TUTORIAL NO: 04

THEORY OF TORSION

1. Define torsion.

Answer:

Torsion is the **twisting of a structural member** about its longitudinal axis when subjected to a **torque or twisting moment**.

- This causes shear stresses to develop over the cross-section of the member, which vary **linearly with the distance from the center** (zero at the center, maximum at the outer surface).
- Torsion also produces an **angular displacement** (angle of twist) along the member's length.

Example: A rotating shaft transmitting power from an engine to a machine experiences torsion.

2. What are the assumptions made in the theory of torsion?

Answer:

The following assumptions are made while deriving the torsion equation

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

1. **Material is homogeneous and isotropic**
   - The shaft material has the same properties in all directions and is uniform throughout.
2. **Elastic behavior (Hooke's law is obeyed)**
   - Stresses are within the elastic limit; shear stress is directly proportional to shear strain.
3. **Uniform twist along the shaft**
   - The shaft twists uniformly; every cross-section rotates through the same angle per unit length.
4. **Plane cross-sections remain plane**
   - Any cross-section that is plane before loading remains plane after twisting (no warping).
5. **Cross-sections remain circular**
   - The circular section does not change its shape (remains truly circular, not oval).

6. **Radial lines remain straight**
   - A radial line on the cross-section before twisting remains straight after twisting (only rotates).
7. **Shear stress acts tangentially**
   - Shear stress on a cross-section is purely tangential and uniformly distributed along the circumference at a given radius.
8. **No effect of bending or direct shear**
   - The shaft is subjected only to torque; bending moments or axial forces are absent.

3. Define polar modulus.

Answer:

The **polar section modulus** is a measure of the **strength of a shaft in torsion**.
It is defined as:

$$Z_p = \frac{J}{r}$$

Where:

- J = Polar moment of inertia of the section
- r = Outer radius of the shaft

For a solid shaft:

$$J = \frac{\pi D^4}{32}, \quad r = \frac{D}{2} \Rightarrow Z_p = \frac{\pi D^3}{16}$$

For a hollow shaft:

$$J = \frac{\pi (D^4 - d^4)}{32}, \quad r = \frac{D}{2} \Rightarrow Z_p = \frac{\pi (D^4 - d^4)}{16D}$$

Thus, **polar modulus represents the torque-carrying capacity per unit shear stress**.

4. Why hollow circular shafts are preferred when compared to solid circular shafts?

Answer:

Hollow shafts are **preferred** because:

1. **Material Saving** – For the same torque transmission capacity, a hollow shaft uses **less material** than a solid shaft → reduced weight and cost.
2. **Better Strength-to-Weight Ratio** – Since most shear stress is near the outer surface, removing material from the core does not reduce strength much.
3. **Higher Torsional Rigidity per Weight** – Hollow shafts give comparable stiffness with less weight.
4. **Design Flexibility** – Hollow shafts can be used to pass lubricants, electrical wires, or control rods through the center.
5. **Better Dynamic Performance** – Lower weight reduces inertia and improves balancing in rotating machinery.

Hence, hollow shafts are widely used in automotive drive shafts, propeller shafts, etc.

5. A hollow shaft with diameter ratio 3/5 is required to transmit 450 kW at 120 rpm. The shearing stress in the shaft must not exceed 60 N/mm and the twist in a length of 2.5 m is not to exceed 1°. Calculate the minimum external diameter of the shaft. C = 80 kN/mm2

Answer:

Given: Hollow shaft, diameter ratio $d/D = 3/5 \Rightarrow k = 0.6$; $P = 450\ \mathrm{kW}$, $n = 120\ \mathrm{rpm}$; $\tau_{\max} \leq 60\ \mathrm{MPa}$; twist $\leq 1^\circ$ over $L = 2.5\ \mathrm{m} = 2500\ \mathrm{mm}$; $G = 80\ \mathrm{kN/mm^2} = 80{,}000\ \mathrm{MPa}$.

Find: Minimum **outer** diameter DDD satisfying **both** stress and twist limits.

Torque from power

$$T = \frac{60P}{2\pi n} \times 10^3 = \frac{60 \times 450 \times 10^3}{2\pi \times 120} \times 10^3$$

$= 3.581 \times 10^7$ N\mm.

(a) Stress condition

$$\tau_{\max} = \frac{16T}{\pi D^3(1-k^4)} \le 60 \Rightarrow D = \left[\frac{16T}{\pi \tau_{\backslash\text{allow}}(1-k^4)}\right]^{1/3}.$$

With $k = 0.6 \Rightarrow (1-k^4) = 0.8704$,

$$D_{\text{str}} = \left[\frac{16(3.581\times10^7)}{\pi(60)(0.8704)}\right]^{1/3} = 151.7 \text{ mm}.$$

(b) Twist condition:

$$(\theta = \tfrac{32TL}{\pi G D^4(1-k^4)} \le 1^\circ = \pi/180)$$

$$D_{\text{stiff}} = \left[\frac{32TL}{\pi G\theta(1-k^4)}\right]^{1/4} = \left[\frac{32(3.581\times10^7)(2500)}{\pi(80{,}000)(\pi/180)(0.8704)}\right]^{1/4} = 165.5 \text{ mm}.$$

Controls: $D \ge \max(151.7, 165.5) = 165.5 \text{ mm}$ ⇒ **Adopt** $D = 170 \text{ mm}$ (standard).
Inner diameter: $d = kD = 0.6 \times 170 = 102 \text{ mm}$.
(At $D = 170$ mm: $\tau \approx 42.6 \text{ MPa}$, $\theta \approx 0.90^\circ$ < limits.)

Answer (Q5):

$$\boxed{D \approx 170 \text{ mm},\ d \approx 102 \text{ mm}}.$$

6. A steel shaft is required to transmit 75 kW power at 100 rpm. and the maximum twisting moment is 30% greater than the mean. Find the diameter of the steel shaft if the maximum stress is 70 N/mm .Also determine the angle of twist in a length of 3m of the shaft. Assume the modulus of rigidity for steel as 90 kN/ mm2

Answer:

Given: Solid steel shaft; $P = 75 \text{ kW}$, $n = 100 \text{ rpm}$; $T_{\max} = 1.3\,T_{\text{mean}}$; $\tau_{\max} \le 70 \text{ MPa}$; $L = 3 \text{ m} = 3000 \text{ mm}$; $G = 90{,}000 \text{ MPa}$.

Find: Diameter D; angle of twist θ over 3 m.

Torque:

$$T_{\text{mean}} = \frac{60P}{2\pi n} \times 10^3 = \frac{60 \times 75 \times 10^3}{2\pi \times 100} \times 10^3$$

= 7.162×106 N\mm,

Tmax = 1.3 Tmean = 9.311×10^6 N\mm.

Diameter (stress at Tmax):

$$\tau_{\max} = \frac{16T_{\max}}{\pi D^3} \le 70 \Rightarrow D = \left[\frac{16T_{\max}}{\pi\tau_{\backslash\text{allow}}}\right]^{1/3} = \left[\frac{16(9.311 \times 10^6)}{\pi(70)}\right]^{1/3} = 87.8 \text{ mm.}$$

Adopt D=90 mm.

Angle of twist (use **mean** torque for service twist):

$$\theta = \frac{T_{\text{mean}}L}{JG} = \frac{32T_{\text{mean}}L}{\pi G D^4} = \frac{32(7.162 \times 10^6)(3000)}{\pi(90{,}000)(90^4)} = 0.0371 \text{ rad} = 2.12^\circ.$$

Answer (Q6):

$$\boxed{D \approx 90 \text{ mm}}, \quad \boxed{\theta \approx 2.1^\circ \text{ (in 3 m)}}.$$

7. Calculate the maximum intensity of shear stress induced and the angle of twist produced in degrees in solid shaft of 100 mm diameter, 12 m long, transmitting 150 kW at 200 rpm. Take G=82kN/mm2

Answer:

Given: Solid shaft $D = 100$ mm, $L = 12$ m $= 12{,}000$ mm; $P = 150$ kW, $n = 200$ rpm; $G = 82{,}000$ MPa.

Find: (i) maximum shear stress, (ii) angle of twist (degrees).

Torque:

$$T = \frac{60 \times 150 \times 10^3}{2\pi \times 200} \times 10^3$$

= 7.162×106 N\mm.

(i) Shear stress (solid):

$$\tau_{\max} = \frac{16T}{\pi D^3} = \frac{16(7.162 \times 10^6)}{\pi(100^3)} = 36.48 \text{ MPa.}$$

(ii) Angle of twist:

$$\theta = \frac{32TL}{\pi G D^4} = \frac{32(7.162 \times 10^6)(12{,}000)}{\pi(82{,}000)(100^4)} = 0.1068 \text{ rad} = 6.12^\circ.$$

Answer (Q7):

$\boxed{\tau_{\max} \approx 36.5 \text{ MPa}}$, $\boxed{\theta \approx 6.12^\circ}$.

8. The internal and external diameter of a hollow shaft is in the ratio of 2:3. The hollow shaft is to transmit a 400 kW power at 120 rpm. The maximum expected torque is 15% greater than the mean valve. If the shear stress is not to exceed 50 Mpa, find section of the shaft which would satisfy the shear stress and twist conditions. Take G=0.85×105 MPa

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Answer:

Given: Hollow shaft with $d : D = 2 : 3 \Rightarrow k = \frac{2}{3} = 0.6667$; $P = 400$ kW, $n = 120$ rpm; $T_{\max} = 1.15\,T_{\text{mean}}$; $\tau_{\text{\allow}} = 50$ MPa; $G = 0.85 \times 10^5 \text{ MPa} = 85{,}000$ MPa.

Asked: "Find the **section** which satisfies both **shear stress** and **twist** conditions."

Torque:

$$T_{\text{mean}} = \frac{60 \times 400 \times 10^3}{2\pi \times 120} \times 10^3$$

=3.183×107 N\mm,

T_{max} = 1.15 T_{mean} = 3.661×10^7 N\mm.

(a) By shear stress at T_{max}:

$$D_{\text{str}} = \left[\frac{16T_{\text{max}}}{\pi\,\tau_{\text{\allow}}(1-k^4)}\right]^{1/3} = \left[\frac{16(3.661 \times 10^7)}{\pi(50)(1-(2/3)^4)}\right]^{1/3} = 166.9 \text{ mm.}$$

Hence **D≈167 mm, d=kD≈111 mm**.

(b) Twist condition needs an allowable twist over a given length LLL (not stated in the snap). Nevertheless, for the above section the **twist per metre at T_{max}** is:

$$\theta_{\text{per m}} = \frac{T_{\text{max}}}{JG} \times 1000 = \frac{(3.661 \times 10^7)\,1000}{\left[\frac{\pi(D^4-d^4)}{32}\right](85{,}000)} \approx 0.404^\circ/\text{m.}$$

So over length L metres, $\theta \approx 0.404\,L^\circ$.

- If the design limit were, say, 1° in L m, this section meets it up to $L \approx 2.47$ m.
- For a different twist limit, use $D_{\text{stiff}} = \left[\frac{32T\,L}{\pi G\,\theta_{\text{\allow}}(1-k^4)}\right]^{1/4}$ and pick $D = \max(D_{\text{str}}, D_{\text{stiff}})$.

Answer (Q8): A section **D≈167 mm, d≈111 mm** satisfies the **stress** limit; stiffness is satisfactory provided the required twist limit/length falls within the above check (or compute D_{stiff} with your specific L, θ$_{\backslash allow}$).

9. A hollow shaft, having an inside diameter is 60% of it is outer diameter is to replace the solid shaft is transmitting in the same power and same speed. Calculate percentage in saving materials, if the materials are to be also the same.

Answer:

Given: Replace a solid shaft by a hollow one with **inside diameter =0.6 D** (same material, same torque & speed).
Asked: Percentage **material saving**.
Let the **hollow** shaft have outer D, inner d=kD with k=0.6. Equating torque capacities at the **same** $\tau\backslash_{\text{allow}}$:

$$D_s^3 = D^3(1 - k^4) \;\Rightarrow\; D_s = D(1 - k^4)^{1/3}.$$

Areas: $A_{\text{solid}} = \frac{\pi D_s^2}{4}, \quad A_{\text{hollow}} = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi D^2}{4}(1 - k^2).$

Material ratio:

$$\frac{A_{\text{hollow}}}{A_{\text{solid}}} = \frac{(1 - k^2)}{(1 - k^4)^{2/3}} = \frac{(1 - 0.36)}{(1 - 0.1296)^{2/3}} = 0.702.$$

Saving $= 1 - 0.702 = \boxed{29.8\%}$.

10. A solid shaft has to transmit the power 105 kW at 2000 rpm. The maximum torque transmitted in each revaluation exceeds the mean by 36%. Find the suitable diameter, if the shear stress is not to exceed 75 N/mm2 and maximum angle of twist is 1.5∘ in a length of 3.30m and G=0.80×10

Answer:

Given: Solid shaft; $P = 105$ kW, $n = 2000$ rpm; $T_{\text{max}} = 1.36\, T_{\text{mean}}$; $\tau\backslash_{\text{allow}} = 75$ MPa; $\theta\backslash_{\text{allow}} = 1.5^\circ$ over $L = 3.30$ m $= 3300$ mm; $G = 0.80 \times 10^5$ MPa $= 80{,}000$ MPa.

Find: Suitable diameter D meeting **both** stress and twist limits.

Torque:

$$T_{\text{mean}} = \frac{60 \times 105 \times 10^3}{2\pi \times 2000} \times 10^3$$

$=5.013\times10^5$ N\mm

Tmax = 1.36Tmean = 6.818×10^5 N\mm.

(a) Stress (at T_{max}):

$$D_{\text{str}} = \left[\frac{16T_{\text{max}}}{\pi\tau_{\text{\allow}}}\right]^{1/3} = \left[\frac{16(6.818 \times 10^5)}{\pi(75)}\right]^{1/3} = 35.9 \text{ mm.}$$

(b) Twist (use mean torque):

$$\theta = \frac{32T_{\text{mean}}L}{\pi G D^4} \le 1.5^\circ = \pi/120$$

$$D_{\text{stiff}} = \left[\frac{32T_{\text{mean}}L}{\pi G\theta_{\text{\allow}}}\right]^{1/4} = \left[\frac{32(5.013 \times 10^5)(3300)}{\pi(80{,}000)(\pi/120)}\right]^{1/4} = 53.26 \text{ mm.}$$

Controls: $D \ge \max(35.9, 53.26) = 53.26 \text{ mm}.$

Adopt D=55 mm (standard). Check:

$$\tau_{\max} = \frac{16T_{\max}}{\pi D^3} = \frac{16(6.818 \times 10^5)}{\pi(55)^3} = 20.9 \text{ MPa} < 75, \quad \theta = \frac{32T_{\text{mean}}L}{\pi G D^4} = 1.32^\circ < 1.5^\circ$$

Answer (Q10):

$$\boxed{D \approx 55 \text{ mm}}.$$

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Short formula sheet you can copy (for quick checks):

- Power ↔ torque: $T = \frac{60P}{2\pi n}$ (in N·m). For N·mm, multiply by 10^3.
- Shear stress: $\tau_{\max} = \frac{16T}{\pi D^3}$ (solid), $\tau_{\max} = \frac{16T}{\pi D^3(1-k^4)}$ (hollow), $k = d/D$.
- Angle of twist: $\theta = \frac{TL}{JG}$ where $J = \frac{\pi D^4}{32}$ (solid) and $J = \frac{\pi(D^4-d^4)}{32} = \frac{\pi D^4(1-k^4)}{32}$ (hollow).
- Work in **N, mm, MPa (= N/mm²)** throughout.