Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023

Subject Code:3140610 Date:17-07-2023

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Prove that i^i is real.	**03**
	(b)	Simplify $\frac{(cos5\theta - isin5\theta)^2(cos7\theta + isin7\theta)^{-3}}{(cos4\theta - isin4\theta)^9(cos\theta + isin\theta)^5}$	**04**
	(c)	Solve $(D^2 + DD' - 6D'^2)z = \sin{(2x + y)}$	**07**
Q.2	**(a)**	Check whether the function $f(z) = e^{\bar{z}}$ is analytic or not at any point.	**03**
	(b)	Find and plot all values of $(8i)^{\frac{1}{3}}$.	**04**
	(c)	Show that the function $u = e^x cosy$ is harmonic. Determine its harmonic conjugate $v(x, y)$ and the analytic function $f(z) = u + iv$.	**07**
		OR	
	(c)	Determine the region in the w-plane into which the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane is mapped under the transformation $w = 4z$.	**07**
Q.3	**(a)**	State: i) Cauchy-Goursat Theorem ii)Liouville Theorem.	**03**
	(b)	Find an upper bound for the absolute value of $\oint_C \frac{e^z}{z+1} dz$, where C is the circle $\|z\| = 4$.	**04**
	(c)	Write Cauchy's Integral formula and hence evaluate $\oint_C \frac{z+1}{z^4+2iz^3} dz$, where C is the circle $\|z\| = 1$.	**07**
		OR	
Q.3	**(a)**	Evaluate $\oint_C \frac{1}{z} dz$, where C is the circle $x = cost$, $y = sint$, $0 \leq t \leq 2\pi$.	**03**
	(b)	Find the values of x and y if $e^z = \sqrt{3} + i$.	**04**
	(c)	Evaluate $\int_C \frac{z+1}{z^4 - 4z^3 + 4z^2} dz$, where C is the circle $\|z - 2 - i\| = 2$.	**07**
Q.4	**(a)**	Identify the type of singularities of $f(z) = \frac{\tan z}{z}$.	**03**
	(b)	Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.	**04**
	(c)	Express $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for the following annular domains. (a) $0 < \|z\| < 1$ (b) $1 < \|z\|$ (c) $0 < \|z - 1\| < 1$.	**07**
		OR	

Q.4	**(a)**	Find the complete integral of $pq = k$, where k is a constant.	**03**
	(b)	Evaluate by the Residue Theorem $\int_C \frac{1}{(z-1)^2(z-3)} dz$, where the contour C is the rectangle defined by $x = 0, x = 4, y = -1, y = 1$.	**04**
	(c)	Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.	**07**
Q.5	**(a)**	Test for singularity of $\frac{1}{z^2+1}$ and hence find the corresponding residues.	**03**
	(b)	Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$.	**04**
	(c)	Using the method of separation of variable, find the solution of $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$.	**07**
		OR	
Q.5	**(a)**	Solve $(D^3 - 2D^2D')z = 2e^{2x}$.	**03**
	(b)	Solve $p^2 + q^2 = x + y$.	**04**
	(c)	Solve $(D^2 + DD' - 6D'^2)z = ycosx$.	**07**