

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-III (NEW) EXAMINATION – WINTER 2024****Subject Code:2130002****Date:18-12-2024****Subject Name:Advance Engineering Mathematics****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		Marks
Q.1	(a) Solve: $x^2 dy + y(x + y)dx = 0$	03
	(b) Find the Laplace transform of $t^2 \sin \pi t$	04
	(c) Find a Fourier series for a periodic function $f(x)$ with period 2, where $f(x) = -1, \quad -1 < x < 0$ $= 1, \quad 0 < x < 1$	07
Q.2	(a) Define Beta function, Gamma function and write the relation between Beta and Gamma function.	03
	(b) Solve: $(x + 1) \frac{dy}{dx} - y = e^{3x}(x + 1)^2$	04
	(c) Find the Fourier series of $f(x) = x + x $, $-\pi < x < \pi$.	07
	OR	
	(c) Solve $y'' + 4y = 8x^2$ by using the method of undetermined coefficients.	07
Q.3	(a) Laplace Find $L^{-1} \left\{ \frac{s^3 + 2s^2 + 2}{s^3(s^2 + 1)} \right\}$	03
	(b) Find particular integral for an equation $(D^2 - 4D + 3)y = \sin 3x \cos 2x$	04
	(c) Solve the IVP $y'' - 2y' = e^t \sin t$, $y(0) = y'(0) = 0$	07
	OR	
Q.3	(a) By using first shifting theorem find $L\{(t + 1)^2 e^t\}$	03
	(b) Solve $\frac{d^2 y}{dx^2} + 4y = \tan 2x$ by using Variation of Parameter.	04
	(c) Using convolution theorem find the inverse transform of $\frac{a}{s^2(s^2 + a^2)}$	07
Q.4	(a) Solve $\frac{dy}{dx} + \frac{2y}{x} = \sin x$	03
	(b) Find the Laplace Transform of $e^{-2t}(\sin 4t + t^2)$	04
	(c) Find the Power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$	07
	OR	
Q.4	(a) (I) Find the Laplace Transform of $\frac{\cos at - \cos bt}{t}$	03
	(II) Find the Laplace transform of $L\{t^2 u(t - 2)\}$	
	(b) Find half-range cosine series for $f(x) = (x - 1)^2$, $0 < x < 1$	04
	(c) Find the series solution of $xy'' + y' + xy = 0$	07

- Q.5** (a) (I) Find the complete integral of $pq = 4z$ **03**
 (II) Find the complete integral of $p - x^2 = q + y^2$
 (b) Form a partial differential equation by eliminating arbitrary constants a and b from equation $z = (x^2 + a)(y^2 + b)$ **04**
 (c) Using Charpti's method solve: $z = pq$ **07**
- OR**
- Q.5** (a) Form a partial differential equation by eliminating arbitrary function from $\phi(x^2 - y^2, xyz) = 0$ **03**
 (b) Solve $x^2p + y^2q = z^2$ **04**
 (c) Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given $u(x, 0) = 4e^{-4x}$ by using Separation of variable method. **07**

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III (NEW) EXAMINATION – WINTER 2023****Subject Code:2130002****Date:02-02-2024****Subject Name:Advance Engineering Mathematics****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		MARKS
Q.1	(a) Define Gamma function, Beta function and Dirac's Delta function	03
	(b) Find the Fourier integral representation, of the function $f(x) = \begin{cases} 1 & x < 1 \\ 0 & x > 1 \end{cases}$	04
	(c) Find Fourier Series expansion of $f(x) = x^2, -\pi < x < \pi$	07
Q.2	(a) Evaluate: $L\left\{t \int_0^t e^{-4t} \sin 3t dt\right\}$.	03
	(b) Find the inverse Laplace transform of $\frac{1}{s(s^2 - 3s + 3)}$.	04
	(c) Solve differential equation, using Laplace Transform $y'' + 6y' = 1, y(0) = 2, y'(0) = 0$.	07
	OR	
	(c) Define Convolution Theorem and use it to find Laplace Transform of $\frac{1}{s^2(s+5)}, \frac{1}{s(s^2+4)}$.	07
Q.3	(a) Solve the following Differential equations: 1) $ye^x dx + (2y + e^x) dy = 0$ 2) $9yy' + 4x = 0$	03
	(b) Solve the following equations : (a) $(D^2 + 3D + 2)y = x e^x \sin x$	04
	(c) 1) Solve by using method of variation of parameters: $y'' + y = \sin x$.	04
	2) Find the wronskian of the given bases 1) $x^4, x^4 \ln x$ 2) $e^{3x}, x e^{3x}$	03
	OR	
Q.3	(a) Use method of undetermined coefficient to solve $y'' + 4y = 8x^2$.	03
	(b) Solve the following equations: (a) $(D^2 - 4D - 5)y = x e^{2x} + 3 \cos 4x$.	04
	(c) Find a general solution of $(4x^2 D^2 + 16xD + 9)y = 0$.	07
Q.4	(a) Find the laplace transform of $\frac{\cos at - \cos bt}{t}$ and $e^{-5t} \sin t$.	03

- (b) Find Ordinary point, Singular point, Regular singular and irregular singular point of the following differential equation
 $x^3(x-1)y'' + 3(x-1)y' + 7xy = 0$ 04
- (c) Find the Power Series solution about $x = 0$ of the equation $y'' + xy = 0$. 07

OR

- Q.4** (a) Find the Laplace Transform of $f(t) = e^t$ $0 < t < 2\pi$ 03
 if $f(t) = f(t + 2\pi)$
- (b) If $y_1 = x$ is one solution of $x^2y'' + xy' - y = 0$, find the second solution. 04
- (c) Find the Series solution of the equations by Frobenius method
 $2x(1-x)y'' + (1-x)y' + 3y = 0$ about $x_0 = 0$ 07
- Q.5** (a) Solve the following Partial Differential Equations 03
 $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$
- (b) Solve the following nonlinear Partial Differential Equations 04
 1) $p^2 + q^2 = x + y$ 2) $pq = p + q$
- (c) Evaluate $(D^2 + DD' - 6D')z = \sin(2x + y)$ 07

OR

- Q.5** (a) Form Partial Differential Equations by eliminating arbitrary function
 $z = f(x + ct) + g(x - ct)$ 03
- (b) Solve the following Partial Differential Equations by Direct integration method 04
 1) $\frac{\partial^2 z}{\partial x \partial y} = \cos x \cos y$
- (c) Solve: $\frac{\partial z}{\partial x} = 4 \frac{\partial z}{\partial y}$ where $z(0, y) = 8e^{-3y}$ using method of separation of variables 07

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- III(NEW) EXAMINATION – WINTER 2022****Subject Code:2130002****Date:16-02-2023****Subject Name:Advance Engineering Mathematics****Time:02:30 PM TO 05:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		MARKS
Q.1	(a) Solve $(y^2 - x^2)dx + (2xy)dy = 0$	03
	(b) Solve $x^2 \frac{dy}{dx} + xy = x^4 y^6$	04
	(c) Solve $y'' + 2y' + 5y = e^{-t} \sin t$, $y(0) = 0, y'(0) = 1$ using Laplace transform.	07
Q.2	(a) Solve $\frac{d^4 y}{dx^4} + 4y = 0$.	03
	(b) Solve $y'' + 4y = \sin 3x$.	04
	(c) Find the Fourier series of $f(x) = x + x^2$ in $-\pi < x < \pi$.	07
	OR	
	(c) Find the Fourier series for the function	07
	$f(x) = \begin{cases} -\pi; & -\pi < x < 0 \\ x; & 0 < x < \pi \end{cases}$	
Q.3	(a) Find $L\{\sin(3t + 2)\}$.	03
	(b) Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.	04
	(c) Find the power series solution of $y'' + x^2 y = 0$.	07
	OR	
Q.3	(a) Find $L\{t \sinh 3t\}$.	03
	(b) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$.	04
	(c) Using the method of variation of parameters, solve	07
	$y'' - 4y' + 4y = \frac{e^{2x}}{x}.$	
Q.4	(a) Solve $y'' + 6y + 9y = e^{3x}$.	03
	(b) Solve $(D^3 - D^2 - 6D)y = x^2 + 1$.	04
	(c) Using convolution theorem find the inverse Laplace transform of	07
	$\frac{1}{(s^2 + a^2)^2}.$	
	OR	
Q.4	(a) Find the convolution of t and e^t .	03
	(b) Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.	04
	(c) Find the Fourier series for the function $f(x) = 2x - x^2$ with period 3 in the range $(0, 3)$.	07
Q.5	(a) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.	03
	(b) Solve $p(1 + q) = qz$.	04

- (c) Solve $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 3y$. 07

OR

- Q.5** (a) Form the partial differential equation from $z = f\left(\frac{x}{y}\right)$. 03

- (b) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that $z = e^y, \frac{\partial z}{\partial x} = 1$ when $x = 0$. 04

- (c) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ given $u(x, 0) = 6e^{-3x}$. 07

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III (NEW) EXAMINATION – WINTER 2021****Subject Code:2130002****Date:15-02-2022****Subject Name:Advance Engineering Mathematics****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		MARKS
Q.1	(a) Solve $yy' - 2x = 0$.	03
	(b) Find $L^{-1} \left[\frac{2s+1}{s(s+1)} \right]$.	04
	(c) Expand $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series in the interval $(0, 2\pi)$.	07
Q.2	(a) Define unit step function and rectangle function.	03
	(b) Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 12e^x$.	04
	(c) Find a power series solution of $y'' + xy = 0$ about the ordinary point $x = 0$.	07
	OR	
	(c) Find the Fourier series of	07
	$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$	
Q.3	(a) Find $L \left[\int_0^t \int_0^t \sin at \, dt \, dt \right]$.	03
	(b) Solve $(D^2 + 9)y = \sin 2x + \cos 4x$.	04
	(c) Find the inverse Laplace transform of $\frac{1}{(s^2+4)^2}$ by Convolution theorem.	07
	OR	
Q.3	(a) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$.	03
	(b) Find $L^{-1} \left[\frac{6+s}{s^2+6s+13} \right]$.	04
	(c) Solve $y'' + 4y = 8x^2$ by method of undetermined coefficients.	07
Q.4	(a) Find the half range sine series of $f(x) = e^x$ in $0 < x < \pi$.	03
	(b) Find the Laplace transform of	04
	1) te^{-t}	
	2) $e^{-at} \cos bt$.	
	(c) Solve $y'' + y = \operatorname{cosec} x$ by method of variation of parameters.	07
	OR	
Q.4	(a) Find the Laplace transform of $\frac{\sin 2t}{t}$.	03
	(b) Solve $(D^2 + 10DD' + 25D'^2)z = e^{3x+2y}$.	04
	(c) Solve by Laplace Transform $y' + 2y = e^{-3t}$ with $y(0) = 1$.	07
Q.5	(a) Form a partial differential equation for the equation $z = (x-3)^2 + (y-4)^2$.	03

- (b) Solve $p - x^2 = q + y^2$. 04
 (c) Solve $z = pq$ by Charpit's method. 07

OR

- Q.5** (a) Solve $z = px + qy - 2\sqrt{pq}$. 03
 (b) Solve $xp + yq = 3z$. 04
 (c) Solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$ by the method of 07
 separation of variables.

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER– III (New) EXAMINATION – WINTER 2019****Subject Code: 2130002****Date: 22/11/2019****Subject Name: Advanced Engineering Mathematics****Time: 02:30 PM TO 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

MARKS

Q.1 (a) Determine the singular points of the differential equation $x(x+1)^2 y'' + (2x-1)y' + x^2 y = 0$ and classify them as regular or irregular. **03**

(b) (i) Compute $\beta\left(\frac{5}{2}, \frac{3}{2}\right)$ **02**

(ii) Define (1) Error Function (2) Beta Function **02**

(c) (i) Solve the I. V. P: $y'' - 4y' + 4y = 0$, $y(0) = 3$ & $y'(0) = 1$ **03**

(ii) Find (a) $L\{e^t(\sin 3t + t^3)\}$ (b) $L^{-1}\left\{\frac{6s-7}{s^2+5}\right\}$ **04**

Q.2 (a) Solve $(1+x)y dx + (1-y)x dy = 0$ **03**

(b) Solve $(D^2 - 5D + 6)y = \sin 3x$ **04**

(c) State the convolution theorem and apply it to evaluate $L^{-1}\left[\frac{1}{s(s+a)^3}\right]$ **07**

OR

(c) Using Laplace Transformation, Solve $y'' + 6y = 1$, $y(0) = 2$, $y'(0) = 0$ **07**

Q.3 (a) Find the Laplace transform of $f(t) = \begin{cases} 0, & 0 \leq t < 2 \\ 3, & t \geq 2 \end{cases}$ **03**

(b) Find the power series solution of $y' = 2xy$. **04**

(c) Obtain Fourier series of the Function $f(x) = \begin{cases} 0, & -2 \leq x < 0 \\ 1, & 0 \leq x \leq 2 \end{cases}$ **07**

OR

Q.3 (a) Find the Inverse Laplace Transform of $\frac{6e^{-2s}}{(s^2+4)}$ **03**

(b) Find the series solution of $y'' + x^2 y = 0$ in power of x . **04**

(c) Obtain Fourier series of the Function $f(x) = x + |x|$, $-\pi < x < \pi$ **07**

Q.4 (a) Solve $\frac{dy}{dx} + y \tan x = \sin 2x$ **03**

(b) Find a sine series for $f(x) = e^x$ in $0 < x < \pi$. **04**

(c) By the Method of Separation of variables, solve $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ where $u(x,0) = 4e^{-3x}$ **07**

OR

Q.4 (a) Solve $y e^x dx + (2y + e^x) dy = 0$, $y(0) = -1$ **03**

(b) Find a cosine series for $f(x) = x^2$ in $0 < x < \pi$. **04**

(c) Using Undetermined co-efficient method, solve the differential equation **07**
 $y'' + y' - 6y = 6x + 3x^2 - 6x^3$

Q.5 (a) Solve $z = px + qy + p^2 q^2$ **03**

(b) Find (1) $L \left[\int_0^t e^{-t} \cos u \, du \right]$ (2) $L^{-1} \left[\frac{2s+2}{s^2+2s+10} \right]$ **04**

(c) Find the general solution of P. D. E : $(x^2 - yz) p + (y^2 - zx) q = z^2 - xy$ **07**

OR

Q.5 (a) Form a Partial differential equation from $f(xy + z^2, x + y + z) = 0$ **03**

(b) Find (1) $L \left[\int_0^t \int_0^t \sin au \, du \, du \right]$ (2) $L^{-1} \left[\frac{s+2}{s^2+4s+8} \right]$ **04**

(c) Using Method of Variation of parameters, Solve $(D^2 - 2D + 1)y = 3x^{\frac{3}{2}} e^x$ **07**
