldinus Theorems. **03**

Answer:

Pappus–Guldinus Theorem I (Surface Area):
The surface area generated by rotating a plane curve about a non-intersecting axis (in its plane) is equal to the **product of the length of the curve and the distance traveled by its centroid**.

$$A = L \cdot 2\pi\bar{y}$$

Where:

- A = surface area
- L = length of the curve
- $\bar{y}$ = distance of centroid from axis of rotation

Pappus–Guldinus Theorem II (Volume):
The volume generated by rotating a plane area about a non-intersecting axis (in its plane) is equal to the **product of the area and the distance traveled by its centroid**.

$$V = A \cdot 2\pi\bar{y}$$

Where:

- V = volume
- A = area of the figure
- $\bar{y}$ = centroidal distance from axis

(b) From first principle obtain the location of center of gravity of semi-circular lamina. **04**

Answer:

Let:

- Radius of semi-circle = R
- Center of full circle is origin; semicircle lies above x-axis (in +y region)
- Axis of symmetry is vertical, so $\bar{x}$=00

We derive $\bar{y}$, the vertical distance of C.G. from base.

Differential Area Element (Strip Method):

- Consider an elemental horizontal strip of thickness dy at distance y
- Length of strip: $2x = 2\sqrt{R^2 - y^2}$
- Area: $dA = 2\sqrt{R^2 - y^2}dy$

Total Area of Semi-Circle:

$$A = \frac{1}{2}\pi R^2$$

C.G. Location:

$$\bar{y} = \frac{1}{A}\int_0^R y \cdot dA = \frac{1}{A}\int_0^R y \cdot 2\sqrt{R^2 - y^2}dy$$

This integral evaluates to:

$$\bar{y} = \frac{4R}{3\pi}$$

✅ **Final Answer:**

$$\boxed{\bar{x} = 0, \quad \bar{y} = \frac{4R}{3\pi}}$$

(c) Calculate the bending stress at point 'C' for the beam shown in Figure-2. Given that the beam cross section 300mm wide × 600mm deep and modulus of elasticity as 25000MPa. **07**

Answer:

Step 1: Use Bending Stress Formula

$$\sigma = \frac{M \cdot y}{I}$$

Where:

- $M = 180 \times 10^6\ N \cdot mm$
- $y = \frac{d}{2} = 300\ mm$
- $I = \frac{bd^3}{12} = \frac{300 \cdot 600^3}{12}$

Step 2: Compute I

$$I = \frac{300 \cdot 600^3}{12} = \frac{300 \cdot 216 \times 10^6}{12} = 5.4 \times 10^9\ mm^4$$

Step 3: Compute Bending Stress

$$\sigma = \frac{180 \times 10^6 \cdot 300}{5.4 \times 10^9} = \frac{54 \times 10^9}{5.4 \times 10^9} = \boxed{10\ \text{MPa}}$$

✅ **Final Answer:**

- **Bending stress at point C** = 10 MPa

OR

Q.3 **(a)** State parallel axes theorem and perpendicular axes theorem. **03**

Answer:

1. Parallel Axes Theorem:

If the moment of inertia of a plane area about an axis **passing through its centroid** is known, the moment of inertia about any **parallel axis** (distance h away) is given by:

$$\boxed{I = I_G + Ah^2}$$

Where:

- IG = Moment of inertia about centroidal axis
- A = Area of the section
- h = Distance between the two axes

2. Perpendicular Axes Theorem:

This applies to **plane laminae only**.

If Ix and Iy are the moments of inertia about two **mutually perpendicular axes in the plane** (both passing through same point), then the moment of inertia about an axis **perpendicular to the plane** (i.e., z-axis) is:

$$\boxed{I_z = I_x + I_y}$$

(b) Using first principle obtain the equation to obtain moment of inertia of a circular section about centroidal x-axis. **04**

Answer:

Let:

- Radius of circle = R

We'll derive moment of inertia Ix about horizontal (centroidal) axis.

Use **integration in polar coordinates**:

- Area element: $dA = r\,dr\,d\theta$
- Distance from x-axis: $y = r\sin\theta$

So:

$$I_x = \int_A y^2 dA = \int_0^{2\pi}\int_0^R (r\sin\theta)^2 \cdot r\,dr\,d\theta$$

$$I_x = \int_0^{2\pi}\int_0^R r^3\sin^2\theta\,dr\,d\theta$$

Split integration:

$$I_x = \left[\int_0^{2\pi}\sin^2\theta\,d\theta\right]\cdot\left[\int_0^R r^3 dr\right]$$

$$\int_0^{2\pi}\sin^2\theta\,d\theta = \pi, \quad \int_0^R r^3 dr = \frac{R^4}{4}$$

So:

$$I_x = \pi \cdot \frac{R^4}{4} = \boxed{\frac{\pi R^4}{4}}$$

But this is for the entire circle **about x-axis passing through center**.

Final Answer:

$$\boxed{I_x = \frac{\pi R^4}{4}}$$

Or, using d as diameter:

$$I_x = \frac{\pi d^4}{64}$$

(c) Calculate the shear stress at point 'D' for the beam shown in Figure-2. Given that the beam cross section 300mm wide × 600mm deep and modulus of elasticity as 25000MPa. **07**

Answer:

Step 1: Use Shear Stress Formula

$$\tau = \frac{V \cdot Q}{I \cdot b}$$

Where:

- V = Shear force at D = $\boxed{60 \text{ kN}} = 60 \times 10^3\ N$
- $b = 300\ mm$ (width)
- $d = 600\ mm$, so $y = 300\ mm$
- $I = \frac{bd^3}{12} = \frac{300 \cdot 600^3}{12} = 5.4 \times 10^9\ mm^4$

Step 2: Compute First Moment of Area Q
We calculate Q for area above the **neutral axis to top fiber**:

- Area above N.A.: $A = b \cdot \frac{d}{2} = 300 \cdot 300 = 90,000\ mm^2$
- Distance of centroid of that area from N.A. = 150 mm

So:

$$Q = A \cdot \bar{y} = 90,000 \cdot 150 = 13.5 \times 10^6\ mm^3$$

Step 3: Calculate Shear Stress

$$\tau = \frac{60 \times 10^3 \cdot 13.5 \times 10^6}{5.4 \times 10^9 \cdot 300} = \frac{810 \times 10^9}{1.62 \times 10^{12}} = 0.5 \text{ MPa}$$

✅ **Final Answer:**

$$\boxed{\tau_D = 0.5 \text{ MPa}}$$

Q.4 (a) Define the following terms; **03**

i) Modulus of elasticity
ii) Poisson's ratio
iii) Modulus of rigidity

Answer:

1. Modulus of Elasticity (E):
Also known as **Young's Modulus**, it is the ratio of **normal stress to normal strain** within elastic limit.

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

Where:

- σ = normal stress
- ε = normal strain

It represents the stiffness of a material. Unit: N/mm² (MPa)

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2. Poisson's Ratio (ν):
It is the ratio of **lateral strain to longitudinal strain** when a material is subjected to axial load.

$$\boxed{\nu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}}$$

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3. Modulus of Rigidity (G):
Also called **Shear Modulus**, it is the ratio of **shear stress to shear strain**.

$$\boxed{G = \frac{\tau}{\gamma}}$$

Where:

- τ = shear stress
- γ = shear strain

(b) With usual notations, derive the relationship among Modulus of elasticity, Poisson's ratio and Bulk modulus. **04**

Answer:

Bulk Modulus (K):
Bulk modulus is defined as:

$$K = \frac{\text{Volumetric Stress}}{\text{Volumetric Strain}}$$

For an isotropic, linear elastic material, the relation between **E**, **ν**, and **K** is:

Derivation:
From theory of elasticity:

$$K = \frac{E}{3(1-2\nu)}$$

✅ **Final Relation:**

$$\boxed{K = \frac{E}{3(1-2\nu)}}$$

Also,

$$\Rightarrow E = 3K(1-2\nu)$$

(c) A solid shaft of 250 mm diameter has the same cross-sectional area as that of hollow shaft of the same material of inside diameter 200 mm. Find the ratio of power transmitted by the two shafts at same angular velocity. Also, compare angle of twist in equal lengths of these shafts when stressed to the same intensity. **07**

Answer:

📌 **Step 1: Equating Areas**

Let outer diameter of hollow shaft be D, inner = 200 mm

Solid shaft area:

$$A_s = \frac{\pi}{4}(250)^2 = 49,087.4\, mm^2$$

Hollow shaft area:

$$A_h = \frac{\pi}{4}(D^2 - 200^2) \Rightarrow \frac{\pi}{4}(D^2 - 40,000) = 49,087.4$$

Solve:

$$D^2 - 40,000 = \frac{49,087.4 \times 4}{\pi} \approx 62,500 \Rightarrow D^2 = 102,500 \Rightarrow D = \sqrt{102,500} \approx \boxed{320\, mm}$$

📌 **Step 2: Ratio of Power Transmitted**

Power is proportional to **Torsional Strength**:

$$T \propto \tau \cdot J/R \Rightarrow \text{Power} \propto \frac{J}{R}$$

🔹 **For Solid Shaft:**

$$J_s = \frac{\pi}{32} \cdot d^4 = \frac{\pi}{32} \cdot 250^4 = 3.05 \times 10^9\, mm^4 \quad R_s = 125\, mm \Rightarrow \frac{J_s}{R_s} = \frac{3.05 \times 10^9}{125} = 24.4 \times 10^6$$

🔹 **For Hollow Shaft:**

$$J_h = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(320^4 - 200^4) = 6.28 \times 10^9\, mm^4 \quad R_h = 160\, mm \Rightarrow \frac{J_h}{R_h} = \frac{6.28 \times 10^9}{160}$$

= 39.25×106

✅ **Ratio of Power:**

$$\frac{P_{\text{hollow}}}{P_{\text{solid}}} = \frac{39.25}{24.4} \approx \boxed{1.61}$$

📌 **Step 3: Ratio of Angle of Twist**

Angle of twist:

$$\theta = \frac{T \cdot L}{G \cdot J} \Rightarrow \theta \propto \frac{1}{J}$$

$$\Rightarrow \frac{\theta_{\text{hollow}}}{\theta_{\text{solid}}} = \frac{J_s}{J_h} = \frac{3.05}{6.28} = \boxed{0.486}$$

✅ **Final Answers:**

- **Outer diameter of hollow shaft = 320 mm**
- **Ratio of power transmitted (hollow/solid)** = 1.61
- **Ratio of angle of twist (hollow/solid)** = 0.486

OR

Q.4 (a) Give assumptions made in the theory of torsion. **03**

Answer:

Assumptions in Theory of Pure Torsion:

1. The shaft is circular in cross-section and remains circular during twisting.
2. The material is homogeneous, isotropic, and obeys Hooke's Law.
3. Cross-sections remain plane and perpendicular to the axis even after twisting.
4. All diameters remain straight and twist uniformly about the longitudinal axis.
5. The torque is applied gradually and does not cause dynamic effects.

(b) For the shaft subjected to pure torsion, derive the torsion equation with usual notations. **04**

Answer:

Let:

- T = Torque
- J= Polar moment of inertia
- τ\ = Shear stress
- R = Radius
- G = Modulus of rigidity
- θ = Angle of twist (radians)
- L = Length of shaft

Derivation:

1. From geometry of deformation (strain):

$$\frac{\theta}{L} = \frac{\gamma}{R} \Rightarrow \gamma = \frac{R \cdot \theta}{L}$$

2. From Hooke's Law for shear:

$$\tau = G \cdot \gamma = G \cdot \frac{R\theta}{L} \Rightarrow \tau = \frac{G \cdot R \cdot \theta}{L}$$

3. From torsional moment:

$$T = \int_A \tau \cdot r \cdot dA = \frac{\tau}{R} \cdot \int_A r^2 dA = \frac{\tau}{R} \cdot J \Rightarrow \boxed{\frac{T}{J} = \frac{\tau}{R}}$$

4. Combine with angle of twist:

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G \cdot \theta}{L}}$$

(c) Calculate the stress and elongation of each part of the bar shown in Figure-3. Also, calculate total change in length of the bar. Take E = 200000MPa. **07**

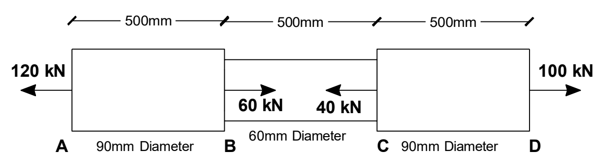

Figure-3

Answer:

Bar Details:

Seg ment	Diameter (mm)	Length (mm)	Internal Force
AB	90	500	120 kN ←
BC	60	500	60 kN ←
CD	90	500	100 kN →

Step 1: Cross-sectional Area

$$A = \frac{\pi}{4} \cdot d^2$$

- **AB & CD:**

  $A = \frac{\pi}{4} \cdot 90^2 = 6361.73\, mm^2$

- **BC:**

  $A = \frac{\pi}{4} \cdot 60^2 = 2827.43\, mm^2$

Step 2: Stress in Each Segment

$$\sigma = P/A$$

- **AB:**
  $\sigma = \frac{120000}{6361.73} = \boxed{18.87\ MPa\ (Tension)}$
- **BC:**
  $\sigma = \frac{60,000}{2827.43} = \boxed{21.22\ MPa\ (Compression)}$
- **CD:**
  $\sigma = \frac{100,000}{6361.73} = \boxed{15.72\ MPa\ (Tension)}$

Step 3: Elongation or Shortening

$$\Delta L = \frac{P \cdot L}{A \cdot E}$$

Use $E = 200,000\ MPa = 200000\ N/mm^2$

- **AB:**

$$\Delta L_{AB} = \frac{120000 \cdot 500}{6361.73 \cdot 200000} = 0.0472\ mm$$

$$\Delta L_{BC} = \frac{60000 \cdot 500}{2827.43 \cdot 200000} = 0.0530\ mm(shortening)$$

- **BC:**
- **CD:**

$$\Delta L_{CD} = \frac{100000 \cdot 500}{6361.73 \cdot 200000} = 0.0394\ mm$$

✅ **Step 4: Total Change in Length**

$$\Delta L_{total} = \Delta L_{AB} - \Delta L_{BC} + \Delta L_{CD}$$

$$= 0.0472 - 0.0530 + 0.0394 = \boxed{0.0336\ mm\ (extension)}$$

✅ **Final Answers:**

Segment	**Stress (MPa)**	**Elongation/Shortening (mm)**
AB	18.87 Tension	0.0472
BC	21.22 Compression	0.0530
CD	15.72 Tension	0.0394

- **Total Change in Length** = 0.0336 mm (Extension)

Q.5 (a) State and explain Hook's law. **03**

Answer:

Hook's Law Statement:
Within the **elastic limit**, **stress is directly proportional to strain**.

$$\boxed{\sigma \propto \varepsilon \quad \Rightarrow \quad \sigma = E \cdot \varepsilon}$$

Where:
- σ = normal stress
- ε = normal strain
- E = modulus of elasticity (Young's Modulus)

Explanation:
Hook's law applies only up to the **elastic limit**, beyond which the material **does not return** to its original shape. This law helps determine deformation in materials when subjected to load.

(b) Explain Mohr's circle method with neat sketch for a section subjected to direct stresses along two perpendicular directions. **04**

Answer:

Mohr's Circle Concept:
Mohr's Circle is a **graphical representation** of the **state of stress** at a point in a 2D stress system. It helps determine:
- Principal stresses
- Maximum shear stress
- Orientation of principal planes

Given:
- Normal stress in x-direction: σ x
- Normal stress in y-direction: σ y
- No shear stress

Construction Steps (with no shear stress):
1. Draw X-axis (normal stress) and Y-axis (shear stress).
2. Plot two points:
   - $A = (\sigma_x, 0)$
   - $B = (\sigma_y, 0)$
3. Find midpoint $C = \left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$
4. Draw a circle with center C and radius $R = \left|\frac{\sigma_x-\sigma_y}{2}\right|$
5. Points where the circle cuts the x-axis give **principal stresses**.

Diagram:
(Draw a horizontal line with two points A and B; center at midpoint, circle passing through both.)

Formulas from Mohr's Circle:

- **Principal stresses:**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{|\sigma_x - \sigma_y|}{2}$$

- **Maximum shear stress:**

$$\tau_{\max} = \frac{|\sigma_x - \sigma_y|}{2}$$

(c) A bar held straight between two rigid supports 5 m apart has initial tensile stress 5 MPa at 30° Celsius. Calculate the stress in wire if temperature reduces to minus 5° C. Take E= 2 × 10^5 MPa and coefficient of thermal expansion as 2 × 10^{-6} per °C. **07**

Answer:

Step 1: Thermal strain (free expansion)

$$\Delta T = -5^\circ - 30^\circ = -35^\circ C$$

$$\varepsilon_{\text{thermal}} = \alpha \cdot \Delta T = 2 \times 10^{-6} \cdot (-35) = -70 \times 10^{-6}$$

Since the bar is **prevented from contracting**, thermal strain is **converted into additional stress**.

Step 2: Thermal Stress Induced

$$\sigma_{\text{thermal}} = E \cdot \varepsilon_{\text{thermal}} = 2 \times 10^5 \cdot (-70 \times 10^{-6}) = -14 \text{ MPa}$$

This is a **compressive stress** of 14 MPa.

Step 3: Final Stress in the Bar

Initial tensile stress = +5 MPa

Thermal compressive stress = -14 MPa

$$\sigma_{\text{final}} = 5 - 14 = \boxed{-9 \text{ MPa (Compressive)}}$$

✅ **Final Answer:**

- **Final stress in the bar =** 9 MPa (Compressive)

OR

Q.5 (a) Define Stress, Strain and Bulk modulus. **03**

Answer:

1. Stress (σ):

Stress is the **internal resisting force per unit area** developed in a material when subjected to external force.

$$\boxed{\sigma = \frac{F}{A}} \quad \text{(Units: N/mm}^2\text{ or MPa)}$$

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2. Strain (ε):

Strain is the **ratio of change in dimension to the original dimension** of a material.

$$\boxed{\varepsilon = \frac{\Delta L}{L}} \quad \text{(It has no units)}$$

3. Bulk Modulus (K):
Bulk modulus is the ratio of **volumetric stress to volumetric strain**. It measures a material's resistance to uniform compression.

$$\boxed{K = \frac{\text{Pressure (p)}}{\text{Volumetric strain}(\varepsilon_v)}} \quad \text{(Units: MPa)}$$

(b) Explain Mohr's circle method with neat sketch for a section subjected to direct stresses along two perpendicular directions and accompanying shear stresses. **04**

Answer:

Given Stresses:

- σ_x, σ_y: normal stresses on x and y faces
- τ xy: shear stress (same on both x and y faces, opposite in direction)

Mohr's Circle Construction:

1. Draw coordinate axes: horizontal for normal stress (σ), vertical for shear stress (τ).
2. Plot points:
   - Point A: (σ_x, τ_{xy})
   - Point B: $(\sigma_y, -\tau_{xy})$
3. Join A and B; midpoint C = center of circle.

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

4. Radius
5. Draw circle using center and radius.

Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm R$$

Maximum Shear Stress:

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(c) Determine the compressive stress developed in a punch of 10 mm diameter, used to make a hole of 10mm diameter in 6mm thick mild steel plate. The shear strength of mild steel is 200MPa. **07**

Answer:

Concept:
To make a hole, **shearing of plate** happens along the **cylindrical area** of punch contact.

- **Shear Area** = π·d· t
- d=10 mm, t=6 mm

Step 1: Shear Force Required

$F = \text{Shear strength} \cdot \text{Shear area} = 200 \cdot \pi \cdot 10 \cdot 6 = 200 \cdot 188.5 = 37,699\ N$

Step 2: Compressive Stress on Punch Face
Face area of punch:

$$A = \frac{\pi}{4} \cdot d^2 = \frac{\pi}{4} \cdot 10^2 = 78.54\ mm^2$$

$$\sigma_c = \frac{F}{A} = \frac{37,699}{78.54} \approx \boxed{480 \text{ MPa}}$$

✅ **Final Answer:**

- **Compressive Stress on Punch =** 480 MPa

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