Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV EXAMINATION – SUMMER 2025

Subject Code:3140610 **Date:17-05-2025**

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	State: (i) Liouville Theorem and (ii) Cauchy Goursat Theorem	**03**
	(b)	Solve $x(y-z)p + y(z-x)q = z(x-y)$	**04**
	(c)	Verify that $u(x,y) = y^3 - 3x^2y$ is harmonic in the whole complex plane and find its harmonic conjugate $v(x,y)$	**07**
Q.2	**(a)**	Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$	**03**
	(b)	Using residue theorem, evaluate $\int_c \frac{\cos \pi z^2}{(z-2)(z-1)}\, dz$ counterclockwise around the circle $c: \lvert z\rvert = 3$	**04**
	(c)	Evaluate $\int_c \frac{z-2}{z(z-1)}\, dz$ where c is the circle $\lvert z\rvert = 3$	**07**
		OR	
	(c)	Evaluate $\int_c \frac{e^{2z}}{(z+1)^4}\, dz$ where c is the circle $\lvert z\rvert = 2$	**07**
Q.3	**(a)**	Solve $(p-q)(z-px-qy) = 1$	**03**
	(b)	Discuss the convergence of $\sum_{n=1}^{\infty}\left(\frac{6n+1}{2n+5}\right)^2 (z-2i)^n$ and also find the radius and circle of convergence	**04**
	(c)	Find the Mobius transformation that maps $z_1 = 0, z_2 = -i, z_3 = 2i \; onto \; w_1 = 5i, w_2 = \infty, w_3 = -\frac{i}{3}$ respectively. What are the invariant points of this transformation.	**07**
		OR	
Q.3	**(a)**	Evaluate $\oint_c e^{z^2}\, dz$ where c is any closed contour. Justify your answer.	**03**
	(b)	Find the image of the infinite strip $0 \le x \le 1$ under the transformation $w = iz + 1$. Sketch the region in the $w - plane$.	**04**
	(c)	Prove that the n^{th} roots of unity are in geometric progression. Also, show that their sum is zero.	**07**
Q.4	**(a)**	Identify the type of singularity of $f(z) = \frac{2-e^z}{z^3}$	**03**
	(b)	Show that $Log(i^3) \neq 3\,\mathrm{Log}\, i$, for the principal branch	**04**
	(c)	Solve $zpq = p + q$ by using charpit's method	**07**
		OR	
Q.4	**(a)**	Find the residue of function $f(z) = \frac{2z+3}{(z+2)^2}$	**03**
	(b)	Using CR equations, prove that $f(z) = \bar{z}$ is nowhere analytic	**04**

	(c)	Determine the Laurent series expansion of $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $(i) 1 < \lvert z \rvert < 3$ $(ii) \lvert z \rvert > 3$ $(iii) 0 < \lvert z+1 \rvert < 2$	**07**
Q.5	**(a)**	Classify second order homogeneous partial differential equations as elliptic, parabolic or hyperbolic i) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\,\partial y} - 12\frac{\partial^2 z}{\partial y^2} = 0$ ii) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\,\partial y} = 0$	**03**
	(b)	Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2\,\partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$	**04**
	(c)	Using the method of separation of variable, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \quad u(x,o) = 6e^{-3x}$	**07**

OR

Q.5	**(a)**	Form a partial differential equation by eliminating the arbitrary function from $z - xy + f(x^2 + y^2)$	**03**
	(b)	Solve $25r - 40s + 16t = 0$	**04**
	(c)	Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ such that $y = a\cos pt$ when $x = l$, and $y = 0$ when $x = 0$	**07**