Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2022

Subject Code:3140610 Date:02-07-2022

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Find an analytic function $f(z)=u+iv$ if $u=x^3-3xy.$	**03**
	(b)	Find the fourth roots of -1.	**04**
	(c)	(i) Find the image of infinite strip $0\le x\le 1$ under the transformation $w=iz+1.$	**03**
		(ii) Separate real and imaginary parts of $f(z)=z^2.$	**04**
Q.2	**(a)**	Evaluate $\int_C (x^2+ixy)dz$ from (1, 1) to (2, 4) along the curve $x=t, y=t^2.$	**03**
	(b)	Determine the mobius transformation that maps $z_1=0, z_2=1, z_3=\infty$ onto $w_1=-1, w_2=-i, w_3=1$ respectively.	**04**
	(c)	(i) Evaluate $\oint_C \frac{e^z}{z(1-z)^3}dz,$ where C is $\lvert z\rvert=\frac{1}{2}.$	**03**
		(ii) Find the radii of convergence of $\sum_{n=1}^{\infty}\frac{z^n}{2^n+1}.$	**04**
		OR	
	(c)	Find the image of $\lvert z-1\rvert=1$ under the mapping $w=\frac{1}{z}.$	**07**
Q.3	**(a)**	Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $\lvert z\rvert=2.$	**03**
	(b)	Find $\operatorname{Res}(f(z),4i)$, where $f(z)=\frac{z}{z^2+16}.$	**04**
	(c)	Expand $f(z)=\frac{1}{(z-1)(z+2)}$ in Laurent's series in the region $(i)\lvert z\rvert<1, (ii)1<\lvert z\rvert<2, (iii)\lvert z\rvert>2.$	**07**
		OR	
Q.3	**(a)**	Evaluate $\oint_C (x^2-y^2+2ixy)dz,$ where C is the circle $\lvert z\rvert=1.$	**03**
	(b)	Evaluate $P.V.\int_{-\infty}^{\infty}\frac{x\cos x}{x^2+9}dx.$	**04**
	(c)	Find Laurent's series that represent $f(z)=\frac{1}{z(z-1)}$ in the region $(i)0<\lvert z\rvert<1, (ii)0<\lvert z-1\rvert<1.$	**07**

Q.4	**(a)**	Solve $\frac{y-z}{yz}p+\frac{z-x}{zx}q=\frac{x-y}{xy}$.	**03**
	(b)	Derive partial differential equation by eliminating arbitrary constants a and b from $z=(x+a)(y+b)$.	**04**
	(c)	(i) Solve $\frac{\partial^3 z}{\partial x^3}=0$.	**03**
		(ii) Find complete integral of $p^2+q^2=z$.	**04**

OR

Q.4	**(a)**	Solve $xp+yq=x-y$.	**03**
	(b)	Form a partial differential equation by eliminating arbitrary function from $z=f(x/y)$.	**04**
	(c)	(i) Solve $\left(D^2-D'^2+D-D'\right)z=0$.	**03**
		(ii) Solve $q=3p^2$ by Charpit's method.	**04**

Q.5	**(a)**	Solve $(r+3s+2t)=x+y$	**03**
	(b)	Solve the p.d.e. $u_{xy}=-u_x$.	**04**
	(c)	Find the deflection $u(x,t)$ of the vibrating string of length π and ends fixed, corresponding to zero velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$.	**07**

OR

Q.5	**(a)**	Solve $\left(D^2+DD'+D'-1\right)z=\sin(x+2y)$.	**03**
	(b)	Solve the p.d.e. $u_x+u_y=2(x+y)u$.	**04**
	(c)	Find the solution of $u_t=c^2u_{xx}$ together with the initial and boundary conditions $u(0,t)=u(l,t)=0$ for all $t\geq 0$ and $u(x,0)=\sin\frac{\pi x}{l}$, $0\leq x\leq l$.	**07**

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