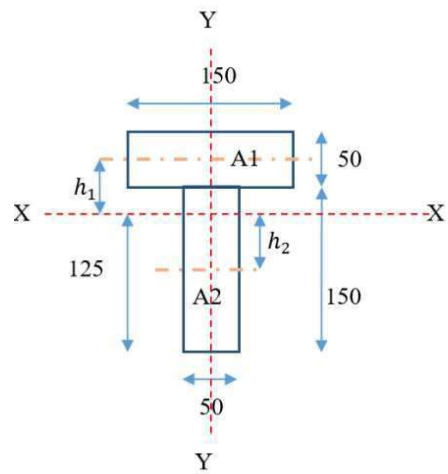
TUTORIAL NO: 02

MOMENT OF INERTIA OF PLANAR CROSS -SECTIONS

1. Find the moment of inertia of a T section shown in fig 4 about X-X and Y-Y axes through the center of gravity of the section. (Fig 1.4)



Answer:

Step 1: Dimensions from the figure

- **Top flange (A1):**

  Width $= 150\,\text{mm}$, Thickness $= 50\,\text{mm}$

- **Web (A2):**

  Width $= 50\,\text{mm}$, Depth $= 150\,\text{mm}$

Step 2: Areas

$$A_1 = 150 \times 50 = 7500\,\text{mm}^2$$

$$A_2 = 50 \times 150 = 7500\,\text{mm}^2$$

$$A_{total} = 15000\,\text{mm}^2$$

Step 3: Centroid (from top edge)

- C.G. of flange from top:

$$y_1 = \frac{50}{2} = 25\,\text{mm}$$

- C.G. of web from top:

$$y_2 = 50 + \frac{150}{2} = 125\,\text{mm}$$

- Overall centroid:

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_{total}}$$

$$\bar{y} = \frac{(7500 \times 25) + (7500 \times 125)}{15000}$$

$$\bar{y} = \frac{187500 + 937500}{15000} = \frac{1125000}{15000} = 75\,\text{mm}$$

✅ C.G. is **75 mm from top edge** (i.e. at the junction of flange and web).

Step 4: Moment of Inertia about X–X axis
Using Parallel Axis Theorem:

$$I_{xx} = \sum \left(I_{xx(CG)} + Ad^2 \right)$$

(a) Flange (A1)

$$I_{xx1(CG)} = \frac{bh^3}{12} = \frac{150 \times 50^3}{12} = 15.625 \times 10^6\,\text{mm}^4$$

Distance of its C.G. from neutral axis:

$$d_1 = |75 - 25| = 50\,\text{mm}$$

$$I_{xx1} = 15.625 \times 10^6 + (7500)(50^2)$$

$$I_{xx1} = 15.625 \times 10^6 + 18.75 \times 10^6 = 34.375 \times 10^6$$

(b) Web (A2)

$$I_{xx2(CG)} = \frac{bh^3}{12} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^6$$

Distance of its C.G. from neutral axis:

$$d_2 = |125 - 75| = 50\,\text{mm}$$

$$I_{xx2} = 14.0625 \times 10^6 + (7500)(50^2)$$

$$I_{xx2} = 14.0625 \times 10^6 + 18.75 \times 10^6 = 32.8125 \times 10^6$$

Total

$$I_{xx} = 34.375 \times 10^6 + 32.8125 \times 10^6$$

$$\boxed{I_{xx} = 67.19 \times 10^6\,\text{mm}^4}$$

Step 5: Moment of Inertia about Y–Y axis
Section is **symmetric** about Y–Y axis, so only centroidal values are required.
(a) Flange (A1)

$$I_{yy1} = \frac{hb^3}{12} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^6$$

(b) Web (A2)

$$I_{yy2} = \frac{hb^3}{12} = \frac{150 \times 50^3}{12} = 7.8125 \times 10^6$$

Total

$$I_{yy} = 14.0625 \times 10^6 + 7.8125 \times 10^6$$

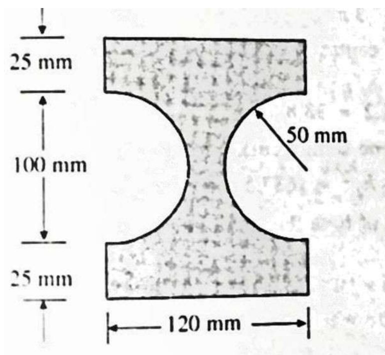
$$\boxed{I_{yy} = 21.88 \times 10^6 \, \text{mm}^4}$$

✅ **Final Answer (Q.1):**

$$I_{xx} = 67.19 \times 10^6 \, \text{mm}^4$$

$$I_{yy} = 21.88 \times 10^6 \, \text{mm}^4$$

2. Figure 1.5 shows the cross section of cast iron beam. Determine the moments of inertia of the section about horizontal and vertical axes passing through the centroid of the section.



Answer:

Step 1: Geometry Understanding

- Outer boundary is a **rectangle**:
  Width =120mm, Total depth =150mm
- Inside, two **semicircular cut-outs** are removed:
  Radius R=50 mm, placed centrally in the vertical depth of 100 mm

Thus, the cross-section = **Rectangle – 2 semicircles**.

Step 2: Areas

- Area of rectangle:

$$A_{rect} = 120 \times 150 = 18000 \, \text{mm}^2$$

- Area of one semicircle:

$$A_{semi} = \tfrac{1}{2}\pi R^2 = \tfrac{1}{2}\pi(50^2) = 3926.99 \, \text{mm}^2$$

- Total cutout (2 semicircles = 1 full circle):

$$A_{cut} = \pi R^2 = 7853.98 \, \text{mm}^2$$

- Net area:

$$A_{net} = A_{rect} - A_{cut} = 18000 - 7853.98 = 10146.02 \, \text{mm}^2$$

Step 3: Centroid Location
Section is **symmetric** both vertically and horizontally.
So centroid lies at **geometric center**:

$$\bar{x} = \frac{120}{2} = 60\,\text{mm}, \quad \bar{y} = \frac{150}{2} = 75\,\text{mm}$$

Step 4: Moment of Inertia about X–X axis (horizontal through C.G.)

$$I_{xx} = I_{xx(rect)} - I_{xx(cut)}$$

- Rectangle about its centroid:

$$I_{xx(rect)} = \frac{bh^3}{12} = \frac{120 \times 150^3}{12} = 33.75 \times 10^6\,\text{mm}^4$$

- Circle (equivalent to 2 semicircles) about its centroidal horizontal axis:

$$I_{xx(circle)} = \frac{\pi R^4}{4} = \frac{\pi(50^4)}{4} = 4.91 \times 10^6\,\text{mm}^4$$

- Net:

$$I_{xx} = 33.75 \times 10^6 - 4.91 \times 10^6$$

$$\boxed{I_{xx} = 28.84 \times 10^6\,\text{mm}^4}$$

Step 5: Moment of Inertia about Y–Y axis (vertical through C.G.)

$$I_{yy} = I_{yy(rect)} - I_{yy(cut)}$$

- Rectangle:

$$I_{yy(rect)} = \frac{hb^3}{12} = \frac{150 \times 120^3}{12} = 21.6 \times 10^6\,\text{mm}^4$$

- Circle (from 2 semicircles) about vertical axis:

$$I_{yy(circle)} = \frac{\pi R^4}{4} = 4.91 \times 10^6\,\text{mm}^4$$

- Net:

$$I_{yy} = 21.6 \times 10^6 - 4.91 \times 10^6$$

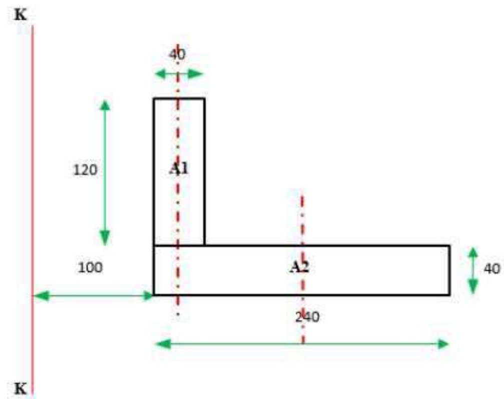
$$\boxed{I_{yy} = 16.69 \times 10^6\,\text{mm}^4}$$

✅ **Final Answer (Q.2)**

$$I_{xx} = 28.84 \times 10^6\,\text{mm}^4$$

$$I_{yy} = 16.69 \times 10^6\,\text{mm}^4$$

3. Figure below shows an area ABCDEF. Compute the moment of inertia of the above area about axis K-K.



Answer:

Step 1: Geometry Breakdown
The given L-section is divided into **two rectangles**:

- **Vertical portion (A1):**
  Width = 40 mm, Height = 120 mm
  Area:

$$A_1 = 40 \times 120 = 4800\,\text{mm}^2$$

- **Horizontal portion (A2):**
  Width = 240 mm, Height = 40 mm
  Area:

$$A_2 = 240 \times 40 = 9600\,\text{mm}^2$$

Step 2: Centroidal Distances from K–K Axis
K–K axis is **100 mm to the left** of the vertical leg (see green arrow).

- Centroid of **A1** (vertical rectangle):
  Distance from K–K:

$$x_1 = 100 + \frac{40}{2} = 120\,\text{mm}$$

- Centroid of **A2** (horizontal rectangle):
  Distance from K–K:

$$x_2 = 100 + \frac{240}{2} = 220\,\text{mm}$$

Step 3: Moments of Inertia about their own Centroidal Y–Y axes
Formula:

$$I_{yy(CG)} = \frac{hb^3}{12}$$

- For **A1**:

$$I_{yy1(CG)} = \frac{120 \times 40^3}{12} = \frac{120 \times 64000}{12} = 640000\,\text{mm}^4$$

- For **A2**:

$$I_{yy2(CG)} = \frac{40 \times 240^3}{12} = \frac{40 \times 13,824,000}{12} = 46,080,000\,\text{mm}^4$$

Step 4: Parallel Axis Theorem

$$I = I_{CG} + Ad^2$$

- For **A1**:

$$I_{yy1} = 640000 + (4800)(120^2)$$

$$I_{yy1} = 640000 + 4800 \times 14400 = 640000 + 69,120,000$$

$$I_{yy1} = 69,760,000\,\text{mm}^4$$

- For **A2**:

$$I_{yy2} = 46,080,000 + (9600)(220^2)$$

$$I_{yy2} = 46,080,000 + 9600 \times 48400$$

$$I_{yy2} = 46,080,000 + 464,640,000$$

$$I_{yy2} = 510,720,000\,\text{mm}^4$$

Step 5: Total Moment of Inertia about K–K

$$I_{yy(K-K)} = I_{yy1} + I_{yy2}$$

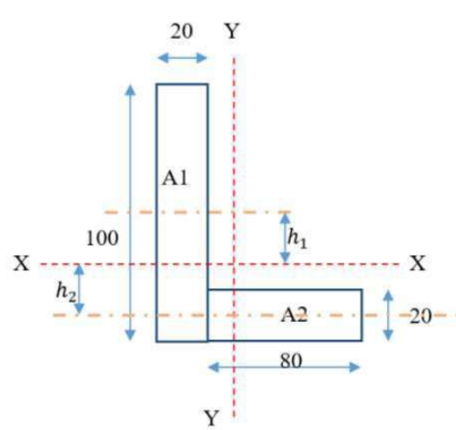
$$I_{yy(K-K)} = 69,760,000 + 510,720,000$$

$$\boxed{I_{K-K} = 580.48 \times 10^6\,\text{mm}^4}$$

✅ **Final Answer (Q.3):**

$$I_{K-K} = 580.48 \times 10^6\,\text{mm}^4$$

4. Find the moment of inertia about the centroid X-X and Y-Y axes of the angle section shown in Figure below



Answer:

Step 1: Geometry Breakdown
Angle section consists of two rectangles:

- **Vertical leg (A1):**
  Width = 20 mm, Height = 100 mm
$$A_1 = 20 \times 100 = 2000 \, \text{mm}^2$$
- **Horizontal leg (A2):**
  Width = 80 mm, Height = 20 mm
$$A_2 = 80 \times 20 = 1600 \, \text{mm}^2$$

Total Area:

$$A = A_1 + A_2 = 2000 + 1600 = 3600 \, \text{mm}^2$$

Step 2: Locate Centroid
Take reference **from outer corner (bottom-left)**.

- Centroid of A1:
  $x_1 = \frac{20}{2} = 10 \, \text{mm}, \; y_1 = \frac{100}{2} = 50 \, \text{mm}$
- Centroid of A2:
  $x_2 = \frac{80}{2} = 40 \, \text{mm}, \; y_2 = 100 + \frac{20}{2} = 110 \, \text{mm}$

Now centroid of full section:

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A} = \frac{(2000 \times 10) + (1600 \times 40)}{3600}$$

$$\bar{x} = \frac{20000 + 64000}{3600} = \frac{84000}{3600} = 23.33 \, \text{mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A} = \frac{(2000 \times 50) + (1600 \times 110)}{3600}$$

$$\bar{y} = \frac{100000 + 176000}{3600} = \frac{276000}{3600} = 76.67 \, \text{mm}$$

✅ Centroid = (23.3 mm, 76.7 mm)

Step 3: Moments of Inertia about X–X axis
Using Parallel Axis Theorem:

$$I_{xx} = \sum \left(I_{xx(CG)} + A d_y^2 \right)$$

- For A1 (20×100):

$$I_{xx1(CG)} = \frac{bh^3}{12} = \frac{20 \times 100^3}{12} = 1.67 \times 10^6$$

$$d_1 = |y_1 - \bar{y}| = |50 - 76.67| = 26.67\,\text{mm}$$

$$I_{xx1} = 1.67 \times 10^6 + 2000(26.67^2) = 1.67 \times 10^6 + 1.42 \times 10^6 = 3.09 \times 10^6$$

- For A2 (80×20):

$$I_{xx2(CG)} = \frac{bh^3}{12} = \frac{80 \times 20^3}{12} = 0.533 \times 10^6$$

$$d_2 = |110 - 76.67| = 33.33\,\text{mm}$$

$$I_{xx2} = 0.533 \times 10^6 + 1600(33.33^2) = 0.533 \times 10^6 + 1.78 \times 10^6 = 2.31 \times 10^6$$

Total:

$$I_{xx} = 3.09 \times 10^6 + 2.31 \times 10^6 = \boxed{5.40 \times 10^6\,\text{mm}^4}$$

Step 4: Moments of Inertia about Y–Y axis

$$I_{yy} = \sum \left(I_{yy(CG)} + Ad_x^2\right)$$

- For A1 (20×100):

$$I_{yy1(CG)} = \frac{hb^3}{12} = \frac{100 \times 20^3}{12} = 0.667 \times 10^6$$

$$d_1 = |10 - 23.33| = 13.33\,\text{mm}$$

$$I_{yy1} = 0.667 \times 10^6 + 2000(13.33^2) = 0.667 \times 10^6 + 0.356 \times 10^6 = 1.02 \times 10^6$$

- For A2 (80×20):

$$I_{yy2(CG)} = \frac{hb^3}{12} = \frac{20 \times 80^3}{12} = 0.853 \times 10^6$$

$$d_2 = |40 - 23.33| = 16.67\,\text{mm}$$

$$I_{yy2} = 0.853 \times 10^6 + 1600(16.67^2) = 0.853 \times 10^6 + 0.444 \times 10^6 = 1.30 \times 10^6$$

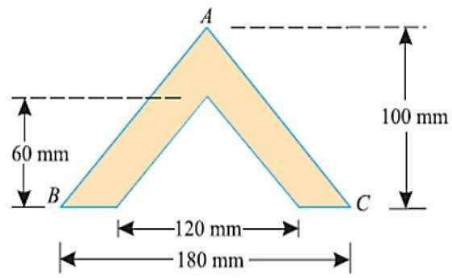
Total:

$$I_{yy} = 1.02 \times 10^6 + 1.30 \times 10^6 = \boxed{2.32 \times 10^6\,\text{mm}^4}$$

✅ **Final Answer (Q.4):**

$$I_{xx} = 5.40 \times 10^6\,\text{mm}^4, \quad I_{yy} = 2.32 \times 10^6\,\text{mm}^4$$

5. A hollow triangular section shown in Figure is symmetrical about its vertical axis. Find the moment of inertia of the section about the base BC.



Answer:

Step 1: Geometry
From the figure:

- **Outer triangle**:

  Base $= 180\,\text{mm},\ Height = 100\,\text{mm}$

- **Inner (removed) triangle**:

  Base $= 120\,\text{mm},\ Height = 60\,\text{mm}$

Since both are similar triangles (same apex A, symmetric about vertical axis), the section is a hollow triangular lamina.

Step 2: Formula
Moment of inertia of a **triangle about its base**:

$$I_{base} = \frac{bh^3}{12}$$

Step 3: Outer Triangle

$$I_{outer} = \frac{180 \times 100^3}{12} = \frac{180 \times 1,000,000}{12} = \frac{180,000,000}{12} = 15.0 \times 10^6\,\text{mm}^4$$

Step 4: Inner Triangle

$$I_{inner} = \frac{120 \times 60^3}{12} = \frac{120 \times 216,000}{12} = \frac{25,920,000}{12} = 2.16 \times 10^6\,\text{mm}^4$$

Step 5: Net Hollow Section

$$I_{BC} = I_{outer} - I_{inner}$$

$$I_{BC} = 15.0 \times 10^6 - 2.16 \times 10^6$$

$$\boxed{I_{BC} = 12.84 \times 10^6\,\text{mm}^4}$$

✅ **Final Answer (Q.5):**

$$I_{BC} = 12.84 \times 10^6 \, \text{mm}^4$$

6. Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm

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Answer:

Assumption (standard):
Depth (overall) D=100 mm, flange width =50 mm, uniform thickness t=15 mm. The section is a symmetric U-channel formed by 3 rectangles: top flange, bottom flange and web **centred** about the vertical axis. (So the channel is symmetric about the vertical mid-line.)
Represent the three rectangles (measured from the bottom outer edge upward):

- Bottom flange: $b = 50, t = 15$.
  $A_1 = 50 \times 15 = 750 \text{ mm}^2$.
  Centroid $y_1 = \dfrac{15}{2} = 7.5 \text{ mm}$.
- Web: thickness $= 15$, height between flanges = $100 - 2 \times 15 = 70$ mm.
  $A_2 = 15 \times 70 = 1050 \text{ mm}^2$.
  Centroid $y_2 = 15 + \dfrac{70}{2} = 15 + 35 = 50 \text{ mm}$.
- Top flange: same as bottom flange: $A_3 = 750 \text{ mm}^2$.
  Centroid $y_3 = 15 + 70 + 7.5 = 92.5 \text{ mm}$.

Total area:

$$A = A_1 + A_2 + A_3 = 750 + 1050 + 750 = 2550 \text{ mm}^2$$

First moment about the bottom edge:

$$\sum Ay = 750(7.5) + 1050(50) + 750(92.5)$$

Calculate:

- $750 \times 7.5 = 5625$
- $1050 \times 50 = 52500$
- $750 \times 92.5 = 750 \times (100 - 7.5) = 75000 - 5625 = 69375$

Sum = $5625 + 52500 + 69375 = 127500$.

Centroid location from bottom:

$$\bar{y} = \frac{127500}{2550} = \frac{127500 \div 50}{2550 \div 50} = \frac{2550}{51} = 50.0 \text{ mm}$$

Result (Q.6):

- Centroid lies on the vertical centreline (symmetric) and at **50.0 mm** from the bottom outer edge (i.e. mid–depth).
- If you need the x-coordinate from the left flange face: x=50/2=25.0 mm. (centreline).

7. An I-section has the following dimensions in mm units: Bottom flange = 300 × 100. Top flange = 150 × 50, Web = 300 × 50. Determine mathematically the position of center of gravity of the section.

Answer:

Given dimensions (mm):

- Bottom flange: 300×100 (width × thickness) → $b_b = 300,\ t_b = 100$
- Web: 300×50 (height × thickness) → $h_w = 300,\ t_w = 50$
- Top flange: 150×50 → $b_t = 150,\ t_t = 50$

Assumption / arrangement: bottom flange at bottom, then web of height 300 sits on top of bottom flange, then top flange sits atop the web. Measure vertical coordinate yyy from the bottom outer face of the bottom flange.

Areas:

$$A_b = 300 \times 100 = 30000\ \text{mm}^2$$

$$A_w = 50 \times 300 = 15000\ \text{mm}^2$$

$$A_t = 150 \times 50 = 7500\ \text{mm}^2$$

Total area:

$$A = 30000 + 15000 + 7500 = 52500\ \text{mm}^2$$

Centroid positions from bottom:

- Bottom flange centroid $y_b = \dfrac{100}{2} = 50\ \text{mm.}$
- Web centroid $y_w = 100 + \dfrac{300}{2} = 100 + 150 = 250\ \text{mm.}$
- Top flange centroid $y_t = 100 + 300 + \dfrac{50}{2} = 100 + 300 + 25 = 425\ \text{mm.}$

First moment:

$$\sum Ay = 30000(50) + 15000(250) + 7500(425)$$

Compute each:

- $30000 \times 50 = 1\,500\,000$
- $15000 \times 250 = 3\,750\,000$
- $7500 \times 425 = 7500 \times (400 + 25) = 3\,000\,000 + 187\,500 = 3\,187\,500$

Sum = $1\,500\,000 + 3\,750\,000 + 3\,187\,500 = 8\,437\,500.$

Centroid from bottom:

$$\bar{y} = \frac{8\,437\,500}{52\,500} = \frac{84\,375}{525} = 160.7142857\ \text{mm}$$

(You can report to 2 d.p.: **160.71 mm** from the bottom outer face.)

Horizontal location: the section is symmetric about the vertical mid-plane (web and flanges are centered), so the centroid lies on the vertical centreline. If measured from left edge of bottom flange:

$$x = \frac{300}{2} = 150 \text{ mm.}$$

Result (Q.7):

$\bar{x}$=vertical centreline (x=150 mm from left flange edge); $\bar{y}$=160.714 mm from bottom

8. Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.

Answer:

Given

- width (breadth) b=30 mm (horizontal dimension)
- depth (height) h=40 mm (vertical dimension)

  Centroidal axes: X−X is horizontal through centroid; Y−Y is vertical through centroid.

Formulas

For a rectangle about its centroidal axes:

$$I_{XX} = \frac{b\,h^3}{12}, \qquad I_{YY} = \frac{h\,b^3}{12}.$$

(These come from integrating y^2 and x^2 over the rectangular area.)

Calculations (digit-by-digit)

1. Compute h^3:

$$h^3 = 40^3 = 40 \times 40 \times 40 = 1600 \times 40 = 64\,000.$$

2. Compute $b\,h^3$:

$$b\,h^3 = 30 \times 64\,000 = 1\,920\,000.$$

3. Divide by 12 for I_{XX}:

$$I_{XX} = \frac{1\,920\,000}{12} = 160\,000 \text{ mm}^4.$$

4. Compute b^3:

$$b^3 = 30^3 = 30 \times 30 \times 30 = 900 \times 30 = 27\,000.$$

5. Compute $h\,b^3$:

$$h\,b^3 = 40 \times 27\,000 = 1\,080\,000.$$

6. Divide by 12 for I_{YY}:

$$I_{YY} = \frac{1\,080\,000}{12} = 90\,000\ \text{mm}^4.$$

Final answer (Q.8)

$$\boxed{I_{XX} = 160{,}000\ \text{mm}^4, \qquad I_{YY} = 90{,}000\ \text{mm}^4}$$

(Notes: I_{XX} about horizontal centroidal axis, I_{YY} about vertical centroidal axis.)

9. Find the moment of inertia of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.

Answer:

External dimensions: breadth Be=60 mm, depth He=80 mm
Internal (hole) dimensions: breadth Bi=30 mm, depth Hi=40 mm
Section is symmetric and concentric → centroid at geometric center. Find I_{XX} and I_{YY} about centroidal axes.

Approach & formula

Treat as: **Outer rectangle** minus **Inner rectangle** (hole). For rectangles about their centroidal axes:

$$I_{XX} = \frac{b\,h^3}{12}, \qquad I_{YY} = \frac{h\,b^3}{12}.$$

So

$$I_{XX(\text{hollow})} = I_{XX(outer)} - I_{XX(inner)} = \frac{B_e H_e^3}{12} - \frac{B_i H_i^3}{12},$$

$$I_{YY(\text{hollow})} = I_{YY(outer)} - I_{YY(inner)} = \frac{H_e B_e^3}{12} - \frac{H_i B_i^3}{12}.$$

Calculations (step-by-step)

For I_{XX}

1. Compute H_e^3:

$$H_e^3 = 80^3 = 80 \times 80 \times 80 = 6400 \times 80 = 512\,000.$$

2. $B_e H_e^3 = 60 \times 512\,000 = 30\,720\,000.$
3. $I_{XX(outer)} = 30\,720\,000 \div 12 = 2\,560\,000 \text{ mm}^4.$
4. Compute H_i^3:

$$H_i^3 = 40^3 = 64\,000.$$

5. $B_i H_i^3 = 30 \times 64\,000 = 1\,920\,000.$
6. $I_{XX(inner)} = 1\,920\,000 \div 12 = 160\,000 \text{ mm}^4.$
7. Net:

$$I_{XX} = 2\,560\,000 - 160\,000 = 2\,400\,000 \text{ mm}^4.$$

For I_{YY}

1. Compute B_e^3:

$$B_e^3 = 60^3 = 216\,000.$$

2. $H_e B_e^3 = 80 \times 216\,000 = 17\,280\,000.$
3. $I_{YY(outer)} = 17\,280\,000 \div 12 = 1\,440\,000 \text{ mm}^4.$
4. Compute B_i^3:

$$B_i^3 = 30^3 = 27\,000.$$

5. $H_i B_i^3 = 40 \times 27\,000 = 1\,080\,000.$
6. $I_{YY(inner)} = 1\,080\,000 \div 12 = 90\,000 \text{ mm}^4.$
7. Net:

$$I_{YY} = 1\,440\,000 - 90\,000 = 1\,350\,000 \text{ mm}^4.$$

Final answer (Q.9)

$$\boxed{I_{XX} = 2.40 \times 10^6 \text{ mm}^4, \qquad I_{YY} = 1.35 \times 10^6 \text{ mm}^4}$$

(Again: axes are centroidal; subtraction is valid because inner rectangle is concentric and aligned with outer.)