Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2024

Subject Code:3140610 **Date:26-11-2024**

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find the principal argument of z= $\frac{-2}{1+i\sqrt{3}}$.	**3**
	(b)	Find the constants a, b, c if $f(z) = x + ay + i(bx + cy)$ is analytic.	**4**
	(c)	Show that $u(x, y) = x^2 - y^2 + x$ is harmonic. Find the corresponding analytic function $f(z) = u + iv$	**7**
Q.2	**(a)**	Sketch the set $0 \le arg z \le \frac{\pi}{4}$ and determine is it domain?	**3**
	(b)	Find the continued product of all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$	**4**
	(c)	Find and sketch the image of the region $\lvert z\rvert > 1$ under the transformation $w = 4z$.	**7**
		OR	
	(c)	Find the bilinear transformation that maps the points $z_1 = 1, z_2 = i, z_3 = -1$ onto $w_1 = -1, w_2 = 0, w_3 = 1$ respectivley. Find the image of $\lvert z\rvert < 1$ under this transformation.	**7**
Q.3	**(a)**	Evaluate $\int_C e^{sinz^2} dz$, where C is $\lvert z\rvert = 1$.	**3**
	(b)	Expand $f(z) = e^z$ in a Taylor series about $z = 0$.	**4**
	(c)	Evaluate $\int_C \frac{z}{(z-1)(z-2)^2} dz$, where C is $\lvert z - 2\rvert = \frac{1}{2}$.	**7**
		OR	
Q.3	**(a)**	Classify the poles of $f(z) = \frac{1}{z^2 - z^6}$.	**3**
	(b)	Find the radius and region of convergence for $\sum_{n=1}^{\infty} \frac{z^n}{2^n+1}$.	**4**
	(c)	Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent's series in the following regions: (i) $\lvert z\rvert < 1$ (ii) $1 < \lvert z\rvert < 2$ (iii) $\lvert z\rvert > 2$.	**7**

Q.4	**(a)**	Form a partial differential equation for the equation $z = ax + by + ct$	**3**
	(b)	Solve $xp + yq = 3z$.	**4**
	(c)	Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.	**7**

OR

Q.4	**(a)**	Solve $z = px + qy - 2\sqrt{pq}$.	**3**
	(b)	Form the partial differential equation of $z = f\left(\frac{x}{y}\right)$.	**4**
	(c)	Solve $px + qy = pq$ by Charpit's method.	**7**

Q.5	**(a)**	Solve $\frac{\partial^2 u}{\partial x \partial y} = e^{-t} cosx$.	**3**
	(b)	Solve $\left(D^2 + 10DD' + 25D'^2\right)z = e^{3x+2y}$.	**4**
	(c)	Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, $u(x,0) = 6e^{-3x}$.	**7**

OR

Q.5	**(a)**	Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial^2 x \partial y} + 2\frac{\partial^3 z}{\partial y^3} = 0$.	**3**
	(b)	Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x}$.	**4**
	(c)	Using separable variable technique, find the acceptable general solution to the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ and find the solution satisfying the condition $u(0,t) = u(\pi,t) = 0 \; for \; t > 0 \; and \; u(x,0) = \pi - x, 0 < x < \pi$.	**7**

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