Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – SUMMER 2024

Subject Code:130002 **Date:11-07-2024**

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	(i) Solve $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$	**04**
		(ii) Solve $\frac{dy}{dx} + \frac{1}{x^2}y = 6e^{\frac{1}{x}}$	**03**
	(b)	(i) Find Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$, using first shifting theorem.	**04**
		(ii) Find inverse Laplace transform of $\frac{s+2}{s^2+4s+8}$	**03**
Q.2	**(a)**	Find Half range cosine series of $f(x) = x^2,\ \ 0 < x < 1$	**07**
	(b)	Obtain Fourier series of $f(x) = x + x^2,\ -\pi < x < \pi$.	**07**
		OR	
	(b)	Find Series solution of $y' - 2xy = 0$	**07**
Q.3	**(a)**	(i) If x is one solution of $x^2y'' + xy' - y = 0$, find the second solution.	**04**
		(ii) Solve $(p - q)(z - px - qy) = 1$	**03**
	(b)	Solve $(D^2 - 2D)y = e^x \sin x$,using method of Undetermined coefficients.	**07**
		OR	
Q.3	**(a)**	(i) Find the Fourier sine integral of $f(x) = e^{-bx}$.	**04**
		(ii) Solve $p^2 - q^2 = x - y$.	**03**
	(b)	Using Power series method, solve $(1 - x^2)y'' - 2xy' + 2y = 0$	**07**
Q.4	**(a)**	(i) Solve $(D^3 + 1)y = 0$	**04**
		(ii) Solve $(D^4 - 2D^3 + D^2)y = 0$	**03**
	(b)	Using method of Variation of parameter, solve $(D^2 + 3D + 2)y = e^{e^x}$	**07**
		OR	
Q.4	**(a)**	(i) Find inverse Laplace transform of $\log\left(1 + \frac{\omega^2}{s^2}\right)$	**04**
		(ii) Find Laplace transform of $(\sin 2t)/t$	**03**
	(b)	Find inverse Laplace transform of $\frac{1}{(s+1)(s^2+1)}$, using Convolution theorem.	**07**
Q.5	**(a)**	(i) Solve $p^2 + q^2 = 1$	**04**
		(ii) Define Gamma function and find its value for 3/2.	**03**
	(b)	Solve $(D^2 + 10DD' + 25D'^2)z = e^{3x+2y}$	**07**
		OR	
Q.5	**(a)**	(i) Solve $yzp - xzq = xy$.	**04**
		(ii) Define Dirac's Delta function and Beta function.	**03**
	(b)	Solve $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$, using method of separation of variables.	**07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III(OLD) EXAMINATION – SUMMER 2023

Subject Code:130002 **Date:21-07-2023**

Subject Name:Advanced Engineering Mathematics

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Find the Fourier series of $f(x) = x + \|x\|$ in the interval $-\pi < x < \pi$	**07**
	(b)	Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = logx.\sin(logx)$	**07**
Q.2	**(a)**	(i) Solve $\frac{dy}{dx} + 2ytanx = sinx$	**03**
		(ii) Solve $(D^2 + a^2)y = cosax$ by the method of variation of parameter	**04**
	(b)	Solve $\frac{d^3y}{dx^3} + 8y = cosh2x$	**07**
		OR	
	(b)	(i) Find the Laplace transform of $e^{-3t}(2cos5t - 3sin5t)$.	**03**
		(ii) Find the Inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$	**04**
Q.3	**(a)**	(i) Define 1. Gamma Function 2. Beta Function 3. Signum Function	**03**
		(ii) Find half range *cosine* series for $f(x) = x,\ 0 < x < 3$.	**04**
	(b)	Find the power series solution of the equation $(x^2 + 1)y$” + xy’- xy = 0 about x = 0	**07**
		OR	
Q.3	**(a)**	(i) Solve $(x + 1)\frac{dy}{dx} - y = e^{3x}(x + 1)^2$	**03**
		(ii) Solve by using Undetermined Coefficient method $(D^2 - 2D + 3)y = x^3 + sinx$	**04**
	(b)	Find the series solution of $2x(x-1)y'' - (x+1)y' + y = 0; x_0 = 0$	**07**

Q.4	**(a)**	Find the Inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$	**07**
	(b)	Solve $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ subject to the condition $u(x,0) = 4e^{-3x}$ by method of separation of variables.	**07**

OR

Q.4	**(a)**	Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}sint$, where $x(0) = 0$ & $x'(0) = 1$.	**07**
	(b)	(i) Solve $p^2 + q^2 = npq$	**03**
		(ii) Solve $pz - qz = z^2 + (x+y)^2$	**04**
Q.5	**(a)**	Find the Inverse Laplace transform of $\frac{s+2}{(s^2+4s+5)^2}$ using Convolution Method.	**07**
	(b)	Obtain the Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$.	**07**

OR

Q.5	**(a)**	(i) Form a partial differential equation by eliminating the arbitrary functions from $xyz = \emptyset(x+y+z)$	**03**
		(ii) Find the Laplace transform of $cos3t.cos2t.cost$	**04**
	(b)	Find the series solution $(x-2)\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + 9y = 0$ about $x_0 = 0$	**07**

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GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (OLD) EXAMINATION – SUMMER 2022

Subject Code:130002 **Date:08-07-2022**

Subject Name:Advanced Engineering Mathematics

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	(i) Define Dirac – Delta function.	**02**
		(ii) Find the inverse laplace transform of in $\frac{s^2+1}{(s+1)(s-2)^2}$	**05**
	(b)	Find the series solution of y"=2y' in powers of x.	**07**
Q.2	**(a)**	(i) Find the fourier sine series of $f(x) = e^x$ in $0 < x < \pi$.	**03**
		(ii) $\frac{dy}{dx} + 2y\ tanx = sinx$	**04**
	(b)	Find the Fourier series of $f(x) = x + \lvert x \rvert$ in the interval $-\pi < x < \pi$.	**07**
		OR	
	(b)	Find the Fourier series for $f(x) = -\pi\ ; \quad \pi < x < 0$ $f(x) = -\pi\ ; \quad \pi < x < 0$ $= x - \pi; \quad 0 < x < \pi$	**07**
Q.3	**(a)**	(i) Find the value of $B(\frac{3}{2}, \frac{1}{2})$.	**02**
		(ii) Show that $\int_0^\infty \frac{\lambda^3 sin\lambda x}{\lambda^4+4} d\lambda = \frac{\pi}{2} e^{-x} cosx$, where x > 0.	**05**
	(b)	Solve $y'' + a^2 y = tanx$ by using Variation of Parameters.	**07**
		OR	
Q.3	**(a)**	(i) $ye^x dx + (2y + e^x)dy = 0$.	**03**
		(ii) $y''' - 3y'' + 3y' - y = 4e^t$	**04**
	(b)	Using the power- series method, Solve $(1 - x^2)y'' - 2xy' + 2y = 0$	**07**
Q.4	**(a)**	(i) Find the Laplace transform of $\frac{e^{-t} sint}{t}$.	**03**
		(ii) Find the inverse Laplace transform of $\log(1 + \frac{w^2}{s^2})$.	**04**
	(b)	Using the convolution method to solve $\frac{s+2}{(s^2+4s+5)^2}$	**07**

OR

Q.4 **(a)** (i) Solve $y''' - 6y' + 11y' - 6y = 0.$ **03**

(ii) $x^2y'' - xy' + y = \sin(logx)$ **04**

(b) Solve $y'' + 4y' + 3y = e^{-t}, y(0) = y'(0) = 1.$ **07**

Q.5 **(a)** (i) Solve $(y + z)p + (z + x)q = x + y$ **03**

(ii) $x^2ydx - (x^3 + xy^2)dy = 0.$ **04**

(b) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$by method of separation variables. **07**

OR

Q.5 **(a)** (i) Form a Partial Differential Equation by eliminating the arbitrary constants from the equation $z = (x - 2)^2 + (y - 3)^2$ **03**

(ii) Solve $(D^2 - 2DD' + D'^2)z = e^x + 2y + x^3$ **04**

(b) Solve $y'' + 2y' + 4y = 2x^2 + 3e^{-x}$ by using method of Undetermined coefficients. **07**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – SUMMER 2021

Subject Code:130002 **Date:03/09/2021**

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	(i) Solve $\frac{dy}{dx} = \sin(x+y)$.	**03**
		(ii) Solve $(1+ x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$.	**04**
	(b)	Solve the differential equation $\frac{d^2y}{dx^2} + x^2y = 0$ by Power method at $x = 0$.	**07**
Q.2	**(a)**	Solve the partial differential equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ subject to condition $u(0,y) = 3e^{-y} - 5e^{-5y}$ by method of Separation Variables.	**07**
	(b)	Solve the differential equation by Frobenius method $$x(x-1)y'' + (3x-1)y' + y = 0.$$	**07**
		OR	
	(b)	(i) Solve: $(D^2 - 5D + 6)y = x + e^{4x}$.	**03**
		(ii) Solve : $(D^2 + a^2)y = cosec(ax)$ by variation parameter method	**04**
Q.3	**(a)**	Find the Fourier series for the function $f(x) = x + x^2,\ -\pi < x < \pi$.	**07**
	(b)	Obtain Fourier series for the function $f(x) = x + 1, \quad -1 < x < 0$ $= x - 1, \quad 0 < x < 1$.	**07**
		OR	
Q.3	**(a)**	Express $f(x) = e^x,\ 0 < x < l$ as a half range Fourier Cosine series with period $2l$.	**07**
	(b)	Find the Fourier series of $f(x) = \sqrt{1 - cosx}$ in the interval $[0, 2\pi]$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$.	**07**
Q.4	**(a)**	(i) State the change of scale property of Laplace Transform. If $L\{f(t)\} = \frac{s}{s^2-k^2}$. find $L\{f(3t).\}$	**03**
		(ii) Find the Laplace Transform of $\frac{e^{-2t}\sin(2t)\cosh(t)}{t}$.	**04**
	(b)	Find the Inverse Laplace Transform of (i) $\frac{3s+7}{s^2-2s-3}$ (ii) $ln\left(1+\frac{1}{s^2}\right)$	**07**
		OR	
Q.4	**(a)**	Solve the initial value problem $y'' + 2y' + y = e^{-t},\ y(0) = -1,\ y'(0) = 1$ by using Laplace transform method.	**07**
	(b)	(i) Find Laplace Transform of $tsin^2 3t$.	**03**
		(ii) State convolution theorem. Use it to find Inverse Laplace Transform of $\frac{1}{(s+1)(s^2+1)}$	**04**

Q.5	**(a)**	(i) Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.	**03**
		(ii) Solve $\left(D^2 - 2DD' + D'^2\right)z = e^{x+2y}$.	**04**
	(b)	(i) Solve $p^2 + q^2 = x + y$.	**03**
		(ii) Solve $(mz - ny)p + (nx - lz)q = ly - mx$.	**04**

OR

Q.5	**(a)**	(i) Solve $z = px + qy + n\sqrt{1 + p^2 + q^2}$.	**03**
		(ii) Solve $(D^2 + 3D + 2)y = e^{2x} sinx$.	**04**
	(b)	(i) Define Heaviside's function	**02**
		(ii) Express the function $f(x) = sinx, \quad 0 \le x \le \pi$ $\qquad = 0, \quad x > \pi$ as a Fourier sine Integral.	**05**

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Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(OLD) EXAMINATION – SUMMER 2019

Subject Code: 130002 **Date: 30/05/2019**

Subject Name: Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Solve $3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$ **03**

(ii) Solve $y' - (1 + 3x^{-1})y = x+2;\ y(1) = e - 1$ **04**

(b) Find the Power series solution of the differential equation $y'' = y'$. **07**

Q.2 **(a)** Using the method of separation of variables solve $u_{xx} = 16\, u_y$. **07**

(b) Find the series solution of the differential equation by Frobenius method **07**

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

OR

(b) (i) Solve $y'' + 4y = 8\cos 2x,\ y(0) = 0,\ y'(0) = 2$ **03**

(ii) Solve $y'' - 4y' - 12y = 7e^{-7x}$ by method of undetermined coefficients. **04**

Q.3 **(a)** Find the Fourier series for the function $f(x) = x^2 + x,\ -\pi \le x \le \pi$. **07**

(b) Find the Fourier series of the function **07**

$$f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$$

OR

Q.3 **(a)** Find the Fourier series with period 3 to represent $f(x) = 2x - x^2$ in the range (0, 3). **07**

(b) Find the half range Fourier cosine series of the function $f(x) = c - x$ in interval (0, c) with period 2c. **07**

Q.4 **(a)** (i) Find the Laplace transform of $e^{-t}(4t^3 + 3\cos 2t + 2e^{-2t})$ **03**

(ii) Prove that **04**

$$L(\sin at) = \frac{a}{s^2 + a^2} \text{ and } L(\cos at) = \frac{s}{s^2 + a^2}$$

$s > 0$, where a is a constant.

(b) Find the Inverse Laplace transform of **07**

(1) $\dfrac{s+3}{(s^2+1)(s^2+9)}$ (2) $\dfrac{2s+3}{s^2 - 2s + 5}$

OR

Q.4 **(a)** (i) Find the Laplace transform of **03**

$$e^{-2t}\int_0^t t\cos t\, dt$$

(ii) Find the Inverse Laplace transform of **04**

$$\frac{1 + e^{-\frac{\pi}{2}s}}{s^2 + 4}$$

(b) Using Laplace transform solve the differential equation y'' + 6y = 1, y(0) = 2, y'(0) = 0 **07**

Q.5 **(a)** (i) Form Partial differential equation by eliminating the arbitrary function from the equation **03**

$$z = y^2 + 2f\left(\frac{1}{x} + logy\right)$$

(ii) Define the following: (1) Beta function (2) Dirac's Delta Function **04**

(b) Express the function as a Fourier Integral **07**

$$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & [x] > 1 \end{cases}$$

OR

Q.5 **(a)** (i) Solve : p + q = pq **03**

(ii) Solve: $x\,(y^2 - z^2)\,p + y\,(z^2 - x^2)\,q = z(x^2 - y^2)$. **04**

(b) Solve the following: **07**

(i) $\dfrac{\partial^3 z}{\partial x^3} - 4\dfrac{\partial^3 z}{\partial x^2 \partial y} + 4\dfrac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$

(ii) $(D - D' - 1)(D - D' - 2)\,z = e^{2x - y}$

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