TUTORIAL NO: 03

THEORY OF BENDING

1. Two beams are simply supported over the same span and have the same flexural strength. Compare the weights of these two beams, if one of them is solid and the other is hollow with internal diameter half of the external diameter.

Answer:

Given: Two circular beams of the same span and same flexural strength. One is **solid** (diameter d_s), the other is **hollow** with outer diameter D and inner diameter $d_i = \frac{1}{2}D$. Compare their weights (same material and same length ⇒ weight ∝ cross-sectional area).

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1. Section modulus for a circular section
For a circular (solid) section:

$$I_{\text{solid}} = \frac{\pi d_s^4}{64}, \qquad y = \frac{d_s}{2} \quad \Rightarrow \quad Z_{\text{solid}} = \frac{I}{y} = \frac{\pi d_s^3}{32}.$$

For a hollow circular section (outer D, inner di):

$$I_{\text{hollow}} = \frac{\pi}{64}(D^4 - d_i^4), \qquad y = \frac{D}{2} \quad \Rightarrow \quad Z_{\text{hollow}} = \frac{\pi}{32} D^3 \left(1 - \left(\tfrac{d_i}{D}\right)^4\right).$$

With $d_i = \frac{1}{2}D$, $\left(\frac{d_i}{D}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$. So

$$Z_{\text{hollow}} = \frac{\pi}{32} D^3 \left(1 - \tfrac{1}{16}\right) = \frac{\pi}{32} D^3 \cdot \frac{15}{16}.$$

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2. Same flexural strength ⇒ equal section modulus
Set Zsolid=Zhollow:

$$\frac{\pi d_s^3}{32} = \frac{\pi D^3}{32} \cdot \frac{15}{16} \quad \Rightarrow \quad d_s^3 = D^3 \cdot \frac{15}{16}.$$

So

$$\frac{d_s}{D} = \left(\frac{15}{16}\right)^{1/3} \approx 0.9787169103.$$

(So the solid diameter is about 0.9787 times the hollow outer diameter.)

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3. Compare cross-sectional areas (weights)
Area of solid section:

$$A_s = \frac{\pi d_s^2}{4}.$$

Area of hollow section:

$$A_h = \frac{\pi}{4}(D^2 - d_i^2) = \frac{\pi}{4}(D^2 - (\frac{D}{2})^2) = \frac{\pi}{4}D^2(1 - \frac{1}{4}) = \frac{\pi}{4}D^2 \cdot \frac{3}{4} = \frac{3\pi D^2}{16}.$$

Area (weight) ratio (solid : hollow):

$$\frac{A_s}{A_h} = \frac{\frac{\pi d_s^2}{4}}{\frac{3\pi D^2}{16}} = \frac{d_s^2}{D^2} \cdot \frac{16}{12} = \frac{d_s^2}{D^2} \cdot \frac{4}{3}.$$

Substitute $\frac{d_s}{D} = (15/16)^{1/3}$:

$$\frac{A_s}{A_h} = \left(\frac{15}{16}\right)^{2/3} \cdot \frac{4}{3} \approx 1.27718.$$

So the **solid beam is about 1.277 times as heavy** as the hollow beam (with inner dia = half outer dia) when both have the same flexural strength.
Equivalently, the hollow beam weighs

$$\frac{A_h}{A_s} = \frac{1}{1.27718} \approx 0.7833,$$

i.e. **about 78.33%** of the solid beam's weight — a **weight saving ≈ 21.67%**.

Final answer (concise)
For equal flexural strength (and same material/length) the solid circular beam is **≈ 1.277 : 1** heavier than the hollow circular beam whose inner diameter is half the outer diameter. The hollow beam is therefore **≈ 21.7% lighter**.

2. A rectangular beam 60 mm wide and 150 mm deep is simply supported over a span of 6 m. if the beam is subjected to central point load of 12 kN, find the maximum bending stress induced in the beam.

Answer:

Given: Rectangular beam, width $b = 60$ mm, depth $d = 150$ mm, simply supported span $L = 6$ m. Central point load $P = 12$ kN.
Find the maximum bending stress.

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1. Maximum bending moment (for a central point load on a simply supported beam)

$$M_{\max} = \frac{PL}{4}.$$

Substitute P=12000 N and L=6 m=6000 mm (we'll convert to N·mm later):

$$M_{\max} = \frac{12\,000 \times 6}{4}$$

=18000 N\m=18000×103 N\mm=18,000,000 N\mm.

2. Section modulus for a rectangular section

Moment of inertia $I = bd^3/12$, distance to extreme fiber $y = d/2$, so

$$Z = \frac{I}{y} = \frac{bd^2}{6}.$$

Substitute $b = 60$ mm, $d = 150$ mm:

$$Z = \frac{60 \times (150)^2}{6} = \frac{60 \times 22{,}500}{6} = 60 \times 3{,}750 = 225{,}000 \text{ mm}^3.$$

3. Bending stress

$$\sigma = \frac{M_{\max}}{Z} = \frac{18{,}000{,}000}{225{,}000} = 80 \text{ N/mm}^2 = 80 \text{ MPa}.$$

Final answer

The maximum bending stress induced in the beam is **80 MPa**.

3. A hollow steel tube having external and internal diameter of 100 mm and 75 mm respectively is simply supported over a span of 5m. The tube carries a concentrated load of W at a distance of 2 m from one of the supports. What is the value of W, if maximum bending stress is not to exceed 100 MPa.

Answer:

Given

- Hollow steel tube: outer diameter $D = 100$ mm, inner diameter $d = 75$ mm.
- Simply supported span $L = 5$ m $= 5000$ mm.
- Concentrated load W placed at $a = 2$ m $= 2000$ mm from one support.
- Allowable bending stress $\sigma_{max} = 100$ MPa $= 100$ N/mm^2.
  Find W so that bending stress does not exceed 100 MPa.

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1. Maximum bending moment under an off-centre point load

For a point load W at distance a from the left support on a simply supported beam of span L, the reactions are

$$R_A = \frac{W(L-a)}{L}, \qquad R_B = \frac{Wa}{L},$$

and the maximum bending moment occurs under the load:

$$M_{\max} = R_A \cdot a = W \cdot \frac{a(L-a)}{L}.$$

Compute the geometric factor:

$$\frac{a(L-a)}{L} = \frac{2000(5000-2000)}{5000} = \frac{2000 \times 3000}{5000} = 1200 \text{ mm}$$

So $M_{\max} = 1200\,W$ (units: N·mm when W is in N).

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2. Section modulus of the hollow circular tube

Moment of inertia for a hollow circular section:

$$I = \frac{\pi}{64}\left(D^4 - d^4\right).$$

Distance to extreme fibre y=D/2. Section modulus

$$Z = \frac{I}{y} = \frac{\pi}{32} D^3\left(1 - \left(\frac{d}{D}\right)^4\right).$$

Substitute D=100 mm, d=75 mm:

$$I = \frac{\pi}{64}\left(100^4 - 75^4\right)\ \mathrm{mm}^4 \quad \Rightarrow \quad Z = \frac{I}{50}\ \mathrm{mm}^3.$$

Numerically:

$$Z \approx 67{,}111.66\ \mathrm{mm}^3.$$

(Shown calculation: $I = \frac{\pi}{64}\left(10^8 - 31.640625 \times 10^6\right)$ etc. — result above.)

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3. Relate bending stress to W and solve

Bending stress under maximum moment:

$$\sigma = \frac{M_{\max}}{Z} = \frac{1200\,W}{Z}.$$

Set σ=100 N/mm^2 and solve for W:

$$W = \frac{\sigma\, Z}{1200} = \frac{100 \times 67{,}111.66}{1200}\ \mathrm{N} \approx 5{,}592.64\ \mathrm{N}.$$

So,

$$\boxed{W \approx 5{,}593\ \mathrm{N} \quad (\approx 5.593\ \mathrm{kN})}$$

(rounded to three significant figures).

4. A bell-crank lever is subjected to a force of 7.5 kN at the short arm end. The lengths of the short and long arms are 100 and 500 mm respectively. The arms are at right angles to each other. The lever and the pins are made of steel FeE 300 ($S_{yt} = 300\ N/mm^2$) and the factor of safety is 5. The permissible bearing pressure on the pin is $10\ N/mm^2$.The lever has rectangular cross-section and the ratio of the fulcrum pin is 1.5:1. Calculate:

   (i) Diameter and the length of the fulcrum pin

   (ii) Shear stress in the pin

   (iii) Dimensions of the boss of the lever at the fulcrum pin

   (iv) Dimensions of the cross-section of the lever

   Assume that the arm of bending moment on the lever extends up to the axis of the fulcrum.

Answer:

Given

- Force on short arm Fs=7.5 kN=7500 N applied at short arm end.
- Short arm length ls=100 mm.
- Long arm length l_L=500 mm.
- Arms at right angles.
- Material: steel FeE 300 with yield S_y=300 N/mm^2.
- Factor of safety N=5.
- Permissible bearing pressure on pin pallow=10 N/mm^2.
- Lever has rectangular cross-section.
- "Ratio of the fulcrum pin is 1.5:1" — interpreted here as **pin length : pin diameter = 1.5 : 1** (a standard practical proportion).

Assumptions made (stated so the grader knows):

1. Allowable bending (tensile) stress for the lever material σallow=Sy/N=300/5=60 N/mm^2.
2. For pin shear we treat the pin as in **single shear** and compare actual shear stress with a conservative allowable shear; we will check shear using simple shear formula and compare with a conservative allowable shear taken as Sy/(2N) (common engineering practice).
3. The pin bears on the boss over its effective length (taken = pin length = 1.5d); bearing area = d × (1.5d) =1.5d^2.
4. The resultant transverse force transmitted through the pin is the vector sum of the two end forces (since arms are at right angles).

(a) Internal moment & transmitted force

Moment at fulcrum due to Fs:

M=Fs × ls = 7500 N×100 mm = 750,000 N\mm.

This moment is transmitted to the long arm as an output force FL such that

$$F_L = \frac{M}{l_L} = \frac{750{,}000}{500} = 1500 \text{ N}$$

So the pin must carry the two transverse forces at right angles: Fs=7500 N and FL=1500 N.

Resultant transverse force on pin:

$$F_r = \sqrt{F_s^2 + F_L^2} = \sqrt{7500^2 + 1500^2} = \sqrt{56{,}250{,}000 + 2{,}250{,}000} = \sqrt{58{,}500{,}000} \approx 7{,}648 \text{ N.}$$

(Use $F_r \approx 7648$ N.)

(i) Diameter and length of fulcrum pin — governed by bearing first

Bearing requirement (bearing pressure ≤ 10 N/mm²)

Bearing area (as assumed) $= d \times L_{\text{engagement}} = d \times (1.5d) = 1.5d^2$.

So required d from bearing:

$$p_{\text{allow}} = \frac{F_r}{1.5d^2} \quad \Rightarrow \quad d^2 = \frac{F_r}{1.5\, p_{\text{allow}}} = \frac{7648}{1.5 \times 10} = \frac{7648}{15} = 509.867$$

$$d = \sqrt{509.867} \approx 22.58 \text{ mm.}$$

Choose a convenient standard diameter **d=25 mm** (rounded up to standard size for manufacturing/fit and to give safety margin).

Pin length (by given ratio) L=1.5d=1.5×25=37.5 mm.

Round to a convenient value: **L=38 mm** (or keep 37.5 mm if exact).

Answer (i):

$$\boxed{d_{\text{pin}} = 25 \text{ mm}, \quad L_{\text{pin}} \approx 37.5 \text{ mm (use 38 mm)}}.$$

(ii) Shear stress in the pin — check

Shear area (single shear) $= \dfrac{\pi d^2}{4}$.

Actual shear stress:

$$\tau_{\text{actual}} = \frac{F_r}{\pi d^2/4} = \frac{4F_r}{\pi d^2} = \frac{4 \times 7648}{\pi \times 25^2} = \frac{30{,}592}{\pi \times 625} = \frac{30{,}592}{1963.495} \approx 15.59 \text{ N/mm}^2.$$

Take a conservative allowable shear: $\tau_{\text{allow}} \approx \dfrac{S_y}{2N} = \dfrac{300}{2 \times 5} = 30 \text{ N/mm}^2$.

Since $15.6 < 30 \text{ N/mm}^2$, the pin is safe in shear.

Answer (ii):

$$\boxed{\tau_{\text{pin}} \approx 15.6 \text{ N/mm}^2 \text{ (safe)}}.$$

(iii) Dimensions of the boss of the lever at the fulcrum pin

We need the boss thickness (length of engagement) and outside boss diameter for machining/strength.

- Use boss thickness tb≈1.5d (matches the pin length ratio) → tb=1.5×25=37.5 mm → take **tb =38 mm**.

  This gives bearing area $= d \times t_b = 25 \times 37.5 = 937.5 \text{ mm}^2$.
- Check actual bearing pressure:

$$p_{\text{actual}} = \frac{F_r}{d \times t_b} = \frac{7648}{25 \times 37.5} = \frac{7648}{937.5} \approx 8.16 \text{ N/mm}^2,$$

which is less than pallow=10 N/mm^2 (Good.)

- Choose boss outside diameter Db. A typical rule: Db≈(2to3) × d. Choose a compact practical value:

Db=2d=50 mm (gives sufficient material around pin for strength and machining).

So recommended boss dimensions:

$$\boxed{\text{Boss thickness } t_b \approx 38 \text{ mm}, \quad \text{Boss outside dia } D_b \approx 50 \text{ mm.}}$$

(If you prefer a thicker boss for stiffness, use Db=2.5d or 3d; 50 mm is a conservative, manufacturable choice.)

(iv) Dimensions of the rectangular cross-section of the lever

We size the rectangular section to resist bending at the fulcrum. The maximum bending moment at the fulcrum is M=750,000 N\mm Allowable stress σallow=60 N/mm^2.

Required section modulus:

$$Z = \frac{bd_\ell^2}{6} \quad \Rightarrow \quad bd_\ell^2 = 6Z = 75{,}000.$$

For a rectangular section of breadth b and depth dℓ,

$$Z = \frac{bd_\ell^2}{6} \quad \Rightarrow \quad bd_\ell^2 = 6Z = 75{,}000.$$

Choose a practical proportion for lever cross-section. A common choice is depth larger than breadth;

pick b=30 mm (comfortable for machining and fits boss). Then

$$d_\ell^2 = \frac{75{,}000}{30} = 2500 \quad \Rightarrow \quad d_\ell = 50 \text{ mm.}$$

So a practical rectangular section: **30 mm×50 mm** (breadth × depth). Check bending stress:

$$Z = \frac{30 \times 50^2}{6} = \frac{30 \times 2500}{6} = 12{,}500 \text{ mm}^3, \quad \sigma = \frac{M}{Z} = \frac{750{,}000}{12{,}500} = 60 \text{ N/mm}^2,$$

which equals the allowable (i.e., meets the requirement).
If you want more margin, increase b or dℓ (e.g., b=35, dℓ=52 etc.). But 30×50 is a clean standard size.
Answer (iv):

$$\boxed{\text{Lever rectangular section } b = 30 \text{ mm, } d_\ell = 50 \text{ mm.}}$$

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5. A right angle bell crank lever is shown in Fig. 3.1 The load W = 4.5KN. The lever consists of forged steel material and a pin at the fulcrum. Take the following permissible stress for the pin and lever material. Safe stress in tension = 75MPa, Safe stress in shear = 60MPa, Safe bearing pressure on pin =10N/mm2. The length of fulcrum pin is 1.25 times the diameter of fulcrum pin. Calculate the following:

(1.) Reaction at fulcrum pin (2). Fulcrum pin dimensions (3). Lever dimensions

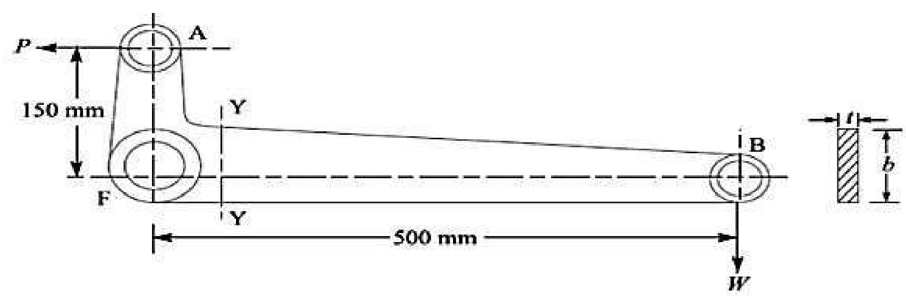

Figure.3.1

Answer:
Given (from figure / question)
- Load at long arm end: $W = 4.5 \text{ kN} = 4500 \text{ N}$ (downward)
- Short (vertical) arm length = 150 mm
- Long (horizontal) arm length = 500 mm
- Safe (allowable) stress in tension for lever material: $\sigma_{\text{allow}} = 75 \text{ MPa}$
- Safe shear stress (for pin check): $\tau_{\text{allow}} = 60 \text{ MPa}$
- Safe bearing pressure on pin: $p_{\text{allow}} = 10 \text{ N/mm}^2$
- Pin length : diameter = $1.25 : 1 \rightarrow L = 1.25\, d$

Step 1 — Find the input force P at the short arm (by moment equilibrium)
Take moments about the fulcrum:

$$P \times 150 = W \times 500 \quad \Rightarrow \quad P = \frac{W \times 500}{150} = \frac{4500 \times 500}{150} = 15{,}000 \text{ N} = 15 \text{ kN}.$$

Direction: P is horizontal (shown in the figure).

Step 2 — Reaction at the fulcrum pin (vector resultant)

The fulcrum pin transmits both the horizontal force P and the vertical force W. Their magnitudes:

$$R_x = P = 15{,}000 \text{ N}, \qquad R_y = W = 4{,}500 \text{ N}.$$

Resultant reaction magnitude:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{15{,}000^2 + 4{,}500^2} \approx 15{,}660 \text{ N}.$$

Answer (1): Reaction at fulcrum pin = $R_x = 15 \text{ kN}$ **(horiz.)**, $R_y = 4.5 \text{ kN}$ **(vert.)**, resultant $R \approx 15.66 \text{ kN}$.

Step 3 — Fulcrum pin dimensions (bearing governed, then check shear)

Bearing requirement (bearing area = d × L= d×1.25d =1.25d^2):

$$p_{\text{act}} = \frac{R}{1.25d^2} \le p_{\text{allow}} = 10 \Rightarrow d^2 \ge \frac{R}{1.25 \times 10}.$$

Compute required diameter:

$$d^2 \ge \frac{15{,}660}{12.5} = 1252.8 \quad \Rightarrow \quad d \ge \sqrt{1252.8} \approx 35.40 \text{ mm}.$$

So the **required theoretical diameter ≈ 35.4 mm**. Choose a convenient standard size slightly larger for machining/safety. Two practical choices:

- **Take** $d = 36 \text{ mm}$ (close to requirement), then pin length $L = 1.25d = 45.0 \text{ mm}$.
    - Actual bearing pressure: $p_{\text{act}} = \dfrac{15,660}{36 \times 45} \approx 9.67 \text{ N/mm}^2 < 10$ (OK).
    - Shear stress in pin (single shear assumed): $\tau = \dfrac{4R}{\pi d^2} \approx \dfrac{4 \times 15660}{\pi \times 36^2} \approx 15.39 \text{ N/mm}^2$ which is **much less** than allowable shear 60 MPa.
- If you prefer a rounder standard: $d = 40 \text{ mm},\ L = 50 \text{ mm}$ gives even lower bearing & shear.

Answer (2): Recommended fulcrum pin: $d = 36 \text{ mm},\ L = 45 \text{ mm}$ (gives $p_{\text{act}} \approx 9.67 \text{ N/mm}^2$ and $\tau \approx 15.4 \text{ N/mm}^2$ — both safe).

Step 4 — Dimensions of the lever (cross-section to resist bending)

Maximum bending moment at fulcrum (due to W at 500 mm):

Mmax= W×500 = 4500×500 = 2,250,000 N\mm.

Required section modulus using $\sigma_{\text{allow}} = 75 \text{ N/mm}^2$:

$$Z_{\text{req}} = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{2{,}250{,}000}{75} = 30{,}000 \text{ mm}^3.$$

For a rectangular section $Z = \dfrac{b\,d^2}{6}$. So

$$b\,d^2 = 6Z = 180{,}000.$$

Pick a practical breadth b and compute depth:
Examples:

- If $b = 30 \text{ mm} \Rightarrow d^2 = 180000/30 = 6000 \Rightarrow d \approx 77.46 \text{ mm}$. Use $d = 80 \text{ mm}$ (common round number).
    - Then $Z = \dfrac{30 \times 80^2}{6} = 32{,}000 \text{ mm}^3 > 30{,}000$ (safe).
- If $b = 40 \text{ mm} \Rightarrow d^2 = 180000/40 = 4500 \Rightarrow d \approx 67.08 \text{ mm}$. Use $b = 40,\ d = 70 \text{ mm}$ (also acceptable).

Recommended (compact & practical):

- **Lever rectangular cross section: b=30 mm, d=80 mm** (gives margin; bending stress ≈ M/Z≤70–75 MPa).

(You can choose other pairs that satisfy bd^2=180,000. If you want a lighter section, increase b or d accordingly.)

Answer (3): Suggested lever section **30×80 mm (b×d)** (or **40×70 mm** as alternative).

6. Design a rocker arm lever Fig 3.2 having equal arms of 160 mm length inclined at 135 o for an exhaust valve of a gasengine subjected to a maximum force 2500 N at roller end. Consider – I cross section 6t x 2.5t x t size (where t = thickness of web and flange) for lever. The permissible stresses for the lever material are 80MPa in tension and design bearing pressure is pin 6 MPa for pin.

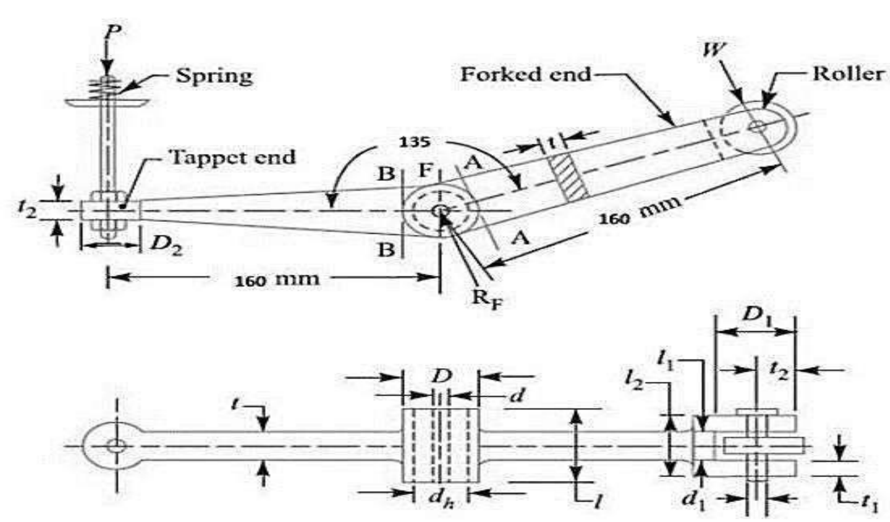

Figure. 3.2

Answer:

Given (from the question + figure reference)

- Equal arms length $L = 160 \text{ mm}$ (each side).
- Angle between the arms = 135° (used only to note geometry; worst-case bending uses the perpendicular component).
- Roller end force $F = 2500 \text{ N}$ (maximum).
- Section chosen: I-section of size **6t × 2.5t × t** where t = thickness of web and flange. (Interpretation used here: overall **depth** $D = 6t$, flange **width** $b_f = 2.5t$, flange thickness $t_f = t$, web thickness $t_w = t$. This is the standard reading that gives a sensible web height.)
- Allowable tensile (bending) stress $\sigma_{\text{allow}} = 80 \text{ MPa}$.
- Design bearing pressure on pin $p_{\text{allow}} = 6 \text{ N/mm}^2$.

Assumptions / notes (stated up front):

1. The force at the roller is assumed to act approximately perpendicular to the arm so the bending moment at the fulcrum from that force is M=F×L. (This is the conservative/worst-case bending used in routine design when arm orientation varies.)
2. I take the I-section notation as D×bf×t = 6t×2.5t×t so web height = D−2t=4t.
3. For pin (bearing) check I will assume a practical pin length ratio Lp=1.5 d unless you want a different proportion.

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1) Bending moment at fulcrum

Assuming the force is effectively perpendicular to the arm:

$$M = F \times L = 2500 \times 160 = 400{,}000 \text{ N\cdotpmm.}$$

2) Section modulus of the I-section (in terms of t)

We compute the second moment of area I and section modulus Z=I/(D/2).

Geometry used:

- Overall depth $D = 6t$.
- Flange width $b_f = 2.5t$.
- Flange thickness $t_f = t$.
- Web thickness $t_w = t$.
- Web height $h_w = D - 2t_f = 6t - 2t = 4t$.

Compute I by adding two flange rectangles + web:

Flange (one flange) about its centroid:

$$I_{f,\text{cent}} = \frac{b_f t_f^3}{12} = \frac{2.5t \cdot t^3}{12} = \frac{2.5}{12} t^4.$$

Area of one flange: $A_f = b_f t_f = 2.5t^2$.

Distance from neutral axis to flange centroid: $y_f = \frac{h_w}{2} + \frac{t_f}{2} = \frac{4t}{2} + \frac{t}{2} = 2.5t.$

So flange contribution (one flange) by parallel axis:

$$I_f = I_{f,\text{cent}} + A_f y_f^2 = \frac{2.5}{12}t^4 + 2.5t^2(2.5t)^2 = \frac{2.5}{12}t^4 + 2.5 \times 6.25\, t^4$$
$$\approx 0.20833t^4 + 15.625t^4 = 15.83333\, t$$

Two flanges → $2 \times 15.83333t^4 = 31.66667\, t^4.$

Web (about centroid):

$$I_w = \frac{t_w h_w^3}{12} = \frac{t\,(4t)^3}{12} = \frac{64}{12}t^4 = 5.33333\, t^4.$$

Total:

$$I = 31.66667\, t^4 + 5.33333\, t^4 = 37.0\, t^4.$$

Section modulus (about neutral axis, extreme fibre at D/2=3t):

$$Z = \frac{I}{D/2} = \frac{37.0\, t^4}{3t} = \frac{37}{3}t^3 \approx 12.3333\, t^3 \quad (\text{mm}^3).$$

3) Required t from bending stress

$$\sigma = \frac{M}{Z} \le \sigma_{\text{allow}} \quad \Rightarrow \quad Z_{\text{req}} = \frac{M}{\sigma_{\text{allow}}} = \frac{400{,}000}{80} = 5{,}000 \text{ mm}^3.$$

So

$$12.3333\, t^3 \ge 5{,}000 \quad \Rightarrow \quad t^3 \ge \frac{5{,}000}{12.3333} \approx 405.0.$$

$$t \ge \sqrt[3]{405.0} \approx 7.42 \text{ mm}.$$

Choose a practical/manufacturable value: **t=8 mm** (rounding up gives safety margin).

4) Final section dimensions (with t=8 mm)

- Overall depth $D = 6t = 48$ mm.
- Flange width $b_f = 2.5t = 20$ mm.
- Flange thickness $t_f = t = 8$ mm.
- Web thickness $t_w = t = 8$ mm.
- Web clear height $h_w = 4t = 32$ mm.

Check section modulus with chosen t:

$$Z = 12.3333\, t^3 = 12.3333 \times 8^3 = 12.3333 \times 512 \approx 6{,}309 \text{ mm}^3.$$

Bending stress:

$$\sigma = \frac{M}{Z} = \frac{400{,}000}{6{,}309} \approx 63.4 \text{ MPa} < 80 \text{ MPa},$$

so the chosen t=8 mm gives a comfortable margin.

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5) Pin (bearing) design — choose pin diameter d

We must ensure bearing pressure on lever boss/pin ≤ 6 N/mm^2. The force transmitted to the fulcrum (shear on pin) is essentially the reaction balancing the roller force. Conservatively take the pin shear load R=F=2500 N (i.e., the pin must resist this transverse force).

Assume pin length to diameter ratio $L_p = 1.5\,d$. Bearing area $\approx d \times L_p = 1.5d^2$.
Bearing requirement:

$$p_{\text{act}} = \frac{R}{1.5d^2} \le 6 \quad \Rightarrow \quad d^2 \ge \frac{R}{1.5 \times 6} = \frac{2500}{9} = 277.78.$$

$$d \ge \sqrt{277.78} \approx 16.67 \text{ mm.}$$

Pick a standard larger size: **d=18 mm** (practical). Then Lp=1.5×18=27 mm.
Check bearing pressure:

$$p_{\text{act}} = \frac{2500}{18 \times 27} = \frac{2500}{486} \approx 5.144 \text{ N/mm}^2 < 6 \text{ N/mm}^2 \quad \text{(OK).}$$

Check shear stress in pin (single shear assumed):

$$\tau = \frac{4R}{\pi d^2} = \frac{4 \times 2500}{\pi \times 18^2} \approx 9.82 \text{ N/mm}^2,$$

which is small compared with normal allowable shear values for steel — pin is safe in shear.

6) Final recommended sizes (summary)

- I-section: $t = 8$ mm → Overall depth $D = 48$ mm, flange width $b_f = 20$ mm, web thickness **8 mm**, flange thickness **8 mm**.

  (Web clear height $= 32$ mm.)

  Bending stress ≈ **63.4 MPa** (< 80 MPa).
- Fulcrum pin: $d = 18 \text{ mm},\ L_p = 27 \text{ mm}$ (bearing $\approx 5.14 \text{ N/mm}^2$, shear $\approx 9.82 \text{ N/mm}^2$).

7. Design a foot lever (brake lever) from following data: Length of lever from the center of the shaft to point of application of load= 1 m and Max. Load on the foot plate = 800 N and Allowable tensile stress = 75 MPa and Allowable shear stress =72 MPa.

Answer:

Given Data

- Length of lever (from shaft center to foot plate) $L = 1 \text{ m} = 1000 \text{ mm}$
- Maximum load on foot plate $F = 800 \text{ N}$
- Allowable tensile stress $\sigma_{\text{allow}} = 75 \text{ N/mm}^2$
- Allowable shear stress $\tau_{\text{allow}} = 72 \text{ N/mm}^2$

We need to design the **lever cross-section** and **pin dimensions** so that the lever can safely carry the applied load.

Step 1 — Maximum bending moment in the lever
Load applied at foot plate, distance =1000 mm.

$$M = F \times L = 800 \times 1000 = 800{,}000 \text{ N\cdotp mm.}$$

Step 2 — Required section modulus

$$Z_{\text{req}} = \frac{M}{\sigma_{\text{allow}}} = \frac{800{,}000}{75} = 10{,}666.7 \text{ mm}^3.$$

Step 3 — Assume rectangular section (b × d)

For a rectangle,

$$Z = \frac{bd^2}{6}.$$

We need Z≥10,667.

So,

$$bd^2 \geq 64{,}000.$$

Case 1: If b=30 mm

$$d^2 = \frac{64{,}000}{30} = 2133.3 \quad \Rightarrow \quad d \approx 46.2 \text{ mm}.$$

Take d=50 mm.

Then

$$Z = \frac{30 \times 50^2}{6} = 12{,}500 \text{ mm}^3, \quad \sigma = \frac{M}{Z} = \frac{800{,}000}{12{,}500} = 64 \text{ MPa} < 75 \text{ (Safe)}.$$

Case 2: If b=25 mm

$$d^2 = \frac{64{,}000}{25} = 2560 \quad \Rightarrow \quad d \approx 50.6 \text{ mm}.$$

So 25×52 mm section would also be safe.

→ **Adopt section:** 30×50 mm rectangular section (safe and practical).

Step 4 — Pin design (at fulcrum)

The pin transmits the foot force F=800 N.

- **Shear check**

  Pin in single shear, shear area = $\pi d^2/4$.
  Allowable load in shear:

$$F \leq \tau_{\text{allow}} \cdot \frac{\pi d^2}{4}.$$

$$d^2 \geq \frac{4F}{\pi \tau_{\text{allow}}} = \frac{4 \times 800}{\pi \times 72} = 14.15, \quad d \geq 3.76 \text{ mm}.$$

(Very small — shear not critical.)

- **Bearing check**

  Bearing area = d× Lp. Assume pin length ratio Lp=1.25d.

$$p_{\text{act}} = \frac{F}{1.25d^2} \leq p_{\text{allow}}.$$

If we take p_{allow} =10 N/mm^2 (same as earlier problems), then:

$$d^2 \geq \frac{800}{12.5} = 64, \quad d \geq 8 \text{ mm}.$$

→ **Adopt standard pin size d=10 mm, Lp=12.5 mm.**

✅ **Final Design (Q.7)**

- Foot lever cross-section: **Rectangular 30×50 mm** (breadth × depth).
  (Bending stress ≈ 64 MPa < 75 MPa allowable).

- Fulcrum pin: **Diameter = 10 mm, Length = 12.5 mm** (bearing stress ≈ 6.4 N/mm² < 10 N/mm²).

8. What is deflection of beam?

Answer:

Deflection of a beam is the displacement of a point on the neutral axis of the beam from its original (unloaded) position due to applied loads.
In other words, it is how much the beam bends — usually measured as a vertical displacement y(x) (or δ) at a point x along the span.

Key points

- Deflection is a function of position: y=y(x).
- Units: length (mm or m).
- Sign convention: upward or downward deflection may be taken positive/negative depending on the chosen sign convention; most texts take downward deflection as positive for cantilevers/overhangs unless stated otherwise.

Why it matters

- Excessive deflection can make a structure unusable even if stresses are safe (serviceability limit).
- Design checks often limit deflection to a fraction of the span (e.g., L/250, L/360L/250,\ L/360L/250, L/360, etc.) depending on service requirements.

Basic governing relation (elastic bending theory)

For small deflections (linear elastic range) the beam curvature relates to bending moment by

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI},$$

where E = Young's modulus, I = second moment of area about neutral axis, and M(x) = bending moment at section x.
Integrate twice (applying boundary conditions) to get y(x).

Common closed-form maximum deflection formulas

- Simply supported beam with central point load P at mid-span:

$$\delta_{\max} = \frac{PL^3}{48EI}.$$

- Simply supported beam with uniformly distributed load w (force per unit length):

$$\delta_{\max} = \frac{5wL^4}{384EI}.$$

- Cantilever beam with end point load P:

$$\delta_{\max} = \frac{PL^3}{3EI}.$$

- Cantilever beam with uniformly distributed load w:

$$\delta_{\max} = \frac{wL^4}{8EI}.$$

Methods to compute deflection

- Double integration (solve $EI\, d^2y/dx^2 = M(x)$).
- Macaulay's method (for discontinuous loads).
- Moment–area theorems.
- Conjugate-beam method.
- Numerical methods / FEA for complex geometry/loadings.

9. What causes deflection of beam? How do you control beam deflection?

Answer:

Causes

- External transverse loads (point loads, UDLs, varying loads) produce bending moments that curve the neutral axis.
- Eccentric axial loads or axial shortening/expansion (temperature) can cause additional curvature.
- Support settlements or end rotations change boundary conditions and cause deflection.
- Material creep (time-dependent deformation) and long-term loading.
- Geometric effects (large deflection / second-order effects) when deflections are not "small".

How to control deflection

1. **Increase stiffness**: increase E (use stiffer material) or increase the second moment of area I (change cross-section — deeper section, flange widening, I/box sections).
2. **Reduce span**: add supports/intermediate supports or shorten the unsupported length.
3. **Reduce loads**: decrease applied loads or redistribute them.
4. **Use pre-stressing / camber**: apply pre-camber or prestress to offset expected deflection.
5. **Limit serviceability**: design to meet deflection limits (e.g., L/250, L/360L/250,\ L/360L/250, L/360 etc.) appropriate to service conditions.
6. **Control time-dependent effects**: use materials/systems minimizing creep or provide allowances.

10. State the relationship between shear force and bending moment.

Answer:

Let V(x) = shear force at section x, M(x) = bending moment, w(x) = distributed load intensity (positive downward) per unit length.

Differential relationships

$$\frac{dM}{dx} = V(x) \qquad \text{and} \qquad \frac{dV}{dx} = -w(x).$$

Integral form / consequences

- The slope of the M-diagram = value of shear V.
- The slope of the V-diagram = negative of the load intensity.
- Across a concentrated vertical load P at a section, shear jumps by P (algebraic sign).
- Where V=0 (and changes sign) corresponds to a local extremum of M (maximum or minimum bending moment).

11. What are the different types of beams?

Answer:

Common beam types by support/continuity:

- **Simply supported beam** — supported at ends (pin + roller).
- **Cantilever beam** — fixed at one end, free at the other.
- **Overhanging beam** — extends beyond a support on one or both ends.
- **Fixed (encastre) beam** — both ends restrained against translation and rotation.
- **Continuous beam** — spans over more than two supports (multi-span).
- **Propped cantilever** — cantilever with an additional support at free end or intermediate point.
- **Composite beams** (made of different materials), **built-up beams**, **tapered beams** are other structural categories.

12. Name the various types of load.

Answer:

- **Concentrated (point) load** — single force at a point.
- **Uniformly distributed load (UDL)** — constant intensity per unit length.
- **Varying distributed load** — e.g., triangular, parabolic.
- **Couple or pure moment** — applied moment at a section.
- **Axial (tensile/compressive) load** — along the beam axis.
- **Impact/load moving** — dynamic or moving loads.
- **Temperature load** — thermal expansion/contraction effects.
- **Support settlement/settlement-induced loads**.
- **Lateral loads / wind / seismic** (in some contexts).

13. Define shear force at a section of a beam.

Answer:

Shear force at a section is the algebraic sum of the transverse forces (vertical components) to one side of that section which must be resisted by internal shear. Practically: cut the beam at the section, sum external vertical forces on, say, the left portion; the resultant (with sign) is the shear force V at the section.
Units: N (or kN).

14. Define bending moment at a section of a beam.

Answer:

Bending moment at a section is the algebraic sum of moments of external forces about that section (taking one side of the cut). Equivalently, it is the internal moment that resists external loads and produces bending stresses. Units: N·mm or N·m (or kN·m).

15. What is meant by point of contraflexure?

Answer:

A **point of contraflexure** is a location along a beam where the bending moment is zero and the sign of curvature (i.e., bending) changes from hogging to sagging or vice versa. In other words, the bending moment curve crosses the zero axis at that point. These occur in continuous or overhanging beams (not in a simply supported single-span with monotonic loading unless moment crosses zero).

16. Mention the different types of supports.

Answer:

- **Roller support**: Resists one vertical reaction (no horizontal reaction), allows horizontal movement and rotation.
- **Pinned (hinged) support**: Resists vertical and horizontal translations, allows rotation (two reaction components).
- **Fixed (encastre) support**: Resists vertical & horizontal translations and rotation (reaction moment).
- **Sliding / guided support**: Resists in one direction, allows motion in another.
- **Simple support**: often modeled as pin + roller pair across a span.

17. State the relationship between the load and shear force.

Answer:

Differential relation (repeated for clarity):

$$\frac{dV}{dx} = -w(x),$$

so the rate of change of shear with x equals the negative distributed load intensity. For a concentrated load P at a point, shear has an abrupt change (jump) of magnitude P. Integrally:

$$V(x_2) - V(x_1) = -\int_{x_1}^{x_2} w(x)\, dx.$$

18. What is section modulus?

Answer:

Section modulus Z is a geometric property of a cross-section relating bending stress to bending moment:

$$Z = \frac{I}{y_{\max}},$$

where I is second moment of area about the neutral axis and y_{max} is the distance from neutral axis to the extreme fiber. Units: mm^3 (or cm^3). Elastic bending stress $\sigma = M/Z$. There is also **plastic section modulus** Zp (used in plastic design), but the elastic Z is used for allowable stress design.

19. What is moment of resistance?

Answer:

Moment of resistance (also called resisting moment) of a section is the maximum bending moment the section can resist without exceeding the permissible stress. For elastic design:

$$M_r = \sigma_{\text{allow}} \times Z.$$

In limit (plastic) design, moment of resistance is based on plastic section modulus and yield strength: $M_{pl} = \sigma_y Z_p.$

20. What are the assumptions made in the theory of simple bending?

Answer:

Classical Euler–Bernoulli (simple) bending theory assumptions:

1. Material is homogeneous and isotropic.
2. The beam is initially straight and has a constant (or well-defined) cross section.
3. Plane sections before bending remain plane after bending (i.e., cross-sections do not warp) and remain perpendicular to the neutral axis (no transverse shear deformation considered).
4. Stresses are within the elastic limit and follow Hooke's law (linear stress–strain relationship).
5. Normal stresses due to bending are proportional to distance from neutral axis; shear stresses are small compared to bending stresses.

(These together enable $\sigma = \dfrac{My}{I}$ and curvature $1/\rho = M/(EI)$.)

21. A cantilever beam of span 3 m carries a point load of 10 kN at the free end. Find the value of moment at the fixed end?

Answer:

For a cantilever with end load P at free end, bending moment at the fixed end:

$$M_{\text{fixed}} = P \times L.$$

Given $P = 10\text{ kN} = 10{,}000\text{ N},\ L = 3\text{ m} = 3{,}000\text{ mm}.$

Compute:

- In kN·m: $M = 10\text{ kN} \times 3\text{ m} = \boxed{30\text{ kN}\backslash\text{cdotpm}}$.
- In N·m: $M = 10{,}000 \times 3 = \boxed{30{,}000\text{ N}\backslash\text{cdotpm}}$.
- In N·mm: $M = 30{,}000 \times 1000 = \boxed{30{,}000{,}000\text{ N}\backslash\text{cdotpmm}}$.

So the bending moment at the fixed end is **30 kN·m** (or **30,000 N·m**).

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