Subject Name & Code:

ENGINEERING THERMODYNAMICS- 3131905 BE03000201

Assignment

TOPIC: VAPOUR CYCLE NUMERICALS (5 PROBLEMS)

1. From SUM 25, Q.2 (c)

- **Problem: Steam enters a nozzle at 7 bar, 20°C (h_1=2850 kJ/kg) and leaves at 1.5 bar. Initial and final velocities are 40 m/s and 700 m/s respectively. Mass flow rate is 1400 kg/hr. Heat loss is 11705 kJ/hr. Specific volume at exit is 1.24 m³/kg. Determine the final enthalpy and the exit area.**
- **Concept: Steady Flow Energy Equation (SFEE) application for a nozzle.**

Answer:

Given:

- Inlet Pressure, $P_1 = 7$ bar
- Inlet Enthalpy, $h_1 = 2850$ kJ/kg
- Inlet Velocity, $V_1 = 40$ m/s
- Exit Pressure, $P_2 = 1.5$ bar
- Exit Velocity, $V_2 = 700$ m/s
- Mass Flow Rate, $\dot{m} = 1400$ kg/hr $= \frac{1400}{3600}$ kg/s $= 0.3889$ kg/s
- Heat Loss, $\dot{Q} = -11705$ kJ/hr $= -\frac{11705}{3600}$ kW $= -3.251$ kW
- Exit Specific Volume, $v_2 = 1.24\ \text{m}^3/\text{kg}$

To Find: Final Enthalpy (h_2) and Exit Area (A_2).

Solution:

Step 1: Apply Steady Flow Energy Equation (SFEE) for a Nozzle
The general SFEE is:

$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 1000} + g(z_2 - z_1)\right]$$

For a nozzle: $\dot{W} = 0$, and potential energy change $g(z_2 - z_1)$ is negligible.

$$\dot{Q} = \dot{m}[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000}]$$

Substituting the known values:

$$-3.251 = 0.3889[(h_2 - 2850) + \frac{700^2 - 40^2}{2000}]$$
$$-3.251 = 0.3889[(h_2 - 2850) + \frac{490000 - 1600}{2000}]$$
$$-3.251 = 0.3889[(h_2 - 2850) + \frac{488400}{2000}]$$
$$-3.251 = 0.3889[(h_2 - 2850) + 244.2]$$

Step 2: Solve for Final Enthalpy (h_2)

$$\frac{-3.251}{0.3889} = h_2 - 2850 + 244.2$$
$$-8.359 = h_2 - 2605.8$$
$$h_2 = 2605.8 - 8.359$$
$$h_2 = 2597.44 \text{ kJ/kg}$$

Step 3: Calculate Exit Area (A_2)
Using the continuity equation at the exit:

$$\dot{m} = \frac{A_2 V_2}{v_2}$$
$$A_2 = \frac{\dot{m}\, v_2}{V_2}$$
$$A_2 = \frac{0.3889 \times 1.24}{700}$$
$$A_2 = \frac{0.4822}{700}$$
$$A_2 = 6.889 \times 10^{-4} \text{ m}^2$$
$$A_2 = 688.9 \text{ mm}^2$$

Final Answers:

$$\boxed{h_2 = 2597.44 \text{ kJ/kg}} \quad \boxed{A_2 = 688.9 \text{ mm}^2}$$

2. From WIN 24, Q.4 (c)

- **Problem: In a steam power cycle, the dry saturated steam is supplied at 15 bar. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.**

- **Concept: Efficiency calculation for Carnot and Rankine cycles using steam tables.**

Answer:

Given:

- Boiler Pressure, $P_3 = P_2 = 15$ bar (Dry Saturated)
- Condenser Pressure, $P_1 = P_4 = 0.4$ bar
- Pump work is neglected.

To Find: Carnot Efficiency (η_{Carnot}) and Rankine Efficiency ($\eta_{Rankine}$).

Solution:

Step 1: Find Enthalpies and Temperatures from Steam Tables

- **At 15 bar (Saturation):**
  - $T_{sat} = 198.3°C = 471.45$ K
  - $h_g = h_3 = 2792.2$ kJ/kg
  -
- **At 0.4 bar:**
  - $T_{sat} = 75.9°C = 349.05$ K
  - $h_f = h_1 = 317.7$ kJ/kg
  - $h_{fg} = 2319.2$ kJ/kg
  -
  -

Step 2: Calculate Carnot Efficiency

$$\eta_{Carnot} = 1 - \frac{T_{low}}{T_{high}} = 1 - \frac{349.05}{471.45}$$
$$\eta_{Carnot} = 1 - 0.7404$$
$$\eta_{Carnot} = 0.2596 \text{ or } 25.96\%$$

Step 3: Calculate Rankine Efficiency
Since pump work is neglected, $h_2 \approx h_1$.

Process 3-4: Isentropic Expansion in Turbine

At $P_4 = 0.4$ bar:

$$s_4 = s_f + x_4 s_{fg}$$

$$6.4448 = 1.0261 + x_4(6.6448)$$
$$x_4(6.6448) = 6.4448 - 1.0261$$
$$x_4(6.6448) = 5.4187$$
$$x_4 = \frac{5.4187}{6.6448} = 0.8154$$

Now, find h_4:

$$h_4 = h_f + x_4 h_{fg} = 317.7 + 0.8154 \times 2319.2$$
$$h_4 = 317.7 + 1890.8 = 2208.5 \text{ kJ/kg}$$

Step 4: Apply Efficiency Formula

$$\eta_{Rankine} = \frac{W_{turbine}}{Q_{supplied}} = \frac{h_3 - h_4}{h_3 - h_2} = \frac{h_3 - h_4}{h_3 - h_1}$$
$$\eta_{Rankine} = \frac{2792.2 - 2208.5}{2792.2 - 317.7}$$
$$\eta_{Rankine} = \frac{583.7}{2474.5}$$
$$\eta_{Rankine} = 0.2358 \text{ or } 23.58\%$$

Final Answers:

$\eta_{Carnot} = 25.96\%$	$\eta_{Rankine} = 23.58\%$

3. From SUM 24, Q.4 (c)

- **Problem: In a Rankine cycle, steam at inlet to turbine is saturated at 35 bar and exhaust pressure is 0.2 bar. For a mass flow rate of 9.5 kg/s, determine: (i) Pump work, (ii) Turbine work, (iii) Rankine efficiency, (iv) Condenser heat flow, (v) Dryness at the end of expansion.**
- **Concept: Comprehensive analysis of the Rankine cycle.**

Answer:

Given:

- Turbine Inlet: Saturated steam at $P_3 = 35$ bar
- Condenser Pressure: $P_1 = 0.2$ bar
- Mass flow rate, $\dot{m} = 9.5$ kg/s

To Find: (i) W_p, (ii) W_t, (iii) $\eta_{Rankine}$, (iv) Q_{cond}, (v) x_4

Solution:

Step 1: Find Properties from Steam Tables

- **At 35 bar (Saturation):**
  - $h_f = h_1' = 1049.8$ kJ/kg, $h_g = h_3 = 2802.3$ kJ/kg
  - , $v_f = v_1' = 0.001235\ \text{m}^3/\text{kg}$
- **At 0.2 bar:**
  - $h_f = 251.5$ kJ/kg, $h_{fg} = 2358.4$ kJ/kg
  - ,

Step 2: Pump Work (Process 1-2)

$$W_p = v_f(P_2 - P_1) = 0.001235 \times (3500 - 200)\ \text{kJ/kg}$$
$$W_p = 0.001235 \times 3300 = 4.0755\ \text{kJ/kg}$$
$$h_2 = h_1 + W_p = 251.5 + 4.0755 = 255.575\ \text{kJ/kg}$$

Step 3: Turbine Work (Process 3-4)

At $P_4 = 0.2$ bar:

$$s_4 = s_f + x_4 s_{fg}$$
$$6.1248 = 0.8321 + x_4(7.0774)$$
$$x_4(7.0774) = 6.1248 - 0.8321 = 5.2927$$
$$x_4 = \frac{5.2927}{7.0774} = 0.7478$$

Now, find h_4:

$$h_4 = h_f + x_4 h_{fg} = 251.5 + 0.7478 \times 2358.4$$
$$h_4 = 251.5 + 1763.6 = 2015.1\ \text{kJ/kg}$$
$$W_t = h_3 - h_4 = 2802.3 - 2015.1 = 787.2\ \text{kJ/kg}$$

Step 4: Rankine Efficiency

$$Q_{in} = h_3 - h_2 = 2802.3 - 255.575 = 2546.725\ \text{kJ/kg}$$
$$W_{net} = W_t - W_p = 787.2 - 4.0755 = 783.1245\ \text{kJ/kg}$$
$$\eta_{Rankine} = \frac{W_{net}}{Q_{in}} = \frac{783.1245}{2546.725} = 0.3075 \text{ or } 30.75\%$$

Step 5: Condenser Heat Flow

$$Q_{cond} = h_4 - h_1 = 2015.1 - 251.5 = 1763.6\ \text{kJ/kg}$$

For the given mass flow rate:

$$\dot{Q}_{cond} = \dot{m} \times Q_{cond} = 9.5 \times 1763.6 = 16754.2\,\text{kW}$$

Final Answers:

$$\boxed{W_p = 4.08\,\text{kJ/kg}} \quad \boxed{W_t = 787.2\,\text{kJ/kg}} \quad \boxed{\eta_{Rankine} = 30.75\%}$$
$$\boxed{Q_{cond} = 16754.2\,\text{kW}} \quad \boxed{x_4 = 0.7478}$$

4. From WIN 23, Q.4 (c)

- **Problem: An ammonia refrigerator works between -6.7°C and 26.7°C. The vapour is dry-saturated at the end of compression. The cooling capacity is 5 tons. Calculate (i) theoretical COP and (ii) Power required to drive the compressor. (Properties of NH_3 are provided in a table).**
- **Concept: Vapour Compression Refrigeration (VCR) cycle analysis.**

Answer:

Given:

- Refrigerant: Ammonia (NH_3)
- Evaporator Temperature, $T_1 = -6.7°C$
- Condenser Temperature, $T_3 = 26.7°C$
- Vapour is dry saturated at compressor inlet.
- Cooling Capacity, $\dot{Q}_{evap} = 5\ \text{TR} = 5 \times 3.517 = 17.585\,\text{kW}$
- Properties from table:

Temp (°C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg·K)	s_g (kJ/kg·K)
-6.7	-29.26	1262.36	-0.1087	4.7401
26.7	124.56	1291.62	0.4264	4.3263

To Find: (i) Theoretical COP, (ii) Power required to drive the compressor (W_{in}).

Solution:

Step 1: Identify State Points on P-h Diagram

- **State 1:** Compressor Inlet (Dry Saturated Vapour at -6.7°C)

- $h_1 = h_g@ -6.7°C = 1262.36$ kJ/kg
-

- **State 2:** Compressor Outlet (Isentropic Compression to 26.7°C)
  -
  - At $T_2 = 26.7°C$, $s_g = 4.3263$. Since $s_2 > s_g$, the state is superheated. We need to find h_2. Since superheated tables are not provided, we assume the state is at the condenser pressure and use the given saturated vapour enthalpy at 26.7°C as an approximation for an ideal cycle, which is a common simplification when superheated data is missing.
  - $h_2 \approx h_g@26.7°C = 1291.62$ kJ/kg
- **State 3:** Condenser Outlet (Saturated Liquid at 26.7°C)
  - $h_3 = h_f@26.7°C = 124.56$ kJ/kg
- **State 4:** Evaporator Inlet (After Throttling)
  - $h_4 = h_3 = 124.56$ kJ/kg (Isenthalpic process)

Step 2: Calculate Refrigerating Effect and Mass Flow Rate
Refrigerating Effect, $q_{evap} = h_1 - h_4$

$$q_{evap} = 1262.36 - 124.56 = 1137.8 \text{ kJ/kg}$$

Mass flow rate, $\dot{m} = \frac{\dot{Q}_{evap}}{q_{evap}}$

$$\dot{m} = \frac{17.585}{1137.8} = 0.01545 \text{ kg/s}$$

Step 3: Calculate Compressor Work
Compressor Work, $w_{in} = h_2 - h_1$

$$w_{in} = 1291.62 - 1262.36 = 29.26 \text{ kJ/kg}$$

Power input, $\dot{W}_{in} = \dot{m} \times w_{in}$

$$\dot{W}_{in} = 0.01545 \times 29.26 = 0.452 \text{ kW}$$

Step 4: Calculate Coefficient of Performance (COP)

$$\text{COP} = \frac{q_{evap}}{w_{in}} = \frac{1137.8}{29.26} = 38.87$$

Alternatively, using power values:

$$\text{COP} = \frac{\dot{Q}_{evap}}{\dot{W}_{in}} = \frac{17.585}{0.452} = 38.90$$

Final Answers:

$$\boxed{\text{COP} = 38.87} \quad \boxed{\dot{W}_{in} = 0.452\ \text{kW}}$$

5. From SUM 23, Q.5 (c)

- **Problem: Steam at 20 bar, 360°C is expanded in a steam turbine to 0.08 bar. It is then condensed to saturated liquid water. The pump feeds back the water into the boiler. Assuming ideal processes, find per kg of steam the network and the cycle efficiency.**
- **Concept: Ideal Rankine cycle efficiency calculation.**

Answer:

Given:

- Boiler Pressure: $P_2 = P_3 = 20$ bar
- Boiler Temperature: $T_3 = 360°\text{C}$ (Superheated)
- Condenser Pressure: $P_1 = P_4 = 0.08$ bar
- Processes are ideal.

To Find: Per kg of steam, (i) Net Work (w_{net}), (ii) Cycle Efficiency (η)

Solution:

Step 1: Find Enthalpies from Steam Tables

- **At 20 bar, 360°C (Superheated):**
  - $h_3 = 3159.3$ kJ/kg,
- **At 0.08 bar (Saturated):**
  - $h_f = h_1 = 173.88$ kJ/kg
  - $h_{fg} = 2403.1$ kJ/kg
  -
  -
  - $v_f = 0.0010084\ \text{m}^3/\text{kg}$

Step 2: Pump Work (Process 1-2)

$$w_p = v_f(P_2 - P_1) = 0.0010084 \times (2000 - 8)\ \text{kJ/kg}$$
$$w_p = 0.0010084 \times 1992 = 2.009\ \text{kJ/kg}$$
$$h_2 = h_1 + w_p = 173.88 + 2.009 = 175.889\ \text{kJ/kg}$$

Step 3: Turbine Work (Process 3-4)

At $P_4 = 0.08$ bar:

$$s_4 = s_f + x_4 s_{fg}$$
$$6.9918 = 0.5926 + x_4(7.6361)$$
$$x_4(7.6361) = 6.9918 - 0.5926 = 6.3992$$
$$x_4 = \frac{6.3992}{7.6361} = 0.8380$$

Now, find h_4:

$$h_4 = h_f + x_4 h_{fg} = 173.88 + 0.8380 \times 2403.1$$
$$h_4 = 173.88 + 2013.0 = 2186.88\ \text{kJ/kg}$$
$$w_t = h_3 - h_4 = 3159.3 - 2186.88 = 972.42\ \text{kJ/kg}$$

Step 4: Net Work and Cycle Efficiency

$$w_{net} = w_t - w_p = 972.42 - 2.009 = 970.411\ \text{kJ/kg}$$
$$q_{in} = h_3 - h_2 = 3159.3 - 175.889 = 2983.411\ \text{kJ/kg}$$
$$\eta = \frac{w_{net}}{q_{in}} = \frac{970.411}{2983.411} = 0.3253 \text{ or } 32.53\%$$

Final Answers:

$w_{net} = 970.41$ kJ/kg	$\eta = 32.53\%$

TOPIC: GAS CYCLE NUMERICALS (5 PROBLEMS)

1. From SUM 25, Q.4 (c)

- **Problem: A diesel engine has a compression ratio of 20. Compression begins at 0.1 MPa and 35°C. The heat added is 1700 kJ/kg. Calculate the maximum pressure and temperature, work done per kg of air, and the cycle efficiency.**
- **Concept: Air-standard Diesel cycle analysis.**

Answer:

Given:

- Compression ratio, $r = 20$
- Initial state: $P_1 = 0.1$ MPa $= 100$ kPa, $T_1 = 35°\text{C} = 308$ K
- Heat added, $q_{in} = 1700$ kJ/kg
- For air: , , , $\gamma = 1.4$

To Find: (i) P_{max}, T_{max}, (ii) w_{net}, (iii) η

Solution:

Step 1: Process 1-2 (Isentropic Compression)

$$T_2 = T_1 \cdot r^{\gamma-1} = 308 \times 20^{0.4}$$
$$T_2 = 308 \times 20^{0.4} = 308 \times 3.314 = 1020.7\ \text{K}$$
$$P_2 = P_1 \cdot r^{\gamma} = 100 \times 20^{1.4} = 100 \times 66.29 = 6629\ \text{kPa} = 6.629\ \text{MPa}$$

Step 2: Process 2-3 (Constant Pressure Heat Addition)

$$q_{in} = C_p(T_3 - T_2)$$
$$1700 = 1.005(T_3 - 1020.7)$$
$$T_3 - 1020.7 = \frac{1700}{1.005} = 1691.5$$
$$T_3 = 1020.7 + 1691.5 = 2712.2\ \text{K}$$
$$P_3 = P_2 = 6.629\ \text{MPa}$$

Cut-off ratio, $\rho = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2712.2}{1020.7} = 2.657$

Step 3: Process 3-4 (Isentropic Expansion)

$$T_4 = T_3(\frac{\rho}{r})^{\gamma-1} = 2712.2 \times (\frac{2.657}{20})^{0.4}$$
$$T_4 = 2712.2 \times (0.13285)^{0.4} = 2712.2 \times 0.4085 = 1107.4\ \text{K}$$

Step 4: Process 4-1 (Constant Volume Heat Rejection)

$$q_{out} = C_v(T_4 - T_1) = 0.718(1107.4 - 308) = 0.718 \times 799.4 = 573.8 \text{ kJ/kg}$$

Step 5: Net Work and Efficiency

$$w_{net} = q_{in} - q_{out} = 1700 - 573.8 = 1126.2 \text{ kJ/kg}$$
$$\eta = \frac{w_{net}}{q_{in}} = \frac{1126.2}{1700} = 0.6625 \text{ or } 66.25\%$$

Final Answers:

$P_{max} = 6.629$ MPa	$T_{max} = 2712.2$ K	$w_{net} = 1126.2$ kJ/kg	$\eta = 66.25\%$

2. From WIN 24, Q.3 (b)

- **Problem: A rigid cylinder containing 0.004 m³ of nitrogen at 1 bar and 300 K is heated reversibly until the temperature becomes 400 K. Determine: (i) The heat supplied, (ii) The entropy change. (Assume nitrogen is a perfect gas, M=28, γ=1.4).**
- **Concept: First law and entropy change for a constant volume process in a closed system.**

Answer:

Given:

- Gas: Nitrogen (N_2), $M = 28$, $\gamma = 1.4$
- Volume, $V = 0.004$ m^3
- Initial state: $P_1 = 1$ bar $= 100$ kPa, $T_1 = 300$ K
- Final temperature, $T_2 = 400$ K
- Process: Reversible heating at constant volume

To Find: (i) Heat supplied (Q), (ii) Entropy change (ΔS)

Solution:

Step 1: Calculate Mass of Gas
Gas constant for N_2,

$$m = \frac{P_1 V}{R T_1} = \frac{100 \times 0.004}{0.297 \times 300} = \frac{0.4}{89.1} = 0.00449 \text{ kg}$$

Step 2: Calculate Heat Supplied
For constant volume process, $Q = mC_v(T_2 - T_1)$

$$Q = 0.00449 \times 0.7425 \times (400 - 300) = 0.00449 \times 0.7425 \times 100$$
$$Q = 0.333\ \text{kJ} = 333\ \text{J}$$

Step 3: Calculate Entropy Change
For constant volume process:

$$\Delta S = mC_v \ln\frac{T_2}{T_1} = 0.00449 \times 0.7425 \times \ln(\frac{400}{300})$$
$$\Delta S = 0.003333 \times \ln(1.333) = 0.003333 \times 0.2877 = 0.000958\ \text{kJ/K} = 0.958\ \text{J/K}$$

Final Answers:

$$\boxed{Q = 333\ \text{J}} \quad \boxed{\Delta S = 0.958\ \text{J/K}}$$

3. From WIN 22, Q.4 (c)

- **Problem: In an engine working on an Otto cycle, air at 1 bar and 30°C is compressed with a compression ratio of 6. Heat is added at constant volume until the temperature rises to 1500°C. Determine (i) air standard efficiency, (ii) pressure and temperature at the end of compression, (iii) heat supplied, (iv) mean effective pressure. (Given Cv and R).**
- **Concept: Air-standard Otto cycle analysis.**

Answer:

Given:

- Compression ratio, $r = 6$
- Initial state: $P_1 = 1$ bar, $T_1 = 30°\text{C} = 303$ K
- Maximum temperature, $T_3 = 1500°\text{C} = 1773$ K
- ,

To Find: (i) η, (ii) P_2, T_2, (iii) q_{in}, (iv) mep

Solution:

Step 1: Process 1-2 (Isentropic Compression)

$$T_2 = T_1 \cdot r^{\gamma-1} = 303 \times 6^{0.4} = 303 \times 2.048 = 620.5\ \text{K}$$
$$P_2 = P_1 \cdot r^{\gamma} = 1 \times 6^{1.4} = 1 \times 12.29 = 12.29\ \text{bar}$$

Step 2: Process 2-3 (Constant Volume Heat Addition)

$$q_{in} = C_v(T_3 - T_2) = 0.718(1773 - 620.5) = 0.718 \times 1152.5 = 827.5 \text{ kJ/kg}$$

Step 3: Process 3-4 (Isentropic Expansion)

$$T_4 = \frac{T_3}{r^{\gamma-1}} = \frac{1773}{6^{0.4}} = \frac{1773}{2.048} = 865.7 \text{ K}$$

Step 4: Process 4-1 (Constant Volume Heat Rejection)

$$q_{out} = C_v(T_4 - T_1) = 0.718(865.7 - 303) = 0.718 \times 562.7 = 404.0 \text{ kJ/kg}$$

Step 5: Air Standard Efficiency and Net Work

$$\eta = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{404.0}{827.5} = 1 - 0.4882 = 0.5118 \text{ or } 51.18\%$$
$$w_{net} = q_{in} - q_{out} = 827.5 - 404.0 = 423.5 \text{ kJ/kg}$$

Step 6: Mean Effective Pressure (mep)

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 303}{100} = 0.869 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{0.869}{6} = 0.1448 \text{ m}^3/\text{kg}$$
$$mep = \frac{w_{net}}{v_1 - v_2} = \frac{423.5}{0.869 - 0.1448} = \frac{423.5}{0.7242} = 584.8 \text{ kPa} = 5.848 \text{ bar}$$

Final Answers:

$$\boxed{\eta = 51.18\%} \quad \boxed{P_2 = 12.29 \text{ bar},\ T_2 = 620.5 \text{ K}}$$
$$\boxed{q_{in} = 827.5 \text{ kJ/kg}} \quad \boxed{mep = 5.848 \text{ bar}}$$

4. From WIN 23, Q.4 (c)

- **Problem: In an engine operated on an Otto cycle, the compression ratio is 6 and compression starts at 30°C and 100 kPa. The maximum temperature of the cycle is 1250°C. Determine: (i) heat supplied per kg of air, (ii) air standard efficiency, (iii) mean effective pressure. (Take Cv = 0.716 and γ = 1.4).**
- **Concept: Air-standard Otto cycle analysis.**

Answer:

Given:

- Compression ratio, $r = 6$
- Initial state: $P_1 = 100\,\text{kPa}$, $T_1 = 30°\text{C} = 303\,\text{K}$
- Maximum temperature, $T_3 = 1250°\text{C} = 1523\,\text{K}$
- , $\gamma = 1.4$

To Find: (i) q_{in}, (ii) η, (iii) mep

Solution:

Step 1: Process 1-2 (Isentropic Compression)

$$T_2 = T_1 \cdot r^{\gamma-1} = 303 \times 6^{0.4} = 303 \times 2.048 = 620.5\,\text{K}$$
$$P_2 = P_1 \cdot r^{\gamma} = 100 \times 6^{1.4} = 100 \times 12.29 = 1229\,\text{kPa}$$

Step 2: Process 2-3 (Constant Volume Heat Addition)

$$q_{in} = C_v(T_3 - T_2) = 0.716(1523 - 620.5) = 0.716 \times 902.5 = 646.2\,\text{kJ/kg}$$

Step 3: Process 3-4 (Isentropic Expansion)

$$T_4 = \frac{T_3}{r^{\gamma-1}} = \frac{1523}{6^{0.4}} = \frac{1523}{2.048} = 743.6\,\text{K}$$

Step 4: Process 4-1 (Constant Volume Heat Rejection)

$$q_{out} = C_v(T_4 - T_1) = 0.716(743.6 - 303) = 0.716 \times 440.6 = 315.5\,\text{kJ/kg}$$

Step 5: Air Standard Efficiency and Net Work

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{6^{0.4}} = 1 - \frac{1}{2.048} = 1 - 0.4883 = 0.5117 \text{ or } 51.17\%$$
$$w_{net} = q_{in} - q_{out} = 646.2 - 315.5 = 330.7\,\text{kJ/kg}$$

Step 6: Mean Effective Pressure (mep)

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 303}{100} = 0.869\,\text{m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{0.869}{6} = 0.1448\,\text{m}^3/\text{kg}$$
$$mep = \frac{w_{net}}{v_1 - v_2} = \frac{330.7}{0.869 - 0.1448} = \frac{330.7}{0.7242} = 456.5\,\text{kPa} = 4.565\,\text{bar}$$

Final Answers:

$q_{in} = 646.2$ kJ/kg	$\eta = 51.17\%$	$mep = 4.565$ bar

5. From SUM 23, Q.4 (c)

- **Problem: An engine working on the Otto cycle is supplied with air at 0.1 MPa, 35°C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure. (For air, Cp=1.005, Cv=0.718, R=0.287 kJ/kg.K).**
- **Concept: Air-standard Otto cycle analysis.**

Answer:

Given:

- Compression ratio, $r = 8$
- Initial state: $P_1 = 0.1$ MPa $= 100$ kPa, $T_1 = 35°\text{C} = 308$ K
- Heat supplied, $q_{in} = 2100$ kJ/kg
- , ,

To Find: $P_{max}, T_{max}, \eta, mep$

Solution:

Step 1: Process 1-2 (Isentropic Compression)

$$T_2 = T_1 \cdot r^{\gamma-1} = 308 \times 8^{0.4} = 308 \times 2.297 = 707.5 \text{ K}$$
$$P_2 = P_1 \cdot r^{\gamma} = 100 \times 8^{1.4} = 100 \times 18.38 = 1838 \text{ kPa}$$

Step 2: Process 2-3 (Constant Volume Heat Addition)

$$q_{in} = C_v(T_3 - T_2)$$
$$2100 = 0.718(T_3 - 707.5)$$
$$T_3 - 707.5 = \frac{2100}{0.718} = 2924.8$$
$$T_3 = 707.5 + 2924.8 = 3632.3 \text{ K}$$
$$\frac{P_3}{P_2} = \frac{T_3}{T_2} \Rightarrow P_3 = P_2 \cdot \frac{T_3}{T_2} = 1838 \times \frac{3632.3}{707.5} = 1838 \times 5.134 = 9436 \text{ kPa} = 9.436 \text{ MPa}$$

Step 3: Process 3-4 (Isentropic Expansion)

$$T_4 = \frac{T_3}{r^{\gamma-1}} = \frac{3632.3}{8^{0.4}} = \frac{3632.3}{2.297} = 1581.2 \text{ K}$$

Step 4: Air Standard Efficiency and Net Work

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{0.4}} = 1 - \frac{1}{2.297} = 1 - 0.4353 = 0.5647 \text{ or } 56.47\%$$
$$w_{net} = \eta \cdot q_{in} = 0.5647 \times 2100 = 1185.9 \text{ kJ/kg}$$

Step 5: Mean Effective Pressure (mep)

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 308}{100} = 0.884 \text{ m}^3/\text{kg}$$
$$v_2 = \frac{v_1}{r} = \frac{0.884}{8} = 0.1105 \text{ m}^3/\text{kg}$$
$$mep = \frac{w_{net}}{v_1 - v_2} = \frac{1185.9}{0.884 - 0.1105} = \frac{1185.9}{0.7735} = 1533.2 \text{ kPa} = 15.332 \text{ bar}$$

Final Answers:

$P_{max} = 9.436$ MPa	$T_{max} = 3632.3$ K	$\eta = 56.47\%$	$mep = 15.332$ bar

TOPIC: ENTROPY & ENERGY (SECOND LAW) NUMERICALS (6 PROBLEMS)

1. From SUM 25, Q.3 (c)

- **Problem: A block of 100 kg of iron at 100°C is immersed in 50 kg of water at 20°C. Determine the entropy change for the combined system of iron and water. (Take Cp_iron=0.42 kJ/kgK, Cp_water=4.18 kJ/kgK).**
- **Concept: Entropy change of systems during an irreversible heat transfer process.**

Answer:

Given:

- Iron: $m_{Fe} = 100\,\text{kg}$, $T_{Fe} = 100°\text{C} = 373\,\text{K}$,
- Water: $m_w = 50\,\text{kg}$, $T_w = 20°\text{C} = 293\,\text{K}$,

To Find: Entropy change of the combined system (ΔS_{total})

Solution:

Step 1: Find Final Equilibrium Temperature (T_f)
Energy balance: $m_{Fe}C_{p,Fe}(T_{Fe} - T_f) = m_w C_{p,w}(T_f - T_w)$

$$100 \times 0.42 \times (373 - T_f) = 50 \times 4.18 \times (T_f - 293)$$
$$42(373 - T_f) = 209(T_f - 293)$$
$$15666 - 42T_f = 209T_f - 61237$$
$$15666 + 61237 = 209T_f + 42T_f$$
$$76903 = 251T_f$$
$$T_f = \frac{76903}{251} = 306.4\,\text{K}$$

Step 2: Calculate Entropy Change of Iron

$$\Delta S_{Fe} = m_{Fe}C_{p,Fe}\ln\frac{T_f}{T_{Fe}} = 100 \times 0.42 \times \ln(\frac{306.4}{373})$$
$$\Delta S_{Fe} = 42 \times \ln(0.8214) = 42 \times (-0.1967) = -8.261\,\text{kJ/K}$$

Step 3: Calculate Entropy Change of Water

$$\Delta S_w = m_w C_{p,w}\ln\frac{T_f}{T_w} = 50 \times 4.18 \times \ln(\frac{306.4}{293})$$
$$\Delta S_w = 209 \times \ln(1.0457) = 209 \times 0.0447 = 9.342\,\text{kJ/K}$$

Step 4: Calculate Total Entropy Change

$$\Delta S_{total} = \Delta S_{Fe} + \Delta S_w = -8.261 + 9.342 = 1.081 \text{ kJ/K}$$

Final Answer:

$$\boxed{\Delta S_{total} = 1.081 \text{ kJ/K}}$$

2. From WIN 24, Q.2 (c)

- **Problem: 300 kJ/s of heat is supplied at 290°C to a heat engine. Heat rejection takes place at 8.5°C. Classify which of the following results are reversible, irreversible, or impossible: (i) 215 kJ/s rejected, (ii) 150 kJ/s rejected, (iii) 75 kJ/s rejected.**
- **Concept: Carnot principle and feasibility of heat engines.**

Answer:

Given:

- $T_H = 290°\text{C} = 563 \text{ K}$, $T_C = 8.5°\text{C} = 281.5 \text{ K}$
- $Q_H = 300 \text{ kJ/s}$
- Cases: (i) $Q_C = 215 \text{ kJ/s}$, (ii) $Q_C = 150 \text{ kJ/s}$, (iii) $Q_C = 75 \text{ kJ/s}$

To Find: Classify each case as Reversible, Irreversible, or Impossible.

Solution:

Step 1: Calculate Carnot (Reversible) Efficiency and Heat Rejection

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{281.5}{563} = 1 - 0.5 = 0.5 \text{ or } 50\%$$
$$W_{max} = \eta_{Carnot} \cdot Q_H = 0.5 \times 300 = 150 \text{ kW}$$
$$Q_{C,rev} = Q_H - W_{max} = 300 - 150 = 150 \text{ kJ/s}$$

Step 2: Analyze Each Case

- **Case (i):** $Q_C = 215 \text{ kJ/s}$
  - $W = Q_H - Q_C = 300 - 215 = 85 \text{ kW}$
  - Since $Q_C > Q_{C,rev}$ and $W < W_{max}$, this is **Irreversible**.
- **Case (ii):** $Q_C = 150 \text{ kJ/s}$
  - $W = 300 - 150 = 150 \text{ kW}$
  - Since $Q_C = Q_{C,rev}$ and $W = W_{max}$, this is **Reversible**.

- **Case (iii):** $Q_C = 75$ kJ/s
  - $W = 300 - 75 = 225$ kW
  - Since $W > W_{max}$, this violates the Second Law and is **Impossible**.

Final Answers:

(i) Irreversible	(ii) Reversible	(iii) Impossible

3. From WIN 24, Q.3 (c)

- **Problem: 5 kg of water at 0°C is exposed to a reservoir at 98°C. Calculate the change of entropy of water, the reservoir, and the universe. (Assume specific heat of water is 4.187 kJ/kg-K).**
- **Concept: Entropy generation for heat transfer through a finite temperature difference.**

Answer:

Given:

- Water: $m = 5$ kg, $T_1 = 0°\text{C} = 273$ K, $T_2 = 98°\text{C} = 371$ K
- Reservoir temperature: $T_{res} = 98°\text{C} = 371$ K
- Specific heat of water,

To Find: ΔS_{water}, $\Delta S_{reservoir}$, $\Delta S_{universe}$

Solution:

Step 1: Calculate Heat Transferred to Water

$$Q = mC_p(T_2 - T_1) = 5 \times 4.187 \times (371 - 273) = 5 \times 4.187 \times 98$$
$$Q = 2051.63 \text{ kJ}$$

Step 2: Calculate Entropy Change of Water

$$\Delta S_{water} = mC_p \ln\frac{T_2}{T_1} = 5 \times 4.187 \times \ln(\frac{371}{273})$$
$$\Delta S_{water} = 20.935 \times \ln(1.358) = 20.935 \times 0.3055 = 6.396 \text{ kJ/K}$$

Step 3: Calculate Entropy Change of Reservoir

$$\Delta S_{reservoir} = -\frac{Q}{T_{res}} = -\frac{2051.63}{371} = -5.530 \text{ kJ/K}$$

Step 4: Calculate Entropy Change of Universe

$$\Delta S_{universe} = \Delta S_{water} + \Delta S_{reservoir} = 6.396 - 5.530 = 0.866 \text{ kJ/K}$$

Final Answers:

$\Delta S_{water} = 6.396$ kJ/K	$\Delta S_{reservoir} = -5.530$ kJ/K	$\Delta S_{universe} = 0.866$ kJ/K

4. From WIN 22, Q.3 (c)

- **Problem: 5 kg of water at 30°C is mixed with 1 kg of ice at 0°C. The process is adiabatic. Calculate the final temperature of the mixture and the change of entropy for the spontaneous mixing process. (Take cp_water=4.187 kJ/kgK, Latent heat of ice=335 kJ/kg).**
- **Concept: Entropy change for an irreversible adiabatic mixing process.**

Answer:

Given:

- Water: $m_w = 5$ kg, $T_w = 30°\text{C} = 303$ K
- Ice: $m_i = 1$ kg, $T_i = 0°\text{C} = 273$ K
- , Latent heat of ice, $h_{fg} = 335$ kJ/kg

To Find: Final temperature (T_f), Total entropy change (ΔS_{total})

Solution:

Step 1: Check if Final Temperature is 0°C
Heat required to melt ice: $Q_{melt} = m_i \cdot h_{fg} = 1 \times 335 = 335$ kJ
Heat available from water cooling to 0°C: $Q_{avail} = m_w C_p (T_w - 273) =$
$5 \times 4.187 \times (303 - 273) = 5 \times 4.187 \times 30 = 628.05$ kJ
Since $Q_{avail} > Q_{melt}$, the final temperature $T_f > 0°C$.

Step 2: Energy Balance to Find T_f

$$[m_w C_p (T_w - T_f)] = [m_i h_{fg}] + [m_i C_p (T_f - T_i)]$$
$$5 \times 4.187 \times (303 - T_f) = 1 \times 335 + 1 \times 4.187 \times (T_f - 273)$$
$$20.935(303 - T_f) = 335 + 4.187(T_f - 273)$$
$$6343.305 - 20.935T_f = 335 + 4.187T_f - 1142.851$$
$$6343.305 - 20.935T_f = 4.187T_f - 807.851$$
$$6343.305 + 807.851 = 4.187T_f + 20.935T_f$$
$$7151.156 = 25.122T_f$$
$$T_f = \frac{7151.156}{25.122} = 284.7 \text{ K} = 11.7°\text{C}$$

Step 3: Entropy Change of Water (Hot Body)

$$\Delta S_w = m_w C_p \ln \frac{T_f}{T_w} = 5 \times 4.187 \times \ln(\frac{284.7}{303})$$
$$\Delta S_w = 20.935 \times \ln(0.9396) = 20.935 \times (-0.0623) = -1.304 \text{ kJ/K}$$

Step 4: Entropy Change of Ice

- **Melting at 273 K:**

$$\Delta S_{melt} = \frac{m_i h_{fg}}{273} = \frac{1 \times 335}{273} = 1.227 \text{ kJ/K}$$

- **Heating from 273 K to 284.7 K:**

$$\Delta S_{heat} = m_i C_p \ln \frac{T_f}{273} = 1 \times 4.187 \times \ln(\frac{284.7}{273})$$
$$\Delta S_{heat} = 4.187 \times \ln(1.0429) = 4.187 \times 0.0420 = 0.1758 \text{ kJ/K}$$

- **Total entropy change of ice:**

$$\Delta S_i = \Delta S_{melt} + \Delta S_{heat} = 1.227 + 0.1758 = 1.4028 \text{ kJ/K}$$

Step 5: Total Entropy Change

$$\Delta S_{total} = \Delta S_w + \Delta S_i = -1.304 + 1.4028 = 0.0988 \text{ kJ/K}$$

Final Answers:

$$\boxed{T_f = 11.7°\text{C}} \quad \boxed{\Delta S_{total} = 0.0988 \text{ kJ/K}}$$

5. From WIN 23, Q.3 (c)

- **Problem: A heat engine receives 1000 kW of heat from a source at 559 K and rejects heat to a sink at 279 K. Comment whether the cycle is possible or irreversible for the following heat rejection amounts: (i) 400 kW, (ii) 490 kW, (iii) 850 kW.**
- **Concept: Application of the Carnot efficiency to check the validity of a heat engine.**

Answer:

Given:

- $T_H = 559\,\text{K}, T_C = 279\,\text{K}$
- $Q_H = 1000\,\text{kW}$
- Cases: (i) $Q_C = 400\,\text{kW}$, (ii) $Q_C = 490\,\text{kW}$, (iii) $Q_C = 850\,\text{kW}$

To Find: Comment on each case (Possible or Impossible).

Solution:

Step 1: Calculate Carnot (Reversible) Efficiency and Heat Rejection

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{279}{559} = 1 - 0.4991 = 0.5009 \text{ or } 50.09\%$$
$$W_{max} = \eta_{Carnot} \cdot Q_H = 0.5009 \times 1000 = 500.9\,\text{kW}$$
$$Q_{C,rev} = Q_H - W_{max} = 1000 - 500.9 = 499.1\,\text{kW}$$

Step 2: Analyze Each Case

- **Case (i):** $Q_C = 400\,\text{kW}$
  - $W = Q_H - Q_C = 1000 - 400 = 600\,\text{kW}$
  - Since $W > W_{max}$, this is **Impossible**.
- **Case (ii):** $Q_C = 490\,\text{kW}$
  - $W = 1000 - 490 = 510\,\text{kW}$
  - Since $W > W_{max}$, this is **Impossible**.
- **Case (iii):** $Q_C = 850\,\text{kW}$
  - $W = 1000 - 850 = 150\,\text{kW}$
  - Since $Q_C > Q_{C,rev}$ and $W < W_{max}$, this is **Possible (Irreversible)**.

Final Answers:

(i) Impossible	(ii) Impossible	(iii) Possible

6. From WIN 22, Q.2 (c) - OR Part

- **Problem: A reversible refrigerator maintains a temperature of 0°C, rejecting heat at 27°C. The heat removal rate is 1500 kJ/min. Find the COP and work input. If a reversible heat engine (source at 400°C, sink at 27°C) runs this refrigerator, find the overall COP of the system.**
- **Concept: Combined performance of reversible heat engine and refrigerator (Carnot COP).**

Answer:

Given:

- **Refrigerator:** $T_{R,C} = 0°\text{C} = 273\ \text{K}$, $T_{R,H} = 27°\text{C} = 300\ \text{K}$, $\dot{Q}_{R,C} = 1500\ \text{kJ/min} = 25\ \text{kW}$
- **Heat Engine:** $T_{H,H} = 400°\text{C} = 673\ \text{K}$, $T_{H,C} = 27°\text{C} = 300\ \text{K}$

To Find: (i) COP of refrigerator and W_{in}, (ii) Overall COP if HE runs R.

Solution:

Part (i): Reversible Refrigerator

Step 1: COP of Reversible Refrigerator

$$\text{COP}_{R,rev} = \frac{T_{R,C}}{T_{R,H} - T_{R,C}} = \frac{273}{300 - 273} = \frac{273}{27} = 10.11$$

Step 2: Work Input to Refrigerator

$$\text{COP}_R = \frac{\dot{Q}_{R,C}}{\dot{W}_{in}} \Rightarrow \dot{W}_{in} = \frac{\dot{Q}_{R,C}}{\text{COP}_{R,rev}} = \frac{25}{10.11} = 2.473\ \text{kW}$$

Part (ii): Combined System

Step 3: Efficiency of Reversible Heat Engine

$$\eta_{HE,rev} = 1 - \frac{T_{H,C}}{T_{H,H}} = 1 - \frac{300}{673} = 1 - 0.4457 = 0.5543$$

Step 4: Heat Required by Heat Engine

$$\eta_{HE} = \frac{\dot{W}_{out}}{\dot{Q}_{H,H}} \Rightarrow \dot{Q}_{H,H} = \frac{\dot{W}_{out}}{\eta_{HE,rev}} = \frac{2.473}{0.5543} = 4.462\ \text{kW}$$

Step 5: Overall COP of Combined System

$$\text{COP}_{overall} = \frac{\dot{Q}_{R,C}}{\dot{Q}_{H,H}} = \frac{25}{4.462} = 5.602$$

Final Answers:

$$\boxed{\text{COP}_R = 10.11} \quad \boxed{\dot{W}_{in} = 2.473\ \text{kW}} \quad \boxed{\text{COP}_{overall} = 5.602}$$