Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) EXAMINATION – WINTER 2022

Subject Code:3140610 Date:16-12-2022

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find the real and imaginary parts of $f(z) = z^2 + \bar{z}$.	**03**
	(b)	Evaluate $\left(1 + i\sqrt{3}\right)^{60} + \left(1 - i\sqrt{3}\right)^{60}$.	**04**
	(c)	Define Harmonic function. Show that $u(x, y) = \sin z$ is analytic everywhere. Also, find *f'(z)* .	**07**
Q.2	**(a)**	Find the image of the region $\lvert z\rvert > 2$ under the transformation $w = 4z$	**03**
	(b)	Find all solution of $e^z = 1 + i$.	**04**
	(c)	Expand $f(z) = \frac{1}{(z-2)(z-3)}$ valid for the region (i) $\lvert z\rvert < 2$ (ii) $2 < \lvert z\rvert < 3$ (iii) $\lvert z\rvert > 3$.	**07**
		OR	
	(c)	Show that $u(x, y) = y^3 - 3x^2y$ is harmonic in some domain D and find the conjugate $v(x, y)$	**07**
Q.3	**(a)**	Check whether the function $f(z) = xy + iy$ is analytic or not at any point.	**03**
	(b)	Evaluate $\oint_C \frac{\cos z}{(z-1)(z-2)}\, dz$ around the circle $C: \lvert z\rvert = 3$.	**04**
	(c)	Evaluate the followings: (i) $\int_C \frac{2z+3}{z^2-4}\, dz$, counter clockwise around the circle $C: \lvert z - 2\rvert=1$. (ii) $\int_C \frac{e^z+z}{z^2-1} dz$, where C $C: \lvert z\rvert=2$	**07**
		OR	
Q.3	**(a)**	Expand $f(z) = \frac{\cos z}{z^2}$ in Laurent's series about *z = 0* and identify the singularity.	**03**
	(b)	Determine the bilinear transformation which maps the points $0, \infty, i$ into $\infty, 1, 0$	**04**
	(c)	Using residue theorem, evaluate $\int_0^{2\pi} \frac{4d\theta}{5+4\sin\theta}$.	**07**
Q.4	**(a)**	Evaluate $\int_C Re(z)dz,$ where *c* is the shortest path from 1+i to 3+2i	**03**
	(b)	Solve the partial differential equation $\left(D^2 - 5DD' + 6D'^2\right)z = e^{x+y}$.	**04**
	(c)	Obtain the complete integral of the followings: (i) $p^2 - q^2 = x - y$. (ii) $z = px + qy - 2\sqrt{pq}$	**07**

OR

Q.4 **(a)** Find the Laurent's series that represents the function $f(z) = z^2 \sin(\frac{1}{z^2})$ in the domain $0 < |z| < \infty$ **03**

(b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ **04**

(c) Find the general solution of the partial differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by method of separation of variables. **07**

Q.5 **(a)** Solve $pq = p + q$ **03**

(b) Find a complete integral of the equation $p^2 y(1 + x^2) = qx^2$ **04**

(c) If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = k sin^3 \left(\frac{\pi x}{l}\right)$, x being the distance from an end point. Find the displacement of the string at any point. **07**

OR

Q.5 **(a)** Solve $\frac{\partial^2 z}{\partial x^2} = \sin x.$ **03**

(b) Solve: $(y + z)p + (z + x)q = x + y$ **04**

(c) A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is $u(x, 0) = x, 0 \leq x \leq 100$. Find the temperature $u(x, 0)$ at any time. **07**

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