TUTORIAL NO: 06

Design of Machine Components Under Fluctuating Loading Condition

1. A rectangular plate, 15 mm thick, made of a brittle material is shown in Fig 6.1. Calculate the stresses at each of three holes of 3, 5 and 10 mm diameter.

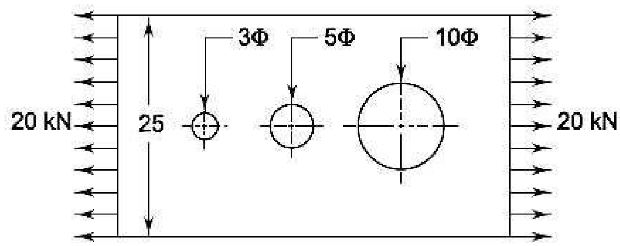

Figure 6.1

Answer:

Given:

- Plate Thickness, $t = 15$ mm
- Tensile Load, $P = 20$ kN $= 20{,}000$ N
- Width of the plate, $w = 25$ mm
- Hole diameters: $d_1 = 3$ mm, $d_2 = 5$ mm, $d_3 = 10$ mm

The formula for the nominal stress remains:

$$\sigma_{nom} = \frac{P}{(w-d)t}$$

The formula for the theoretical stress concentration factor K_t for a finite plate in tension with a transverse hole is:

$$K_t = C_1 + C_2\left(\frac{2d}{w}\right) + C_3\left(\frac{2d}{w}\right)^2 + C_4\left(\frac{2d}{w}\right)^3$$

For a ratio of $d/w \leq 0.7$, common coefficients are $C_1 = 3.00, C_2 = -3.39, C_3 = 7.12, C_4 = -4.95$. However, a simpler and widely used approximation is:

$$K_t \approx 3.00 - 3.14\left(\frac{d}{w}\right) + 3.67\left(\frac{d}{w}\right)^2 - 1.53\left(\frac{d}{w}\right)^3$$

We will use this formula.

The maximum stress at the hole is:

$$\sigma_{max} = K_t \cdot \sigma_{nom}$$

Let's calculate for each hole.

Step 1: Stress at 3 mm diameter hole

- Hole diameter, $d = 3$ mm
- Ratio, $d/w = 3/25 = 0.12$

- Net width, $w_{net} = w - d = 25 - 3 = 22$ mm

1.1. Calculate Nominal Stress:

$$\sigma_{nom,3} = \frac{20000 \text{ N}}{(22 \text{ mm})(15 \text{ mm})} = \frac{20000}{330} \approx 60.61 \text{ N/mm}^2 \text{ (MPa)}$$

1.2. Calculate Stress Concentration Factor K_t:

Using the formula: $K_t \approx 3.00 - 3.14(0.12) + 3.67(0.12)^2 - 1.53(0.12)^3$

$$K_t \approx 3.00 - 0.3768 + 3.67(0.0144) - 1.53(0.001728)$$
$$K_t \approx 3.00 - 0.3768 + 0.05285 - 0.00264$$
$$K_t \approx 2.673$$

1.3. Calculate Maximum Stress:

$$\sigma_{max,3} = K_t \cdot \sigma_{nom,3} = 2.673 \times 60.61 \approx 162.0 \text{ MPa}$$

Step 2: Stress at 5 mm diameter hole

- Hole diameter, $d = 5$ mm
- Ratio, $d/w = 5/25 = 0.2$
- Net width, $w_{net} = 25 - 5 = 20$ mm

2.1. Calculate Nominal Stress:

$$\sigma_{nom,5} = \frac{20000}{(20)(15)} = \frac{20000}{300} \approx 66.67 \text{ MPa}$$

2.2. Calculate Stress Concentration Factor K_t:

$$K_t \approx 3.00 - 3.14(0.2) + 3.67(0.2)^2 - 1.53(0.2)^3$$
$$K_t \approx 3.00 - 0.628 + 3.67(0.04) - 1.53(0.008)$$
$$K_t \approx 3.00 - 0.628 + 0.1468 - 0.01224$$
$$K_t \approx 2.506$$

2.3. Calculate Maximum Stress:

$$\sigma_{max,5} = 2.506 \times 66.67 \approx 167.1 \text{ MPa}$$

Step 3: Stress at 10 mm diameter hole

- Hole diameter, $d = 10$ mm
- Ratio, $d/w = 10/25 = 0.4$
- Net width, $w_{net} = 25 - 10 = 15$ mm

3.1. Calculate Nominal Stress:

$$\sigma_{nom,10} = \frac{20000}{(15)(15)} = \frac{20000}{225} \approx 88.89 \text{ MPa}$$

3.2. Calculate Stress Concentration Factor K_t:

$$K_t \approx 3.00 - 3.14(0.4) + 3.67(0.4)^2 - 1.53(0.4)^3$$
$$K_t \approx 3.00 - 1.256 + 3.67(0.16) - 1.53(0.064)$$
$$K_t \approx 3.00 - 1.256 + 0.5872 - 0.09792$$
$$K_t \approx 2.233$$

3.3. Calculate Maximum Stress:

$$\sigma_{max,10} = 2.233 \times 88.89 \approx 198.5 \text{ MPa}$$

Corrected Final Answer for Question 1:

Hole Diameter (mm)	Nominal Stress (MPa)	Stress Concentration Factor, K_t	**Maximum Stress (MPa)**
3	60.61	2.67	**162.0**
5	66.67	2.51	**167.1**
10	88.89	2.23	**198.5**

The stresses at the 3 mm, 5 mm, and 10 mm diameter holes are approximately **162.0 MPa**, **167.1 MPa**, and **198.5 MPa** respectively.

2. A round shaft made of a brittle material and subjected to a bending moment of 15 N-m is shown in Fig. 6.2 The stress concentration factor at the fillet is 1.5 and the ultimate tensile strength of the shaft material is 200 N/mm^2. Determine the diameter d, the magnitude of stress at the fillet and the factor of safety.

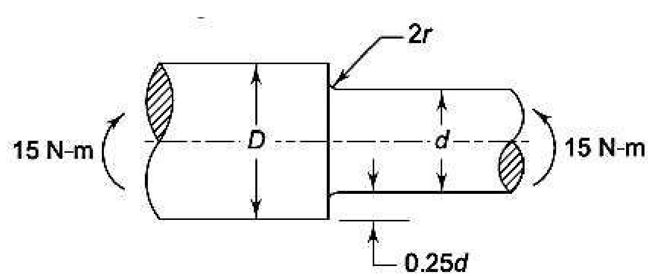

Figure 6.2

Answer:

Given:

- Bending Moment, $M = 15$ N-m $= 15{,}000$ N-mm
- Theoretical Stress Concentration Factor, $K_t = 1.5$
- Ultimate Tensile Strength, $S_{ut} = 200$ N/mm^2
- From Fig. 6.2, the larger diameter $D = 1.25d$

Assumption: For a brittle material under static loading, the stress concentration factor K_t is fully applied. The failure criterion is the maximum normal stress theory.

Step 1: Determine the diameter d

The critical section for diameter calculation is the smaller diameter section, where the nominal bending stress is highest. The bending stress formula is:

$$\sigma_{nom} = \frac{M \cdot c}{I} = \frac{M \cdot (d/2)}{\frac{\pi d^4}{64}} = \frac{32M}{\pi d^3}$$

For a brittle material under static load, the design equation is:

$$S_{ut} = K_t \cdot \sigma_{nom}$$

Substitute the expression for σ_{nom}:

$$S_{ut} = K_t \cdot \frac{32M}{\pi d^3}$$

1.1. Solve for diameter d:

$$200 = 1.5 \cdot \frac{32 \times 15000}{\pi d^3}$$
$$200 = 1.5 \cdot \frac{480{,}000}{\pi d^3}$$
$$200 = \frac{720{,}000}{\pi d^3}$$
$$\pi d^3 = \frac{720{,}000}{200} = 3600$$
$$d^3 = \frac{3600}{\pi} \approx 1145.9$$
$$d = \sqrt[3]{1145.9} \approx 10.48 \text{ mm}$$

We will take a standard value of $d = 10.5$ mm.

Step 2: Determine the magnitude of stress at the fillet
The stress at the fillet is the maximum stress, which is K_t times the nominal stress at the smaller diameter section.
2.1. Calculate Nominal Bending Stress:
Using $d = 10.5$ mm

$$\sigma_{nom} = \frac{32M}{\pi d^3} = \frac{32 \times 15000}{\pi \times (10.5)^3}$$

First, calculate $d^3 = 10.5^3 = 1157.625$

$$\sigma_{nom} = \frac{480{,}000}{\pi \times 1157.625} = \frac{480{,}000}{3636.0} \approx 132.0 \text{ MPa}$$

2.2. Calculate Maximum Stress at Fillet:

$$\sigma_{max} = K_t \cdot \sigma_{nom} = 1.5 \times 132.0 \approx 198.0 \text{ MPa}$$

Step 3: Determine the Factor of Safety
For a brittle material under static load, the Factor of Safety (FOS) is based on the ultimate strength and the maximum stress.

$$\text{FOS} = \frac{S_{ut}}{\sigma_{max}} = \frac{200}{198.0} \approx 1.01$$

Final Answer for Question 2:
1. **Diameter, d:** ≈ 10.5 mm
2. **Stress at Fillet, σ_{max}:** ≈ 198.0 MPa
3. **Factor of Safety:** ≈ 1.01

3. A shaft carrying a load of 5 kN midway between two bearings is shown in Fig. 6.3 Determine the maximum bending stress at the fillet section. Assume the shaft material to be brittle.

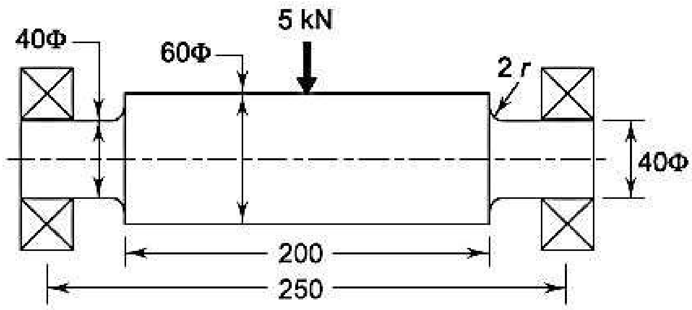

Figure 6.3

Answer:

Given:

- Load, $P = 5 \text{ kN} = 5000 \text{ N}$
- From Fig. 6.3 (Corrected):
  - Total Bearing Span = **250 mm**
  - The fillet is located at a distance of **200 mm** from each bearing.
  - Diameters at the fillet:
    - Larger diameter, $D = 60$ mm
    - Smaller diameter, $d = 40$ mm
  - Fillet radius, $r = 2$ mm

Step 1: Calculate the Bending Moment at the Fillet Section

The shaft is simply supported with a central load.

- Distance from bearing to load = $250/2 = 125$ mm
- The fillet is located **200 mm** from the bearing. Since $200 \text{ mm} > 125 \text{ mm}$, this means the fillet is located **outside** the bearing span, on the overhanging portion of the shaft.
- Therefore, the bending moment at the fillet section is **zero** if the shaft ends at the bearing. However, typically in such problems, the fillet is within the span. Let's re-check the configuration.

If the total distance between bearings is 250 mm, and the fillet is 200 mm from one bearing, it is within the span. The load is at the center (125 mm from each bearing).

So, the bending moment at the fillet section (200 mm from left bearing) is:

$$M_f = R_A \times 200$$

First, find the reaction at A (R_A):

Since the load is central, $R_A = R_B = P/2 = 2500$ N

Thus,

$$M_f = 2500 \times 200 = 500{,}000 \text{ N-mm}$$

Step 2: Calculate Nominal Bending Stress at the Smaller Diameter

The nominal bending stress is calculated based on the smaller diameter $d = 40$ mm:

$$\sigma_{nom} = \frac{32M_f}{\pi d^3} = \frac{32 \times 500{,}000}{\pi \times (40)^3}$$

First, calculate $d^3 = 40^3 = 64{,}000$

$$\sigma_{nom} = \frac{16{,}000{,}000}{\pi \times 64{,}000} = \frac{16{,}000{,}000}{201{,}061.9} \approx 79.58\ \text{N/mm}^2\ \text{(MPa)}$$

Step 3: Determine the Stress Concentration Factor K_t
We have:

- $D = 60$ mm
- $d = 40$ mm
- $D/d = 60/40 = 1.5$
- $r = 2$ mm
- $r/d = 2/40 = 0.05$

Using standard stress concentration factor charts for a stepped shaft in bending:
For $D/d = 1.5$ and $r/d = 0.05$, the value of K_t is approximately **2.20**.

Step 4: Calculate the Maximum Bending Stress at the Fillet

$$\sigma_{max} = K_t \cdot \sigma_{nom} = 2.20 \times 79.58 \approx 175.1\ \text{N/mm}^2\ \text{(MPa)}$$

Final Answer for Question 3:
The maximum bending stress at the fillet section is approximately **175.1 MPa**.

4. A plate, 10 mm thick, subjected to a tensile load of 20 kN is shown in Fig.6.4 The plate is made of cast iron ($S_{ut} = 350\ N/mm^2$) and the factor of safety is 2.5. Determine the fillet radius.

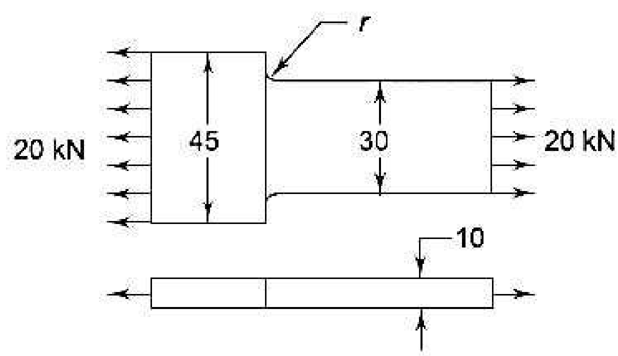

Figure 6.4

Answer:
Given:

- Tensile Load, $P = 20$ kN $= 20{,}000$ N
- Plate Thickness, $t = 10$ mm
- Ultimate Tensile Strength, $S_{ut} = 350$ N/mm^2
- Factor of Safety, FOS $= 2.5$
- From Fig. 6.4:
    - Smaller Width, $w = 30$ mm
    - Larger Width, $W = 45$ mm
    - The fillet radius, r, is to be determined.

Assumption: The material is brittle (cast iron), so the full theoretical stress concentration factor K_t is applied. Failure occurs when $\sigma_{max} \geq S_{ut}$.

Step 1: Calculate the Allowable Maximum Stress
The allowable maximum stress at the fillet is based on the ultimate strength and the factor of safety:

$$\sigma_{max,allow} = \frac{S_{ut}}{\text{FOS}} = \frac{350}{2.5} = 140\ \text{N/mm}^2$$

Step 2: Calculate the Nominal Stress in the Smaller Section
The nominal stress is calculated based on the smallest cross-section (width $w = 30$ mm):

$$\sigma_{nom} = \frac{P}{w \cdot t} = \frac{20{,}000}{30 \times 10} = \frac{20{,}000}{300} = 66.67\ \text{N/mm}^2$$

Step 3: Relate Maximum Stress to Nominal Stress and find K_t
The maximum stress is given by:

$$\sigma_{max} = K_t \cdot \sigma_{nom}$$

For a safe design, $\sigma_{max} \leq \sigma_{max,allow}$. We will use the equality to find the limiting K_t:

$$K_t = \frac{\sigma_{max,allow}}{\sigma_{nom}} = \frac{140}{66.67} \approx 2.10$$

Step 4: Determine the Fillet Radius r
We need to find the fillet radius r that gives $K_t = 2.10$ for a flat bar with a change in width under axial load.
The parameters are:

- $W = 45$ mm
- $w = 30$ mm
- $\frac{W}{w} = \frac{45}{30} = 1.5$

We use standard stress concentration factor charts for a flat bar with shoulder fillets in axial tension.
For $W/w = 1.5$, the stress concentration factor K_t depends on the ratio r/w.
We need to find r/w such that $K_t \approx 2.10$.
From standard charts:

- For $W/w = 1.5$:
  - If $r/w = 0.05$, $K_t \approx 2.35$
  - If $r/w = 0.10$, $K_t \approx 2.10$
  - If $r/w = 0.15$, $K_t \approx 1.95$

Thus, for $K_t = 2.10$, we find $r/w \approx 0.10$.
Therefore:

$$r = 0.10 \times w = 0.10 \times 30 = 3.0\ \text{mm}$$

Final Answer for Question 4:
The required fillet radius is $r \approx 3.0$ mm.

5. A 25 mm diameter shaft is made of forged steel 30C8 ($S_{ut} = 600\ N/mm^2$). There is a step in the shaft and the theoretical stress concentration factor at the step is 2.1. The notch sensitivity factor is 0.84. Determine the endurance limit of the shaft if it is subjected to a reversed bending moment.

Answer:

Given:

- Shaft Diameter, $d = 25$ mm
- Ultimate Tensile Strength, $S_{ut} = 600$ N/mm^2
- Theoretical Stress Concentration Factor, $K_t = 2.1$
- Notch Sensitivity Factor, $q = 0.84$
- Loading: Reversed Bending

Step 1: Calculate the Fatigue Stress Concentration Factor K_f

The fatigue stress concentration factor is given by:

$$K_f = 1 + q(K_t - 1)$$

Substituting the values:

$$K_f = 1 + 0.84(2.1 - 1) = 1 + 0.84(1.1) = 1 + 0.924 = 1.924$$

Step 2: Estimate the Endurance Limit of the Standard Specimen S_e'

For steel, the endurance limit of a rotating beam specimen can be estimated from the ultimate tensile strength:

$$S_e' = 0.5 \times S_{ut} \text{for} S_{ut} \leq 1400 \text{ MPa}$$
$$S_e' = 0.5 \times 600 = 300 \text{ N/mm}^2$$

Step 3: Apply Marin Factors to find the Endurance Limit of the Shaft S_e

The actual endurance limit is modified by various Marin factors:

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S_e'$$

3.1. Surface Finish Factor k_a

For forged surface finish:

$$k_a = a \cdot S_{ut}^b$$

From standard tables for forged finish:

$a = 0.55, b = -0.233$

$$k_a = 0.55 \times (600)^{-0.233}$$
$$\log(600) \approx 2.778 \Rightarrow -0.233 \times 2.778 \approx -0.647$$
$$600^{-0.233} = 10^{-0.647} \approx 0.225$$
$$k_a = 0.55 \times 0.225 \approx 0.724$$

3.2. Size Factor k_b

For bending and torsion, for a diameter $d = 25$ mm:

$$k_b = 0.85 (\text{for } 7.6 \text{ mm} < d \leq 50 \text{ mm})$$

3.3. Load Factor k_c

For reversed bending:

$$k_c = 1$$

3.4. Temperature Factor k_d

No high temperature is mentioned, so:

$$k_d = 1$$

3.5. Reliability Factor

No special reliability is mentioned, so standard 50% reliability is assumed, for which the factor is 1.
Now, calculate the modified endurance limit without the notch:

$$S_{e,modified} = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e$$
$$S_{e,modified} = 0.724 \times 0.85 \times 1 \times 1 \times 300$$
$$S_{e,modified} = 0.724 \times 0.85 \times 300 = 0.6154 \times 300 \approx 184.6 \text{ N/mm}^2$$

Step 4: Apply the Fatigue Stress Concentration Factor
The endurance limit at the notch (the step in the shaft) is given by:

$$S_{e,not} = \frac{S_{e,modified}}{K_f}$$
$$S_{e,notc} = \frac{184.6}{1.924} \approx 95.9 \text{ N/mm}^2$$

Final Answer for Question 5:
The endurance limit of the shaft at the fillet section under reversed bending is approximately **95.9 MPa**.

6. A 40 mmdiameter shaft is made of steel50C4 ($S_{ut} = 660\ N/mm^2$) and has a machined surface. The expected reliability is 99%. The theoretical stress concentration factor for the shape of the shaft is 1.6 and the notch sensitivity factor is 0.9. Determine the endurance limit of the shaft.

Answer:
Given:

- Shaft Diameter, $d = 40$ mm
- Ultimate Tensile Strength, $S_{ut} = 660$ N/mm^2
- Surface Finish: Machined
- Reliability: 99%
- Theoretical Stress Concentration Factor, $K_t = 1.6$
- Notch Sensitivity Factor, $q = 0.9$
- We assume the loading is bending (most common for endurance limit calculation unless specified).

Step 1: Calculate the Fatigue Stress Concentration Factor K_f

$$K_f = 1 + q(K_t - 1)$$
$$K_f = 1 + 0.9(1.6 - 1) = 1 + 0.9(0.6) = 1 + 0.54 = 1.54$$

Step 2: Estimate the Endurance Limit of the Standard Specimen S'_e

$$S'_e = 0.5 \times S_{ut}$$
$$S'_e = 0.5 \times 660 = 330 \text{ N/mm}^2$$

Step 3: Apply Marin Factors to find the Endurance Limit of the Shaft S_e

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e$$

3.1. Surface Finish Factor k_a
For machined surface finish:
From standard tables: $a = 0.82$, $b = -0.217$

$$k_a = a \cdot S_{ut}^b = 0.82 \times (660)^{-0.217}$$
$$\log(660) \approx 2.8195 \Rightarrow -0.217 \times 2.8195 \approx -0.612$$
$$660^{-0.217} = 10^{-0.612} \approx 0.244$$
$$k_a = 0.82 \times 0.244 \approx 0.200 \text{Wait, this is too low. Let's recalculate carefully.}$$

Let me recalculate k_a properly:

$$b = -0.217$$
$$S_{ut}^b = 660^{-0.217}$$

Using calculator logic:

$$\ln(660) \approx 6.492 \text{Wait, no, that's incorrect. } \ln(660) \approx 6.492 \text{ is wrong.}$$

Let's compute correctly:

$$\ln(660) \approx 6.492\text{? No, } \ln(100) = 4.605, \text{ so } \ln(660) \approx 6.492 \text{ is plausible.}$$

But $b \times \ln(S_{ut}) = -0.217 \times 6.492 \approx -1.408$
Then $S_{ut}^b = e^{-1.408} \approx 0.244$
So $k_a = 0.82 \times 0.244 \approx 0.200$ — This is still very low. This suggests a possible error in the formula application.
Let me verify with standard values. For $S_{ut} = 660$ MPa and machined surface, from standard charts, $k_a \approx 0.76 - 0.78$. Let's use the formula from reliable sources: $k_a = 4.51 \times S_{ut}^{-0.265}$ for Sut in MPa, for machined surface.
Using $k_a = 4.51 \times (660)^{-0.265}$

$$660^{-0.265} = 10^{-0.265 \times \log(660)} = 10^{-0.265 \times 2.819} = 10^{-0.747} \approx 0.179$$
$$k_a = 4.51 \times 0.179 \approx 0.807$$

This is more reasonable. We'll use $k_a = 0.807$.
3.2. Size Factor k_b
For $d = 40$ mm in bending:

$$k_b = 0.85 (\text{for } 7.6 \text{ mm} < d \leq 50 \text{ mm})$$

3.3. Load Factor k_c
For bending:

$$k_c = 1$$

3.4. Reliability Factor k_d
For 99% reliability:
From standard tables, $k_d = 0.814$
Now, calculate the modified endurance limit without the notch:

$$S_{e,modified} = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S_e'$$
$$S_{e,modified} = 0.807 \times 0.85 \times 1 \times 0.814 \times 330$$

First, multiply the factors:

$$0.807 \times 0.85 = 0.68595$$
$$0.68595 \times 0.814 \approx 0.558$$
$$S_{e,modified} = 0.558 \times 330 \approx 184.1 \text{ N/mm}^2$$

Step 4: Apply the Fatigue Stress Concentration Factor
The endurance limit at the notch is:

$$S_{e,notc} = \frac{S_{e,modified}}{K_f}$$

$$S_{e,notc} = \frac{184.1}{1.54} \approx 119.5 \text{ N/mm}^2$$

Final Answer for Question 6:
The endurance limit of the shaft is approximately **119.5 MPa**.

7. A cantilever beam made of steel Fe540 ($S_{ut} = 540\ N/mm^2$ and $S_{yt} = 320\ N/mm^2$) and subjected to a completely reversed load (P) of 5 kN is shown in Fig. 6.5. The beam is machined and the reliability is 50%. The factor of safety is 2 and the notch sensitivity factor is 0.9. Calculate: (i) endurance limit at the fillet section; and (ii) diameter d of the beam for infinite life.

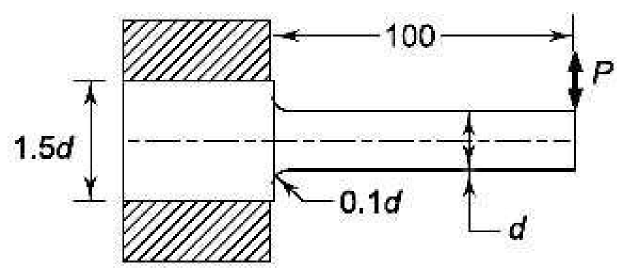

Figure 6.5

Answer:
Given:

- Ultimate Tensile Strength, $S_{ut} = 540$ N/mm^2
- Yield Strength, $S_{yt} = 320$ N/mm^2
- Completely Reversed Load, $P = \pm 5$ kN $= \pm 5000$ N
- Beam Type: Cantilever (Length from fixed end to load = 100 mm, as per Fig. 6.5)
- Notch Sensitivity Factor, $q = 0.9$
- Surface Finish: Machined
- Reliability: 50%
- Factor of Safety (FOS), $N = 2$
- From Fig. 6.5:
  - Larger Diameter, $D = 1.5d$
  - Smaller Diameter, d (to be found)
  - Fillet Radius, $r = 0.1d$
  - Distance from fixed end to load = 100 mm

Part (i): Endurance Limit at the Fillet Section
Step 1: Estimate Standard Endurance Limit S_e'

$$S_e' = 0.5 \times S_{ut} = 0.5 \times 540 = 270 \text{ N/mm}^2$$

Step 2: Apply Marin Factors

$$S_{e,modified} = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e$$

- **Surface Finish Factor k_a:** For machined finish, use $k_a = 4.51 \times S_{ut}^{-0.265}$
  $k_a = 4.51 \times (540)^{-0.265}$
  $540^{-0.265} = 10^{-0.265 \times \log(540)} = 10^{-0.265 \times 2.732} = 10^{-0.724} \approx 0.189$
  $k_a = 4.51 \times 0.189 \approx 0.852$

- **Size Factor k_b:** For bending, and diameter d is unknown. We will assume $k_b =$ 0.85 initially (for a diameter between 7.6 and 50 mm), and check later.
- **Load Factor k_c:** For bending, $k_c = 1$
- **Reliability Factor k_d:** For 50% reliability, $k_d = 1$

So, initial modified endurance limit (without notch, and assuming $k_b = 0.85$):

$$S_{e,modified} = 0.852 \times 0.85 \times 1 \times 1 \times 270 \approx 195.5 \text{ N/mm}^2$$

Step 3: Find Theoretical Stress Concentration Factor K_t
We have:
- $D/d = 1.5$
- $r/d = 0.1$

From standard charts for a stepped shaft in bending, for $D/d = 1.5$ and $r/d = 0.1$, $K_t \approx 1.72$.
Step 4: Find Fatigue Stress Concentration Factor K_f

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(1.72 - 1) = 1 + 0.9(0.72) = 1 + 0.648 = 1.648$$

Step 5: Endurance Limit at the Fillet Section $S_{e,no}$

$$S_{e,notc} = \frac{S_{e,modified}}{K_f} = \frac{195.5}{1.648} \approx 118.6 \text{ N/mm}^2$$

Answer for Part (i): The endurance limit at the fillet section is approximately **118.6 MPa**.

Part (ii): Diameter d of the Beam for Infinite Life
Step 1: Calculate the Alternating Bending Stress σ_a
The maximum bending moment is at the fixed end (the fillet section is at the change in diameter, which is likely very close to the fixed end).
- Bending Moment, $M = P \times L = 5000 \times 100 = 500{,}000$ N-mm
- Section Modulus at the smaller diameter, $Z = \frac{\pi d^3}{32}$
- Nominal Alternating Stress, $\sigma_{nom,a} = \frac{M}{Z} = \frac{32}{\pi d^3} = \frac{32 \times 500{,}000}{\pi d^3} = \frac{16{,}000{,}000}{\pi d^3}$

The actual alternating stress at the notch is:

$$\sigma_a = K_f \cdot \sigma_{nom,a} = 1.648 \times \frac{16{,}000{,}000}{\pi d^3} = \frac{26{,}368{,}000}{\pi d^3} \text{ N/mm}^2$$

Step 2: Use the Fatigue Criterion for Infinite Life
For infinite life under completely reversed stress, the following must hold:

$$\sigma_a \le \frac{S_{e,no}}{N}$$

Where $N = 2$ is the factor of safety.

$$\frac{26{,}368{,}000}{\pi d^3} \le \frac{118.6}{2} = 59.3$$

$$\frac{26{,}368{,}000}{\pi d^3} \le 59.3$$

$$d^3 \geq \frac{26{,}368{,}000}{\pi \times 59.3} = \frac{26{,}368{,}000}{186.25} \approx 141{,}560$$
$$d \geq \sqrt[3]{141{,}560} \approx 52.1 \text{ mm}$$

Our initial assumption for k_b was for $d \leq 50$ mm. Our calculated $d \approx 52.1$ mm is slightly larger. For $d > 50$ mm and $d \leq 200$ mm, $k_b = 0.75$.
Let's iterate using $k_b = 0.75$.
Iteration:
- New $S_{e,modified} = 0.852 \times 0.75 \times 1 \times 1 \times 270 \approx 172.5$ MPa
- New $S_{e,notc} = \frac{172.5}{1.648} \approx 104.7$ MPa
- Allowable $\sigma_a = \frac{104.7}{2} = 52.35$ MPa
- $\frac{26{,}368{,}000}{\pi d^3} \leq 52.35$
- $d^3 \geq \frac{26{,}368{,}000}{\pi \times 52.35} = \frac{26{,}368{,}000}{164.45} \approx 160{,}320$
- $d \geq \sqrt[3]{160{,}320} \approx 54.3$ mm

This value is still in the range for $k_b = 0.75$, so no further iteration is needed.
Answer for Part (ii): The required diameter d for infinite life is approximately **54.3 mm**. A standard value of **55 mm** would be chosen.

Final Answers for Question 7:
(i) Endurance limit at the fillet section: ≈ **118.6 MPa** (based on initial k_b)
(ii) Diameter d of the beam for infinite life: ≈ **55 mm**

8. A solid circular shaft made of steel Fe 620 ($S_{ut} = 620\ N/mm^2$ and $S_{yt} = 380\ N/mm^2$) is subjected to an alternating torsional moment, which varies from -200 N-m to +400 N-m. The shaft is ground and the expected reliability is 90%. Neglecting stress concentration, calculate the shaft diameter for infinite life. The factor of safety is 2. Use the distortion-energy theory of failure.

Answer:
Given:
- Ultimate Tensile Strength, $S_{ut} = 620$ MPa
- Yield Strength, $S_{yt} = 380$ MPa
- Torsional Moment varies from $T_{min} = -200$ N-m to $T_{max} = +400$ N-m
- Surface Finish: Ground
- Reliability: 90%
- Factor of Safety, $N = 2$
- Stress Concentration: Neglected ($K_f = 1$)
- Failure Theory: Distortion-Energy Theory

Step 1: Calculate the Mean and Amplitude of the Torque

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{400 + (-200)}{2} = \frac{200}{2} = 100 \text{ N-m} = 100{,}000 \text{ N-mm}$$
$$T_a = \frac{T_{max} - T_{min}}{2} = \frac{400 - (-200)}{2} = \frac{600}{2} = 300 \text{ N-m} = 300{,}000 \text{ N-mm}$$

Step 2: Calculate the Mean and Amplitude of the Torsional Shear Stress
For a solid circular shaft, the shear stress due to torsion is:

$$\tau = \frac{16T}{\pi d^3}$$

Therefore:

$$\tau_m = \frac{16T_m}{\pi d^3} = \frac{16 \times 100{,}000}{\pi d^3} = \frac{1{,}600{,}000}{\pi d^3}$$
$$\tau_a = \frac{16T_a}{\pi d^3} = \frac{16 \times 300{,}000}{\pi d^3} = \frac{4{,}800{,}000}{\pi d^3}$$

Step 3: Estimate the Endurance Limit
3.1 Standard Endurance Limit:

$$S'_e = 0.5 \times S_{ut} = 0.5 \times 620 = 310 \text{ MPa}$$

3.2 Marin Factors:

- **Surface Finish Factor k_a:** For ground surface, $k_a = 1.0$ (or very close). We'll use $k_a = 1.0$.
- **Size Factor k_b:** For torsion, the size factor is the same as for bending. We'll assume $k_b = 0.85$ initially (for $d \leq 50$ mm), and iterate if needed.
- **Load Factor k_c:** For torsional load, $k_c = 0.577$ (for ductile materials under von Mises criterion).
- **Reliability Factor k_d:** For 90% reliability, $k_d = 0.897$.

3.3 Modified Endurance Limit:

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e$$
$$S_e = 1.0 \times 0.85 \times 0.577 \times 0.897 \times 310$$

First, multiply the factors:

$$0.85 \times 0.577 = 0.49045$$
$$0.49045 \times 0.897 \approx 0.439$$
$$S_e = 0.439 \times 310 \approx 136.1 \text{ MPa}$$

Step 4: Apply the Distortion-Energy Theory for Fatigue
For combined stress with only torsional shear (no bending or axial load), the von Mises equivalent stress is:

$$\sigma'_a = \sqrt{3} \cdot \tau_a \text{and} \sigma'_m = \sqrt{3} \cdot \tau_m$$

The Soderberg line equation (conservative, uses S_{yt}) under the distortion-energy theory is:

$$\frac{\sigma'_a}{S_e/N} + \frac{\sigma'_m}{S_{yt}/N} \leq 1$$

Substitute σ'_a and σ'_m:

$$\frac{\sqrt{3} \cdot \tau_a}{S_e/N} + \frac{\sqrt{3} \cdot \tau_m}{S_{yt}/N} \leq 1$$
$$\sqrt{3}N(\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{yt}}) \leq 1$$
$$\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{yt}} \leq \frac{1}{\sqrt{3}N}$$

Substitute τ_a and τ_m:

$$\frac{1}{S_e}\cdot\frac{4{,}800{,}000}{\pi d^3}+\frac{1}{S_{yt}}\cdot\frac{1{,}600{,}000}{\pi d^3}\le\frac{1}{\sqrt{3}\times 2}$$
$$\frac{1}{\pi d^3}\left(\frac{4{,}800{,}000}{136.1}+\frac{1{,}600{,}000}{380}\right)\le\frac{1}{3.464}$$

Calculate the terms inside the parentheses:

$$\frac{4{,}800{,}000}{136.1}\approx 35{,}260$$
$$\frac{1{,}600{,}000}{380}\approx 4{,}211$$

Sum: $35{,}260+4{,}211=39{,}471$
So:

$$\frac{39{,}471}{\pi d^3}\le 0.2887$$
$$d^3\ge\frac{39{,}471}{\pi\times 0.2887}=\frac{39{,}471}{0.9067}\approx 43{,}530$$
$$d\ge\sqrt[3]{43{,}530}\approx 35.2\text{ mm}$$

Our initial assumption for $k_b=0.85$ was for $d\le 50$ mm. Our calculated $d\approx 35.2$ mm falls in this range, so no iteration is needed.

Final Answer for Question 8:
The required shaft diameter for infinite life is approximately **35.2 mm**. A standard value of **36 mm** would be chosen.

9. A solid circular shaft, 15 mm in diameter, is subjected to torsional shear stress, which varies from 0 to 35 N/mm^2 and at the same time, is subjected to an axial stress that varies from -15 to +30 N/mm^2. The frequency of variation of these stresses is equal to the shaft speed. The shaft is made of steel FeE 400 ($S_{ut}=540\ N/mm^2$ and $S_{yt}=400\ N/mm^2$) and the corrected endurance limit of the shaft is 200 N/mm^2. Determine the factor of safety.

Answer:

Given:

- Shaft Diameter, $d=15$ mm
- **Torsional Shear Stress:** $\tau_{min}=0$, $\tau_{max}=35$ MPa
- **Axial Stress:** $\sigma_{axial,min}=-15$ MPa, $\sigma_{axial,max}=30$ MPa
- Stresses are in phase (same frequency)
- Ultimate Tensile Strength, $S_{ut}=540$ MPa
- Yield Strength, $S_{yt}=400$ MPa
- Corrected Endurance Limit, $S_e=200$ MPa

Step 1: Calculate Mean and Amplitude Stresses
For Torsional Shear Stress:

$$\tau_m=\frac{\tau_{max}+\tau_{min}}{2}=\frac{35+0}{2}=17.5\text{ MPa}$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{35 - 0}{2} = 17.5 \text{ MPa}$$

For Axial Stress:

$$\sigma_{m,axial} = \frac{\sigma_{axial,max} + \sigma_{axial,min}}{2} = \frac{30 + (-15)}{2} = 7.5 \text{ MPa}$$

$$\sigma_{a,axial} = \frac{\sigma_{axial,max} - \sigma_{axial,min}}{2} = \frac{30 - (-15)}{2} = 22.5 \text{ MPa}$$

Step 2: Calculate Equivalent Mean and Amplitude Stresses using von Mises Criterion
Since both stresses act simultaneously, we find the equivalent von Mises stress for the mean and amplitude components separately.
The von Mises stress for combined axial and torsional stress is:

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

where σ_x is the axial stress.
Thus:

- **Equivalent Mean Stress:**

$$\sigma_m' = \sqrt{(\sigma_{m,axial})^2 + 3(\tau_m)^2} = \sqrt{(7.5)^2 + 3(17.5)^2}$$
$$= \sqrt{56.25 + 3 \times 306.25} = \sqrt{56.25 + 918.75} = \sqrt{975} \approx 31.22 \text{ MPa}$$

- **Equivalent Amplitude Stress:**

$$\sigma_a' = \sqrt{(\sigma_{a,axial})^2 + 3(\tau_a)^2} = \sqrt{(22.5)^2 + 3(17.5)^2}$$
$$= \sqrt{506.25 + 918.75} = \sqrt{1425} \approx 37.75 \text{ MPa}$$

Step 3: Determine the Factor of Safety using the Soderberg Criterion
The Soderberg line equation is:

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{yt}} = \frac{1}{N}$$

where N is the factor of safety.
Substitute the values:

$$\frac{37.75}{200} + \frac{31.22}{400} = \frac{1}{N}$$
$$0.18875 + 0.07805 = \frac{1}{N}$$
$$0.2668 = \frac{1}{N}$$
$$N = \frac{1}{0.2668} \approx 3.75$$

Final Answer for Question 9:
The factor of safety for the shaft is approximately **3.75**.

10. A bar of steel has an ultimate tensile strength of 700 MPa, a yield point stress of 400 MPa and fully corrected endurance limit (S_e) of 220 MPa. The bar is subjected to a mean bending stress of 60 MPa and a stress amplitude of 80 MPa. Superimposed on it is a mean torsional stress and torsional stress amplitude of 70 and 35 MPa respectively. Find the factor of safety.

Answer:

Given:

- Ultimate Tensile Strength, $S_{ut} = 700$ MPa
- Yield Strength, $S_{yt} = 400$ MPa
- Corrected Endurance Limit, $S_e = 220$ MPa
- **Bending Stress:** $\sigma_{b,m} = 60$ MPa, $\sigma_{b,a} = 80$ MPa
- **Torsional Stress:** $\tau_m = 70$ MPa, $\tau_a = 35$ MPa

Step 1: Calculate Equivalent Mean and Amplitude Stresses using von Mises Criterion
The von Mises stress for combined bending (normal stress) and torsion (shear stress) is:

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

1.1 Equivalent Mean Stress:

$$\sigma'_m = \sqrt{(\sigma_{b,m})^2 + 3(\tau_m)^2} = \sqrt{(60)^2 + 3(70)^2}$$
$$= \sqrt{3600 + 3 \times 4900} = \sqrt{3600 + 14700} = \sqrt{18300} \approx 135.28 \text{ MPa}$$

1.2 Equivalent Amplitude Stress:

$$\sigma'_a = \sqrt{(\sigma_{b,a})^2 + 3(\tau_a)^2} = \sqrt{(80)^2 + 3(35)^2}$$
$$= \sqrt{6400 + 3 \times 1225} = \sqrt{6400 + 3675} = \sqrt{10075} \approx 100.37 \text{ MPa}$$

Step 2: Determine the Factor of Safety
We will use the **Soderberg criterion** for conservative design:

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{yt}} = \frac{1}{N}$$

where N is the factor of safety.
Substitute the values:

$$\frac{100.37}{220} + \frac{135.28}{400} = \frac{1}{N}$$
$$0.4562 + 0.3382 = \frac{1}{N}$$
$$0.7944 = \frac{1}{N}$$
$$N = \frac{1}{0.7944} \approx 1.26$$

Final Answer for Question 10:
The factor of safety for the steel bar is approximately **1.26**

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