Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2024

Subject Code:3140610 Date:01-07-2024

Subject Name: Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find $|z|$, $\operatorname{Arg} z$ and $\arg z$ for $z = \frac{-i}{1-i}$	**03**
	(b)	Define a bilinear transformation. Find the Mobius transformation which sends the points $z = -1, 0, 1$ of the z-plane onto the points $w = -1, -i, 1$ respectively in the w-plane.	**04**
	(c)	Define an analytic function. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is a harmonic function. Determine also its harmonic conjugate $v(x, y)$ and the corresponding analytic function $f(z) = u + iv$.	**07**
Q.2	**(a)**	Evaluate $\int_c \frac{e^z}{(z-3)^2} dz$, where C: $|z| = 2$.	**03**
	(b)	Evaluate $\int_c \frac{1}{z^3(z+4)} dz$, where C: $|z| = 2$.	**04**
	(c)	Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$, along the shortest path joining the two points $z = 0$ and $z = 2 + i$.	**07**
		OR	
	(c)	Find the radius of convergence of	
		(i) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} (z - 3i)^n$	**04**
		(ii) $\sum_{n=0}^{\infty} (6 + 8i) z^n$	**03**
Q.3	**(a)**	Find the Maclaurin's series for $f(z) = \frac{1}{1-z}$ about $z = 0$.	**03**
	(b)	Expand the function $f(z) = \frac{1}{4z-z^2}$ in Laurent's series in powers of z under the condition $0 < |z| < 4$.	**04**
	(c)	State Cauchy's Residue theorem. Using it integrate $f(z) = \frac{1}{z^3-z^4}$ counter clockwise around the circle $|z| = \frac{1}{2}$.	**07**
		OR	
Q.3	**(a)**	Expand $f(z) = \frac{1}{1-z}$ in powers of z, where $|z| > 1$.	**03**
	(b)	Prove that $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)^2} dx = \frac{\pi}{e}$, by using complex integrals.	**04**
	(c)	Using Cauchy's Residue theorem, evaluate $\int_C \frac{z+1}{z^2-2z} dz$ counter clockwise around the circle $|z| = 3$.	**07**

Q.4	**(a)**	Form a partial differential equation by eliminating arbitrary constants from $(x-a)(y-b)=x^2+y^2+z^2$	**03**
	(b)	Using Lagrange's method, solve the partial differential equation $y^2p-xyq=x(z-2y)$.	**04**
	(c)	Using Charpit's method, solve the partial differential equation $px+qy=pq$.	**07**

OR

Q.4	**(a)**	Form a partial differential equation by eliminating arbitrary function from $z=xy+f(x^2+y^2)$	**03**
	(b)	Solve the non-linear partial differential equation $p^2+q^2=npq$.	**04**
	(c)	Using Charpit's method, solve the partial differential equation $px+qy+p^2+q^2-z=0$	**07**

Q.5	**(a)**	Classify the partial differential equation $u_{xx}+2u_{xy}+u_{yy}=0$, as hyperbolic, parabolic or elliptic.	**03**
	(b)	Solve the partial differential equation $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}=2e^{2x}$	**04**
	(c)	Find the solution of the one dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ satisfying the conditions $u(0,t)=u(\pi,t)=0$; for $t>0$ and $u(x,0)=\pi-x$; for $0<x<\pi$.	**07**

OR

Q.5	**(a)**	Solve the partial differential equation $\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial y^3}=0$	**03**
	(b)	Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ by using the method of separation of variables.	**04**
	(c)	Find the solution of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ satisfying the conditions $u(0,t)=u(L,t)=0$; for $t>0$, $u(x,0)=\frac{\pi x}{L}$; for $0<x<L$ and $u_t(x,0)=0$; for $0<x<L$.	**07**

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