

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – WINTER 2023****Subject Code:3140610****Date: 06-02-2024****Subject Name: Complex Variables and Partial Differential Equations****Time: 10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q. 1**
- (a) 1. State De Moivre's theorem on complex numbers. 01
 2. Write the formula of $\sin h(x+y)$ 01
 3. Find the modulus of $z = \frac{1}{\sqrt{3}+i}$ 01
- (b) Prove that $\sinh^{-1} z = \log \left\{ z + \sqrt{z^2 + 1} \right\}$ 04
- (c) Find all the roots of $x^{12} - 1 = 0$ and identify the roots which are also roots of $x^4 + x^2 + 1 = 0$. 07
- Q. 2**
- (a) Define limit of a complex function. Evaluate $\lim_{z \rightarrow i} \frac{z-i}{z^2+1}$ 03
- (b) Show that $f(z) = z \operatorname{Im}(z)$ is differentiable only at $z = 0$ and $f'(0) = 0$. 04
- (c) State the necessary and sufficient conditions for the complex function $f(z) = u + iv$ to be analytic function. Find the analytic function $f(z) = u + iv$, if $u = x^3 - 3xy^2$. Also determine its imaginary part v . 07
- OR**
- (c) Define harmonic function. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic. Find also its harmonic conjugate v , where $f(z) = u + iv$. 07
- Q. 3**
- (a) State the following on complex analysis: 03
 (i) Cauchy–Goursat theorem (ii) Cauchy's integral formula (iii) Liouville's theorem.
- (b) Evaluate $\int_c (x^2 - iy^2) dz$ along the parabola $y = 2x^2$ from the points $(-2, 8)$ to $(2, 8)$. 04
- (c) 1. Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 3$. 03
 2. Evaluate $\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$, where c is the circle $|z| = 3$. 04
- OR**
- Q. 3** (a) Find the upper bound for $\int_c e^z dz$ where c is the line segment joining the points $(0, 0)$ to $(1, 2\sqrt{2})$. 03

- (b) Find Laurent's series expansion in the power of z that represents the function $f(z) = \frac{1}{z^2(1-z)}$ in the domain $|z| > 1$. 04
- (c) 1. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$ 03
2. Find residues at each pole for $f(z) = \frac{1}{(z-3)(z-1)^2}$ 04
- Q. 4** (a) Form the partial differential equation from $z = x y + f(x^2 + y^2)$ 03
- (b) Solve $p^2 - q^2 = x - y$ 04
- (c) Solve $(x^2 - y^2 - z^2)p + 2xy q = 2xz$ 07
- OR**
- Q. 4** (a) Form the partial differential equation from $(x-a)(y-b) - z^2 = x^2 + y^2$. 03
- (b) Solve $p(1+q) = qz$ 04
- (c) Apply Charpit's method to solve $px + qy = pq$ 07
- Q. 5** (a) Discuss the classification of the second order linear partial differential equation. 03
- (b) Solve the equation $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ by the method of separation of variables given that $u(x,0) = 6e^{-3x}$ 04
- (c) Solve $\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial y^3} = e^{x+2y}$ 07
- OR**
- Q. 5** (a) Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ as parabolic, elliptic or hyperbolic? 03
- (b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables given that $u(x,0) = \sin \pi x$ 04
- (c) The vibration of an elastic string is governed by the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$. 07
- The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for $t > 0$.
