Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – WINTER 2023

Subject Code:130002 Date:02-02-2024

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Find the Fourier series of $f(x) = \|x\|$ in the interval $-\pi < x < \pi$	**07**
	(b)	Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2 \ln x$	**07**
Q.2	**(a)**	(i) Solve $\frac{dy}{dx} - y = e^{2x}$	**03**
		(ii) Solve $(D^2 + a^2)y = \operatorname{cosec} ax$ by the method of variation of parameter	**04**
	(b)	Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \sec x$	**07**
		OR	
	(b)	Solve by using Undetermined Coefficient method $(D^2 + 4)y = 8x^2$	**07**
Q.3	**(a)**	Find power series solution of $y'' + y = 0$	**07**
	(b)	(i) Define 1. Gamma Function 2. Beta Function 3. Signum Function	**03**
		(ii) Show that $\int_0^\infty \frac{\cos \omega x}{\omega^2+1} = \frac{\pi}{2}e^{-x} \quad if \quad x > 0$	**04**
		OR	
Q.3	**(a)**	(i) Find the Laplace transform of $e^{-3t}(2cos5t - 3sin5t)$.	**03**
		(ii) Find the Inverse Laplace transform of $\log\left(\frac{s+a}{s-a}\right)$.	**04**
	(b)	Find power series solution of $y'' + x^2y = 0$	**07**
Q.4	**(a)**	Using the convolution method to find $L^{-1}\left(\frac{1}{s(s^2+4)}\right)$.	**07**
	(b)	Solve $y'' + 4y = 0$, $y(0) = 1$ and $y'(0) = 6$	**.07**
		OR	
Q.4	**(a)**	(i) Find the Laplace transform of $\frac{e^{-t}\sin t}{t}$	**03**
		(2) Solve $y''' - 6y'' + 11y' - 6y = 0$	**04**
	(b)	Obtain the Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$.	**07**
Q.5	**(a)**	Form a partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2, z - xy) = 0$	**07**
	(b)	Solve $u_x = 4u_y$ subject to the condition $u(0, y) = 8e^{-3y}$ by method of separation of variables	**07**
		OR	
Q.5	**(a)**	Form a partial differential equation by eliminating the arbitrary constants from $z = (x - a)^2 + (y - b)^2$	**07**
	(b)	Solve $(D^2 + 6DD' + 9D'^2)z = 6x + 2y$	**07**

****

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(OLD) EXAMINATION – WINTER 2022

Subject Code:130002 Date:16-02-2023

Subject Name:Advanced Engineering Mathematics

Time:02:30 PM TO 05:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 (a) (i)Solve $\frac{dy}{dx}+y\tan x=\sin 2x$ **03**

(ii) Find the solution of differential equation $ye^x dx+(2y+e^x)dy=0,$ where $y(0)=-1$ **04**

(b) Find the series solution of differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$ **07**

Q.2 (a) (i) Solve $x^2ydx=(x^3+y^3)dy$ **03**

(ii) Find the solution by variation of parameter **04**

$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^x$

(b) Solve $\frac{d^2y}{dx^2}-y=x\sin x+e^x+x^2e^x$ **07**

OR

(b) (i) Find the Laplace transform of $te^t\sin t$ **03**

(ii) Find the inverse Laplace transform of $\frac{s^3}{s^4-81}$ **04**

Q.3 (a) (i) Define 1 Error function, 2 Gamma function, 3 Heaviside's unit step function. **03**

(ii) Obtain the half range sine series for $f(x)=2x,$ in $0\prec x\prec 1$ **04**

(b) Solve $\left(1+x\right)^2\frac{d^2y}{dx^2}+\left(1+x\right)\frac{dy}{dx}+y=4\cos\left(\log\left(1+x\right)\right)$ **07**

OR

Q.3 (a) (i) Solve $\frac{dy}{dx}+\tan x\tan y=\cos x\sec y$ **03**

(ii) Find the solution using method of undetermined coefficients. **04**

$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=3e^{-2x}$

(b) Find the series solution by Frobinus method near $x=0$ **07**

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

Q.4 **(a)** (i) Solve $p^2 - q^2 = x - y$ **03**

(ii) solve $p(1+q) = qz$ **04**

(b) Using Laplace transformation solve the initial value problem **07**

$y'' + 2y' + 5y = e^{-t}\sin t, \quad y(0) = 0, y'(0) = 1$

OR

Q.4 **(a)** **(i) Find the Laplace transformation of** $t^2 e^{3t}\sin 4t$ **03**

(ii)State convolution theorem and using it find the inverse Laplace transformation of $\dfrac{1}{s\left(s^2+4\right)}$ **04**

(b) Solve $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$ subject to the condition $u(x,0) = 4e^{-4x}$ **07**

Q.5 **(a)** (i) State second shifting theorem and using it find the inverse Laplace transformation of $\dfrac{e^{-3s}}{s^2+8s+25}$. **03**

(ii) Find the Inverse L aplace transformation of $\tan^{-1}\left(\dfrac{2}{s}\right)$ **04**

(b) Obtain the Fourier series of $f(x) = x - x^2$, in the interval $-\pi \prec x \prec \pi$ **07**

OR

Q.5 **(a)** (i) Form a partial differential equation by eliminating the arbitrary function. **03**

$z = xy + f(x^2 + y^2)$

(ii) Find the Laplace transformation of $\dfrac{1-\cos t}{t}$ **04**

(b) Find the Fourier integral representation of the function **07**

$$f(x) = \begin{matrix} 1, |x| \prec 1 \\ 0, |x| \succ 1 \end{matrix} \quad \text{and evaluate} \int_0^\infty \frac{\sin\omega}{\omega} d\omega$$

****

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – WINTER 2021

Subject Code:130002 Date:15-02-2022

Subject Name: Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	(i) Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$	**03**
		(ii) Find the half range sine series of $f(x) = e^x$ in $0 < x < \pi$	**04**
	(b)	Find the series solution of $(1+x^2)y'' + xy' - xy = 0$	**07**
Q.2	**(a)**	(i) Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$	**03**
		(ii) Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = cosec x$ by the method of variation of parameter	**04**
	(b)	Solve $\frac{d^3y}{dx^3} + 8y = cosh 2x$	**07**
		OR	
	(b)	(i) Find the Laplace transform of $sinh^5 t$.	**03**
		(ii) Find the Inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$	**04**
Q.3	**(a)**	(i) Define 1. Gamma Function 2. Beta Function 3. Signum Function	**03**
		(ii) Find the Fourier *cosine* series for $f(x) = x^2, 0 < x < c$.	**04**
	(b)	Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = logx.\sin(logx)$	**07**
		OR	
Q.3	**(a)**	(i) Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$	**03**
		(ii) Solve by using Undetermined Coefficient method $(D^2 - 2D + 3)y = x^3 + sinx$	**04**
	(b)	Find the series solution of $2x(x-1)y'' - (x+1)y' + y = 0; x_0 = 0$	**07**
Q.4	**(a)**	Find the Inverse Laplace transform of $slog(\frac{s^2+a^2}{s^2+b^2})$	**07**
	(b)	Solve $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ subject to the condition $u(x,0) = 4e^{-3x}$ by method of separation of variables.	**07**
		OR	
Q.4	**(a)**	Solve the initial value problem using Laplace Transform $y'' + 3y' + 2y = e^t, \quad y(0) = 1, y'(0) = 0.$	**07**

	(b)	(i) Solve $p^2 + q^2 = npq$	**03**
		(ii) Solve $pz - qz = z^2 + (x+y)^2$	**04**
Q.5	**(a)**	State the convolution theorem and verify it for $f(t) = t$ and $g(t) = e^{2t}$.	**07**
	(b)	Obtain the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 \le x \le 2\pi$. Hence Deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2}$	**07**

OR

Q.5	**(a)**	(i) Form a partial differential equation by eliminating the arbitrary functions from $xyz = \emptyset(x+y+z)$	**03**
		(ii) Find the Laplace Transform of $f(t) = sinat$	**04**
	(b)	Find the fourier integral representation of the function $$f(x) = \begin{cases} 2\ ; & |x| < 2 \\ 0\ ; & |x| > 2 \end{cases}$$	**07**

****

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (Old) EXAMINATION – WINTER 2019

Subject Code: 130002 **Date: 22/11/2019**

Subject Name: Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	(i) Solve $ye^x\,dx + (2y + e^x)\,dy = 0$	**03**
		(ii) Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$	**04**
	(b)	Obtain Fourier series of $f(x) = x^2$ in the interval$(0, 4)$.	**07**
Q.2	**(a)**	(i) Use method of Undetermined coefficients and find general solution of $y'' + 10y' + 25y = e^{-5x}$	**07**
	(b)	Find general solution of $(D^2 + 2D - 35)y = 37\sin 5x$	**07**
		OR	
	(b)	Solve by Variation of parameter method $(D^2 + 9)y = tan3x$	**07**
Q.3	**(a)**	Find Fourier series of $f(x) = e^{ax}$ in $(0, 2\pi), a > 0$	**07**
	(b)	Find Fourier series of $f(x) = \begin{cases} x & ,0 \le x \le 2 \\ 4 - x & ,2 \le x \le 4 \end{cases}$	**07**
		OR	
Q.3	**(a)**	Find the Series solution of $y'' - 2y' = 0$	**07**
	(b)	Express the function $f(x) = \begin{cases} \sin x\,, & 0 \le x \le \pi \\ 0\,, & x > \pi \end{cases}$ as a Fourier sine integral and show that $$\int_0^\infty \frac{\sin \omega x \sin \pi\omega}{1 - \omega^2}\,d\omega = \frac{\pi}{2}\sin x \quad , \quad 0 \le x \le \pi$$	**07**
Q.4	**(a)**	(i) Find Laplace transform of $e^t\left(1 + \sqrt{t}\right)^4$	**03**
		(ii) Find the inverse Laplace transform of $\frac{2s+2}{s^2+2s+10}$	**04**
	(b)	State Convolution theorem and using it find inverse Laplace transform of $\frac{1}{(s-2)(s+2)^2}$	**07**
		OR	
Q.4	**(a)**	(i) Find Laplace transform of $e^{-3t}\,u(t-2)$	**03**
		(ii) Find inverse Laplace transform of $\frac{e^{-2s}}{(s+4)^3}$	**04**
	(b)	Solve initial value problem using Laplace transform method $$y'' - 3y' + 2y = 12e^{-2t}, \quad y(0) = 2, y'(0) = 6$$	**07**
Q.5	**(a)**	(i) Form Partial differential equation for the equation $z = ax + by + ct$	**03**
		(ii) Find Laplace transform of $f(t) = \begin{cases} cos\,t & ,0 < t < 2\pi \\ 0 & ,\ t > 2\pi \end{cases}$	**04**

(b) Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x\,\partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$ **07**

OR

Q.5 **(a)** Find the Series solution of $4xy'' + 2y' + y = 0$ **07**

(b) Using method of Separation of variables solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ given that $u(0, y) = 8\,e^{-3y}$ **07**

****

Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – WINTER 2018

Subject Code:130002 **Date:17/11/2018**

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Solve: $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)dx + x\sec^2\frac{y}{x}\,dy = 0$ **04**

(ii) Find Laplace transform of $\left(\cos at - \cos bt\right)/t$ **03**

(b) (i) Solve: $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}.$ **04**

(ii) Find Inverse Laplace transform of $\frac{3\left(s^2-1\right)^2}{s^5}$ **03**

Q.2 **(a)** (i) Find Half-Range cosine series for $f(x) = 1;\ 0 \le x \le 1.$ **04**

$= x;\ 1 \le x \le 2.$

(ii) Solve: $(y+z)p + (x+z)q = x + y$ **03**

(b) Obtain Fourier series for $f(x) = x - x^2;\ -1 < x < 1$ **07**

OR

(b) Find series solution of $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} = 0.$ **07**

Q.3 **(a)** (i) Solve $2r + 5s + 2t = 0.$ **04**

(ii) Solve: $\left(2xy + y - \tan y\right)dx + \left(x^2 - x\tan^2 y + \sec^2 y\right)dy = 0.$ **03**

(b) Obtain Fourier series for $f(x) = |\sin x|;\ -\pi < x < \pi.$ **07**

OR

Q.3 **(a)** (i) Define periodic function. Find Laplace transform of $f(t) = t^2;\ 0 \le t \le 2\quad f(t+2) = f(t).$ **04**

(ii) Solve: $\left(x^2-1\right)\frac{dy}{dx} + 2xy = 1.$ **03**

(b) State Convolution theorem. Use it find Inverse Laplace transform of $\frac{s}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$ **07**

Q.4 **(a)** (i) Solve: $\frac{d^4x}{dt^4} + 4x = 0.$ **04**

(ii) Evaluate: Prove that $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}dt = \ln\left(\frac{b}{a}\right).$ **03**

(b) (i) Solve $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$. using method of separable variable method. **04**

(ii) Solve $p^2 + q^2 = 1$ **03**

OR

Q.4 **(a)** (i) Solve: $\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} - 6y = 0.$ **04**

(ii) Find Inverse Laplace transform of $\frac{5s^2 - 2s - 19}{(s+3)(s-1)^2}$ **03**

(b) Find series solution of $(1+x^2)y'' + xy' - y = 0.$ **07**

Q.5 **(a)** (i) Express the function $f(x) = 1;\ |x| < 1$ as Fourier integral. **05**

$= 0$; Otherwise

(ii) Define Dirac-Delta function **02**

(b) Find solution of $y'' - 2y' + 2y = e^x \tan x$ using method of variation of parameter. **07**

OR

Q.5 **(a)** (i) Solve: $(x^2D^2 + 4xD + 2)y = x + \log x.$ **05**

(ii) Define Gamma function and find its value for 3/2. **02**

(b) Find solution of $(D^2 + D)y = x^2 + 2x + 4.$ using method of undetermined coefficient **07**

****