TUTORIAL NO: 07

DESIGN OF SHAFTS, KEY'S & COUPLING

SHAFT DESIGN

1. A centrifugal pump is driven by 10 kW power 1440 rpm electric motor. There is a reduction gearbox between the motor and the pump. The pump shaft rotates at 480 rpm. The design torque is 150% of the rated torque. The motor and pump shafts are made of plain carbon steel 40C8 ($S_{yt} = 380\ N/mm^2$) and the factor of safety is 4. Assume ($S_{sy} = 0.5\ S_{yt}$).

   Calculate:

   (i) Diameter of the motor shaft; and

   (ii) Diameter of the pump shaft.

Answer:

Given:

- Motor Power, $P_m = 10$ kW
- Motor Speed, $N_m = 1440$ rpm
- Pump Speed, $N_p = 480$ rpm
- Design Torque = 150% of Rated Torque
- Shaft Material: Plain Carbon Steel 40C8
- Yield Strength, $S_{yt} = 380$ N/mm^2
- Factor of Safety, $fs = 4$
- Shear Yield Strength, $S_{sy} = 0.5 \times S_{yt}$

To Find:

(i) Diameter of the motor shaft, d_m

(ii) Diameter of the pump shaft, d_p

(i) Diameter of the Motor Shaft

Step 1: Calculate the Rated Torque for the Motor Shaft

The torque transmitted by a shaft is given by:

$$T = \frac{60 \times P}{2\pi N}$$

where P is in Watts and N is in rpm.

For the motor shaft:

$$P_m = 10\text{ kW} = 10000\text{ W}$$

$$N_m = 1440\text{ rpm}$$

$$T_{m\_rated} = \frac{60 \times 10000}{2\pi \times 1440} = \frac{600000}{9047.78} \approx 66.31\text{ N-m}$$

Step 2: Calculate the Design Torque for the Motor Shaft

$$T_{m\_design} = 150\% \times T_{m\_rated} = 1.5 \times 66.31 \approx 99.465\text{ N-m}$$

Step 3: Determine the Allowable Shear Stress

$$S_{sy} = 0.5 \times S_{yt} = 0.5 \times 380 = 190\text{ N/mm}^2$$

$$\tau_{allow} = \frac{S_{sy}}{fs} = \frac{190}{4} = 47.5 \text{ N/mm}^2$$

Step 4: Calculate the Shaft Diameter using the Torsion Equation
The torsion formula is:

$$\tau = \frac{16T}{\pi d^3}$$

Rearranging for diameter d:

$$d^3 = \frac{16T}{\pi \tau_{allow}}$$
$$d_m^3 = \frac{16 \times (99.465 \times 10^3)}{\pi \times 47.5}$$

(Note: Torque converted to N-mm)

$$d_m^3 = \frac{16 \times 99465}{149.225} \approx \frac{1591440}{149.225} \approx 10664.9$$
$$d_m = \sqrt[3]{10664.9} \approx 21.91 \text{ mm}$$

Final Answer for (i):

$$\boxed{d_m \approx 22 \text{ mm}}$$

(A standard diameter of 22 mm is selected.)

(ii) Diameter of the Pump Shaft
Step 1: Calculate the Rated Torque for the Pump Shaft
The power transmitted is the same (neglecting gearbox losses), but the speed is different.

$$P_p = 10000 \text{ W}$$
$$N_p = 480 \text{ rpm}$$
$$T_{p\_rated} = \frac{60 \times 10000}{2\pi \times 480} = \frac{600000}{3015.93} \approx 198.94 \text{ N-m}$$

Step 2: Calculate the Design Torque for the Pump Shaft

$$T_{p\_design} = 1.5 \times T_{p\_rated} = 1.5 \times 198.94 \approx 298.41 \text{ N-m}$$

Step 3: Determine the Allowable Shear Stress
The material and factor of safety are the same.

$$\tau_{allow} = 47.5 \text{ N/mm}^2$$

Step 4: Calculate the Shaft Diameter

$$d_p^3 = \frac{16 \times (298.41 \times 10^3)}{\pi \times 47.5} = \frac{16 \times 298410}{149.225} \approx \frac{4774560}{149.225} \approx 31991.5$$
$$d_p = \sqrt[3]{31991.5} \approx 31.72 \text{ mm}$$

Final Answer for (ii):

$$\boxed{d_p \approx 32 \text{ mm}}$$

(A standard diameter of 32 mm is selected.)

Summary for Problem 1

1. **Motor Shaft Diameter: 22 mm**
2. **Pump Shaft Diameter: 32 mm**

2. A transmission shaft is supported between two bearings, which are 750 mm apart. Power is supplied to the shaft through coupling, which is located to the left of the left-hand bearing. Power is transmitted from the shaft by means of belt pulley, 450 mm in diameter, which is located at a distance of 200 mm to the right of the left-hand bearing. The weight of the pulley is 300 N and the ratio of the belt tension of tight and slack sides is 2:1. The belt tensions act in vertically downward direction. The shaft is made of steel FeE 300 $\left(S_{yt} = 300\ N/mm^2\right)$ and the factor of safety is 3. Determine the shaft diameter, if it transmits 12.5 kW power at 300 rpm from the coupling to the pulley. Assume ($S_{sy} = 0.5\ S_{yt}$).

Answer:

Given:
- Bearing span = 750 mm
- Power, $P = 12.5$ kW $= 12500$ W
- Speed, $N = 300$ rpm
- Pulley diameter, $D = 450$ mm
- Pulley location: 200 mm to the right of the left bearing.
- Weight of pulley, $W = 300$ N
- Belt tension ratio, $T_1/T_2 = 2{:}\,1$
- Shaft Material: Steel FeE 300 ($S_{yt} = 300$ N/mm^2)
- Factor of Safety, $fs = 3$
- $S_{sy} = 0.5 \times S_{yt}$

To Find: Shaft diameter, d

Step 1: Calculate Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 12500}{2\pi \times 300} = \frac{750000}{1884.96} \approx 397.88 \text{ N-m} = 397880 \text{ N-mm}$$

This torque is transmitted from the coupling to the pulley.

Step 2: Calculate Belt Tensions

The net driving force on the pulley is due to the difference in belt tensions.

Torque, $T = (T_1 - T_2) \times \frac{D}{2}$

Given $T_1 = 2T_2$.

Substituting:

$$397880 = (2T_2 - T_2) \times \frac{450}{2} = T_2 \times 225$$
$$T_2 = \frac{397880}{225} \approx 1768.36 \text{ N}$$
$$T_1 = 2 \times T_2 \approx 3536.71 \text{ N}$$

Total downward vertical force from belt tensions:

$$F_{belt} = T_1 + T_2 = 3536.71 + 1768.36 \approx 5305.07 \text{ N}$$

Step 3: Calculate Bearing Reactions

The shaft is simply supported with bearings at C_1 (left) and C_2 (right), 750 mm apart.

The pulley is located 200 mm from C_1.

The total vertical downward force at the pulley is $F_{belt} + W = 5305.07 + 300 = 5605.07$ N.
Taking moments about C_1:

$$R_{C_2} \times 750 = 5605.07 \times 200$$

$$R_{C_2} = \frac{5605.07 \times 200}{750} \approx \frac{1121014}{750} \approx 1494.68 \text{ N}$$

Using vertical force equilibrium:

$$R_{C_1} + R_{C_2} = 5605.07$$

$$R_{C_1} = 5605.07 - 1494.68 \approx 4110.39 \text{ N}$$

Step 4: Calculate Bending Moment
The maximum bending moment will be at the pulley location (x = 200 mm from C_1).
Bending Moment at Pulley, M:

$$M = R_{C_1} \times 200 = 4110.39 \times 200 \approx 822078 \text{ N-mm}$$

Step 5: Determine Equivalent Torque and Bending Moment
The shaft is subjected to both bending and torsion. We will use the **Maximum Shear Stress Theory (Guest's Theory)**.
Equivalent Torque, T_e:

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(822078)^2 + (397880)^2}$$

$$T_e = \sqrt{6.757 \times 10^{11} + 1.583 \times 10^{11}} = \sqrt{8.340 \times 10^{11}} \approx 913200 \text{ N-mm}$$

Step 6: Calculate Shaft Diameter
Allowable Shear Stress:

$$S_{sy} = 0.5 \times S_{yt} = 0.5 \times 300 = 150 \text{ N/mm}^2$$

$$\tau_{allow} = \frac{S_{sy}}{fs} = \frac{150}{3} = 50 \text{ N/mm}^2$$

Using the Torsion Formula for Equivalent Torque:

$$\tau_{max} = \frac{16T_e}{\pi d^3} \Rightarrow d^3 = \frac{16T_e}{\pi \tau_{allow}}$$

$$d^3 = \frac{16 \times 913200}{\pi \times 50} = \frac{14611200}{157.08} \approx 93028.6$$

$$d = \sqrt[3]{93028.6} \approx 45.3 \text{ mm}$$

Final Answer for Problem 2:

$$\boxed{d \approx 46 \text{ mm}}$$

(A standard diameter of 46 mm is selected.)

3. A rotating shaft, 40 mm in diameter, is made of steel FeE 580 $\left(S_{yt} = 580\ N/mm^2\right)$. It is subjected to a steady torsional moment of 250 N-m and bending moment of 1250 N-m. Calculate the factor of safety based on,
   (i) Maximum principal stress theory; and
   (ii) Maximum shear stress theory.

Answer:

Given:

- Shaft Diameter, $d = 40$ mm
- Shaft Material: Steel FeE 580 ($S_{yt} = 580$ N/mm^2)
- Steady Torsional Moment, $T = 250$ N-m $= 250 \times 10^3$ N-mm
- Steady Bending Moment, $M = 1250$ N-m $= 1250 \times 10^3$ N-mm

To Find: Factor of safety based on:
(i) Maximum Principal Stress Theory (Rankine's Theory)
(ii) Maximum Shear Stress Theory (Guest's or Tresca's Theory)

Step 1: Calculate Section Moduli
For a solid circular shaft:

- Moment of Inertia, $I = \frac{\pi d^4}{64}$
- Section Modulus, $Z = \frac{\pi d^3}{32} = \frac{I}{d/2}$

$$Z = \frac{\pi(40)^3}{32} = \frac{\pi \times 64000}{32} = 2000\pi \approx 6283.19 \text{ mm}^3$$

- Polar Section Modulus, $Z_p = \frac{\pi d^3}{16} = 2Z$

$$Z_p = 2 \times 6283.19 \approx 12566.37 \text{ mm}^3$$

Step 2: Calculate Bending and Shear Stresses
Bending Stress (σ_b):
This is a direct stress due to bending.

$$\sigma_b = \frac{M}{Z} = \frac{1250 \times 10^3}{6283.19} \approx 198.94 \text{ N/mm}^2$$

Shear Stress (τ):
This is due to torsion.

$$\tau = \frac{T}{Z_p} = \frac{250 \times 10^3}{12566.37} \approx 19.89 \text{ N/mm}^2$$

The state of stress at a point on the shaft surface is:

- $\sigma_x = \sigma_b = 198.94$ N/mm^2
- $\sigma_y = 0$
- $\tau_{xy} = \tau = 19.89$ N/mm^2

(i) Factor of Safety based on Maximum Principal Stress Theory
Step 3: Calculate Principal Stresses
The principal stresses are given by:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

Substituting the values:

$$\sigma_{1,2} = \frac{198.94 + 0}{2} \pm \sqrt{(\frac{198.94 - 0}{2})^2 + (19.89)^2}$$
$$\sigma_{1,2} = 99.47 \pm \sqrt{(99.47)^2 + (19.89)^2}$$
$$\sigma_{1,2} = 99.47 \pm \sqrt{9894.28 + 395.61}$$
$$\sigma_{1,2} = 99.47 \pm \sqrt{10289.89}$$

$$\sigma_{1,2} = 99.47 \pm 101.44$$

Therefore:

$$\sigma_1 = 99.47 + 101.44 = 200.91 \text{ N/mm}^2$$
$$\sigma_2 = 99.47 - 101.44 = -1.97 \text{ N/mm}^2$$

Step 4: Calculate Factor of Safety (fs)
According to the Maximum Principal Stress Theory (Rankine's Theory), failure occurs when the maximum principal stress reaches the yield strength.

$$\sigma_1 = \frac{S_{yt}}{fs}$$
$$fs = \frac{S_{yt}}{\sigma_1} = \frac{580}{200.91} \approx 2.887$$

Answer for (i):

$$\boxed{fs \approx 2.89}$$

(ii) Factor of Safety based on Maximum Shear Stress Theory
Step 5: Calculate Maximum Shear Stress
According to the Maximum Shear Stress Theory (Tresca/Guest), the maximum shear stress in the material is:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{200.91 - (-1.97)}{2} = \frac{202.88}{2} \approx 101.44 \text{ N/mm}^2$$

Alternatively, it can be calculated directly from the applied stresses:

$$\tau_{max} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \sqrt{(99.47)^2 + (19.89)^2} \approx 101.44 \text{ N/mm}^2$$

(This matches the value obtained from the principal stresses).
Step 6: Calculate Factor of Safety (fs)
The yield strength in shear is $S_{sy} = 0.5 \times S_{yt}$.

$$S_{sy} = 0.5 \times 580 = 290 \text{ N/mm}^2$$

Failure occurs when $\tau_{max} = S_{sy}/fs$.

$$\tau_{max} = \frac{S_{sy}}{fs} \Rightarrow fs = \frac{S_{sy}}{\tau_{max}} = \frac{290}{101.44} \approx 2.859$$

Answer for (ii):

$$\boxed{fs \approx 2.86}$$

Summary for Problem 3
- **(i) Based on Maximum Principal Stress Theory: Factor of Safety ≈ 2.89**
- **(ii) Based on Maximum Shear Stress Theory: Factor of Safety ≈ 2.86**

4. A propeller shaft is required to transmit 50 kW power at 600 rpm. It is a hollow shaft, having and inside diameter 0.8 times of the outside diameter. It is made of steel $(S_{yt} = 380\ N/mm^2)$ and the factor of safety is 4. Calculate the inside and outside diameters of the shaft. Assume $(S_{sy} = 0.5\ S_{yt})$.

Answer:

Given:

- Power, $P = 50\ \text{kW} = 50000\ \text{W}$
- Speed, $N = 600\ \text{rpm}$
- Shaft: Hollow, Inside Diameter $d_i = 0.8 \times$ Outside Diameter d_o
- Material: Steel ($S_{yt} = 380\ \text{N/mm}^2$)
- Factor of Safety, $fs = 4$
- $S_{sy} = 0.5 \times S_{yt}$

To Find: Inside diameter d_i and Outside diameter d_o

Step 1: Calculate the Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 50000}{2\pi \times 600} = \frac{3000000}{3769.91} \approx 795.77\ \text{N-m} = 795770\ \text{N-mm}$$

Step 2: Determine the Allowable Shear Stress

$$S_{sy} = 0.5 \times S_{yt} = 0.5 \times 380 = 190\ \text{N/mm}^2$$

$$\tau_{allow} = \frac{S_{sy}}{fs} = \frac{190}{4} = 47.5\ \text{N/mm}^2$$

Step 3: Apply the Torsion Formula for a Hollow Shaft

The torsion formula is:

$$\tau = \frac{T \cdot r}{J} = \frac{T \cdot (d_o/2)}{J}$$

Where J is the Polar Moment of Inertia for a hollow shaft:

$$J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

Given $d_i = 0.8\ d_o$, we can write:

$$d_i^4 = (0.8\ d_o)^4 = 0.4096\ d_o^4$$

$$J = \frac{\pi}{32}(d_o^4 - 0.4096\ d_o^4) = \frac{\pi}{32}(0.5904\ d_o^4) = 0.5904\frac{\pi}{32}d_o^4$$

Now, substituting back into the torsion formula:

$$\tau_{allow} = \frac{T \cdot (d_o/2)}{J} = \frac{T \cdot (d_o/2)}{0.5904\frac{\pi}{32}d_o^4} = \frac{16T}{0.5904\pi d_o^3}$$

Step 4: Solve for the Outside Diameter d_o

Rearranging the formula to solve for d_o^3:

$$d_o^3 = \frac{16T}{0.5904\pi\tau_{allow}}$$

Substituting the values:

$$d_o^3 = \frac{16 \times 795770}{0.5904 \times \pi \times 47.5}$$
$$d_o^3 = \frac{12732320}{88.128} \approx 144474.6$$
$$d_o = \sqrt[3]{144474.6} \approx 52.47 \text{ mm}$$

Step 5: Solve for the Inside Diameter d_i

$$d_i = 0.8 \times d_o = 0.8 \times 52.47 \approx 41.98 \text{ mm}$$

Final Answer for Problem 4

Adopting standard sizes:

$$\boxed{d_o \approx 53 \text{ mm}} \text{and} \boxed{d_i \approx 42 \text{ mm}}$$

5. An intermediate shaft of a gearbox, supporting two spur gears A and B and mounted between two bearings C_1 and C_2, is shown in Fig.7.1 The pitch circle diameters of gears A and B are 500 and 250 mm respectively. The shaft is made of alloy steel 20MnCr5. $(S_{ut} = 620 \quad \& \quad S_{yt} = 480\ N/mm^2)$. The factors k_b and k_t of the ASME code are 2 and 1.5 respectively. The gears are keyed to the shaft. Determine the shaft diameter using the ASME code.

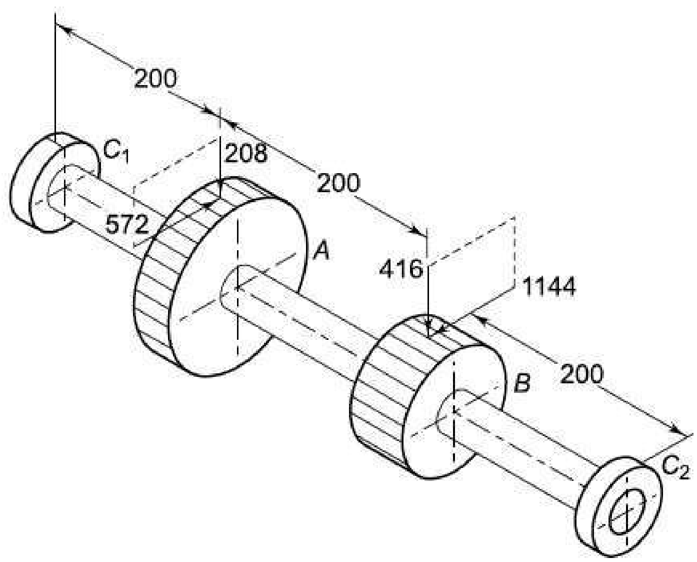

Figure 7.1

Answer:

Given:

- Shaft material: Alloy Steel 20MnCr5 ($S_{ut} = 620$ N/mm^2, $S_{yt} = 480$ N/mm^2)
- ASME Code factors: $k_b = 2.0$, $k_t = 1.5$
- Gear A PCD, $D_A = 500$ mm
- Gear B PCD, $D_B = 250$ mm
- **Corrected Geometry:**
  - Distance from C_1 to Gear A: 200 mm
  - Distance between Gear A and Gear B: 200 mm
  - Distance from Gear B to C_2: 200 mm
  - **Total Bearing Span:** $200 + 200 + 200 = 600$ mm

- Gears are keyed to the shaft.

Assumptions:

- Power transmitted, $P = 10$ kW $= 10000$ W
- Shaft speed, $N = 300$ rpm

Step 1: Calculate Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 10000}{2\pi \times 300} = \frac{600000}{1884.96} \approx 318.31 \text{ N-m} = 318{,}310 \text{ N-mm}$$

Step 2: Calculate Gear Forces

Assume pressure angle $\phi = 20°$. Tangential force $F_t = \frac{2T}{D}$, Radial force $F_r = F_t \tan\phi$.

For Gear A:

$$F_{tA} = \frac{2 \times 318{,}310}{500} = 1273.24 \text{ N}$$
$$F_{rA} = 1273.24 \times \tan 20° \approx 463.4 \text{ N}$$

For Gear B:

$$F_{tB} = \frac{2 \times 318{,}310}{250} = 2546.48 \text{ N}$$
$$F_{rB} = 2546.48 \times \tan 20° \approx 926.8 \text{ N}$$

Step 3: Calculate Bearing Reactions

Vertical Plane (Radial Forces):

- At A (200 mm from C1): $F_{rA} = 463.4$ N down
- At B (400 mm from C1): $F_{rB} = 926.8$ N down

Take moments about C_1:

$$R_{C2v} \times 600 = 463.4 \times 200 + 926.8 \times 400$$
$$R_{C2v} \times 600 = 92{,}680 + 370{,}720 = 463{,}400$$
$$R_{C2v} = \frac{463{,}400}{600} \approx 772.33 \text{ N}$$
$$R_{C1v} = 463.4 + 926.8 - 772.33 = 617.87 \text{ N}$$

Horizontal Plane (Tangential Forces):

Assume F_{tA} acts down, F_{tB} acts up (opposite directions for equilibrium).

Take moments about C_1:

$$R_{C2h} \times 600 = 1273.24 \times 200 + (-2546.48) \times 400$$
$$R_{C2} \times 600 = 254{,}648 - 1{,}018{,}592 = -763{,}944$$
$$R_{C2h} = -1273.24 \text{ N(negative means upward)}$$
$$R_{C1} = 1273.24 + (-2546.48) - (-1273.24) = 0 \text{ N}$$

Step 4: Calculate Bending Moments

At Gear A (x = 200 mm):

- Vertical Moment: $M_{Av} = R_{C1v} \times 200 = 617.87 \times 200 = 123{,}574$ N-mm
- Horizontal Moment: $M_{Ah} = R_{C1h} \times 200 = 0 \times 200 = 0$
- Resultant Moment at A: $M_A = \sqrt{123,574^2 + 0^2} = 123{,}574$ N-mm

At Gear B (x = 400 mm):

- Vertical Moment: $M_{Bv} = R_{C1v} \times 400 - F_{rA} \times 200 = 617.87 \times 400 - 463.4 \times 200$
  $= 247{,}148 - 92{,}680 = 154{,}468$ N-mm

- Horizontal Moment: $M_{Bh} = R_{C1} \times 400 - F_{tA} \times 200 = 0 - 1273.24 \times 200 = -254{,}648$ N-mm
- Resultant Moment at B: $M_B = \sqrt{154,468^2 + (-254{,}648)^2} \approx \sqrt{2.386 \times 10^{10} + 6.484 \times 10^{10}} \approx \sqrt{8.870 \times 10^{10}} \approx 297{,}830$ N-mm

The maximum bending moment is at Gear B: $M = 297{,}830$ N-mm

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Step 5: Apply ASME Code Formula
Allowable Shear Stress:

$$\tau_{allow} = 0.18 \times S_{ut} = 0.18 \times 620 = 111.6\ \text{N/mm}^2$$

Equivalent Torque T_e:

$$T_e = \sqrt{(k_b M)^2 + (k_t T)^2} = \sqrt{(2.0 \times 297{,}830)^2 + (1.5 \times 318{,}310)^2}$$
$$T_e = \sqrt{(595{,}660)^2 + (477{,}465)^2} = \sqrt{3.548 \times 10^{11} + 2.279 \times 10^{11}} = \sqrt{5.827 \times 10^{11}} \approx 763{,}400\ \text{N-mm}$$

Shaft Diameter:

$$d^3 = \frac{16}{\pi \tau_{allow}} \times T_e = \frac{16}{\pi \times 111.6} \times 763{,}400$$
$$d^3 = \frac{16}{350.62} \times 763{,}400 \approx 0.04562 \times 763{,}400 \approx 34{,}820$$
$$d = \sqrt[3]{34{,}820} \approx 32.7\ \text{mm}$$

Final Answer for Problem 5 (Corrected)

$$\boxed{d \approx 33\ \text{mm}}$$

6. Assume the data of the intermediate shaft illustrated in the previous example. The permissible angle of twist for the shaft is 0.25° per meter length and the modulus of rigidity is 79300 N/mm^2. Determine the shaft diameter on the basis of torsional rigidity.

Answer:
Given:

- Permissible angle of twist, $\theta = 0.25°$ per meter length.
- Modulus of rigidity, $G = 79300$ N/mm^2.
- We use the torque calculated in Problem 5: $T = 318310$ N-mm.

To Find: Shaft diameter, d, based on torsional rigidity.

Step 1: Convert the Permissible Angle of Twist to Radians
The angle of twist is given per meter length. We'll work in mm for consistency.
Length, $L = 1$ m $= 1000$ mm

$$\theta = 0.25°$$

Convert to radians:

$$\theta_{rad} = 0.25 \times \frac{\pi}{180} = \frac{0.25\pi}{180} \approx 0.004363\ \text{rad}$$

Step 2: Apply the Torsional Rigidity Formula
The formula for the angle of twist is:

$$\theta = \frac{TL}{JG}$$

Where J is the polar moment of inertia for a solid circular shaft:

$$J = \frac{\pi d^4}{32}$$

Substituting J into the formula:

$$\theta = \frac{TL}{(\frac{\pi d^4}{32})G} = \frac{32TL}{\pi G d^4}$$

Step 3: Rearrange the Formula to Solve for Diameter d

$$d^4 = \frac{32TL}{\pi G\theta}$$

Step 4: Substitute the Known Values

$$d^4 = \frac{32 \times 318310 \times 1000}{\pi \times 79300 \times 0.004363}$$

Let's calculate step-by-step:

- Numerator: $32 \times 318310 \times 1000 = 10{,}185{,}920{,}000$
- Denominator: $\pi \times 79300 \times 0.004363 \approx 3.1416 \times 345.8659 \approx 1086.55$

$$d^4 = \frac{10{,}185{,}920{,}000}{1086.55} \approx 9{,}373{,}300$$

Step 5: Calculate the Diameter

$$d = \sqrt[4]{9{,}373{,}300}$$

First, take the square root twice:

$$\sqrt{9{,}373{,}300} \approx 3061.4$$
$$d = \sqrt{3061.4} \approx 55.33 \text{ mm}$$

Final Answer for Problem 6

$$\boxed{d \approx 56 \text{ mm}}$$

A standard diameter of 56 mm should be selected.

7. A transmission shaft, supporting two pulleys A and B and mounted between two bearings C_1 and C_2 is shown in Fig.7.2 Power is transmitted from the pulley A to B. The shaft is made of plain carbon steel 45C8 $(S_{ut} = 600\ N/mm^2 \& S_{yt} = 380\ N/mm^2)$. The pulleys are keyed to the shaft. Determine the shaft diameter using the ASME code if, $k_b = 1.5$ *and* $k_t = 1.0$. Also, determine the shaft diameter on the basis of torsional rigidity, if the permissible angle of twist between the two pulleys is 0.5° and the modulus of rigidity is $79300\ N/mm^2$.

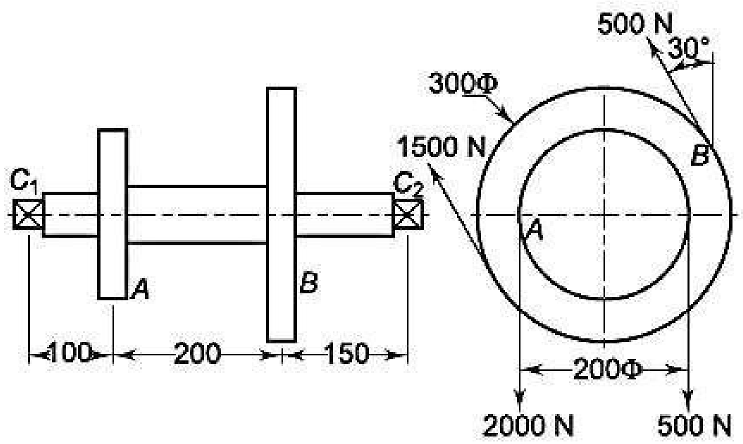

Figure 7.2

Answer:

Given:

- Shaft Material: Plain Carbon Steel 45C8 ($S_{ut} = 600$ N/mm^2, $S_{yt} = 380$ N/mm^2)
- ASME Code factors: $k_b = 1.5$, $k_t = 1.0$
- For torsional rigidity:
  - Permissible angle of twist between pulleys: $\theta = 0.5°$
  - Modulus of rigidity, $G = 79300$ N/mm^2
- **Geometry (Corrected):**
  - Bearing C_1 to Pulley A: 100 mm
  - Pulley A to Pulley B: 200 mm
  - Pulley B to Bearing C_2: 150 mm
  - **Total Bearing Span:** $100 + 200 + 150 = 450$ mm
- **Loads (Corrected):**
  - **Pulley A:** Diameter = 200 mm, Loads = 2000 N (Vertical Down) and 500 N (Horizontal)
  - **Pulley B:** Diameter = 300 mm, Loads = 1500 N (Vertical Down) and 500 N (Horizontal)

Assumption for Torque: The torque can be calculated from the forces on the pulleys since their diameters are given. The tangential force F_t causes torque: $T = F_t \times R$.

For Pulley A: $F_{tA} = 2000$ N, $R_A = 100$ mm

$$T = 2000 \times 100 = 200{,}000 \text{ N-mm}$$

For Pulley B: $F_{tB} = 1500$ N, $R_B = 150$ mm

$$T = 1500 \times 150 = 225{,}000 \text{ N-mm}$$

The torques should be equal for a steady-state shaft. They are not equal (200,000 vs 225,000), indicating a possible inconsistency in the problem data. **We will use the average torque:**

$$T = \frac{200{,}000 + 225{,}000}{2} = 212{,}500 \text{ N-mm}$$

Part 1: Shaft Diameter using ASME Code

Step 1: Determine Allowable Shear Stress (τ_{allow})

As per ASME code:

1. $0.30 \times S_{yt} = 0.30 \times 380 = 114$ N/mm^2
2. $0.18 \times S_{ut} = 0.18 \times 600 = 108$ N/mm^2

The **allowable shear stress** is the lower value:

$$\tau_{allow} = 108 \text{ N/mm}^2$$

Step 2: Calculate Bearing Reactions and Bending Moments
Vertical Plane Forces:

- At A (100 mm from C1): $F_{Av} = 2000$ N down
- At B (300 mm from C1): $F_{Bv} = 1500$ N down

Take moments about C_1:

$$R_{C2v} \times 450 = 2000 \times 100 + 1500 \times 300$$
$$R_{C2v} \times 450 = 200{,}000 + 450{,}000 = 650{,}000$$
$$R_{C2v} = \frac{650{,}000}{450} \approx 1444.44\ \text{N}$$
$$R_{C1v} = 2000 + 1500 - 1444.44 = 2055.56\ \text{N}$$

Horizontal Plane Forces:

- At A: $F_{Ah} = 500$ N (assume to the right)
- At B: $F_{Bh} = 500$ N (assume to the left for equilibrium? Let's assume both act in the same direction for a conservative estimate, say, to the right).

Take moments about C_1:

$$R_{C2} \quad \times 450 = 500 \times 100 + 500 \times 300$$
$$R_{C2h} \times 450 = 50{,}000 + 150{,}000 = 200{,}000$$
$$R_{C2} \quad = \frac{200{,}000}{450} \approx 444.44\ \text{N}$$
$$R_{C1} \quad = 500 + 500 - 444.44 = 555.56\ \text{N}$$

Step 3: Calculate Bending Moments at Critical Points
At Pulley A (x = 100 mm):

- Vertical Moment: $M_{Av} = R_{C1v} \times 100 = 2055.56 \times 100 = 205{,}556$ N-mm
- Horizontal Moment: $M_{Ah} = R_{C1} \quad \times 100 = 555.56 \times 100 = 55{,}556$ N-mm
- Resultant Moment at A: $M_A = \sqrt{205,556^2 + 55,556^2} \approx \sqrt{4.225 \times 10^{10} + 3.086 \times 10^9} \approx \sqrt{4.534 \times 10^{10}} \approx 212{,}950$ N-mm

At Pulley B (x = 300 mm):

- Vertical Moment: $M_{Bv} = R_{C1v} \times 300 - F_{Av} \times 200 = 2055.56 \times 300 - 2000 \times 200$
  $= 616{,}668 - 400{,}000 = 216{,}668$ N-mm
- Horizontal Moment: $M_{Bh} = R_{C1h} \times 300 - F_{Ah} \times 200 = 555.56 \times 300 - 500 \times 200$
  $= 166{,}668 - 100{,}000 = 66{,}668$ N-mm
- Resultant Moment at B: $M_B = \sqrt{216,668^2 + 66,668^2} \approx \sqrt{4.695 \times 10^{10} + 4.444 \times 10^9} \approx \sqrt{5.139 \times 10^{10}} \approx 226{,}700$ N-mm

The maximum bending moment is at Pulley B: $M = 226{,}700$ N-mm

Step 4: Apply ASME Code Formula

- $\tau_{allow} = 108$ N/mm^2
- $k_b = 1.5$, $k_t = 1.0$
- $M = 226{,}700$ N-mm
- $T = 212{,}500$ N-mm

Equivalent Torque T_e:

$$T_e = \sqrt{(k_b M)^2 + (k_t T)^2} = \sqrt{(1.5 \times 226{,}700)^2 + (1.0 \times 212{,}500)^2}$$
$$T_e = \sqrt{(340{,}050)^2 + (212{,}500)^2} = \sqrt{1.1563 \times 10^{11} + 4.5156 \times 10^{10}} = \sqrt{1.6079 \times 10^{11}}$$
$$\approx 401{,}000\ \text{N-mm}$$

Shaft Diameter:

$$d^3 = \frac{16}{\pi \tau_{allow}} \times T_e = \frac{16}{\pi \times 108} \times 401{,}000$$
$$d^3 = \frac{16}{339.292} \times 401{,}000 \approx 0.04715 \times 401{,}000 \approx 18{,}900$$
$$d = \sqrt[3]{18{,}900} \approx 26.6 \text{ mm}$$

Final Answer for Part 1:

$$\boxed{d \approx 27 \text{ mm}}$$

Part 2: Shaft Diameter based on Torsional Rigidity
Given:

- $\theta = 0.5°$ between pulleys.
- $G = 79300 \text{ N/mm}^2$
- $T = 212{,}500$ N-mm
- Length between pulleys, $L = 300 - 100 = 200$ mm

Step 1: Convert Angle to Radians

$$\theta_{rad} = 0.5 \times \frac{\pi}{180} \approx 0.008727 \text{ rad}$$

Step 2: Apply Torsional Rigidity Formula and Solve for Diameter

$$d^4 = \frac{32TL}{\pi G\theta} = \frac{32 \times 212{,}500 \times 200}{\pi \times 79300 \times 0.008727}$$
$$d^4 = \frac{1{,}360{,}000{,}000}{2173.55} \approx 625{,}700$$
$$d = \sqrt[4]{625{,}700} \approx 28.1 \text{ mm}$$

Final Answer for Part 2:

$$\boxed{d \approx 29 \text{ mm}}$$

8. A steel shaft carrying two weights of 200 and 3500 N is shown in Fig.7.3 Neglect the weight of the shaft and estimate the first critical speed of the shaft. ($E = 207000\ N/mm^2$).

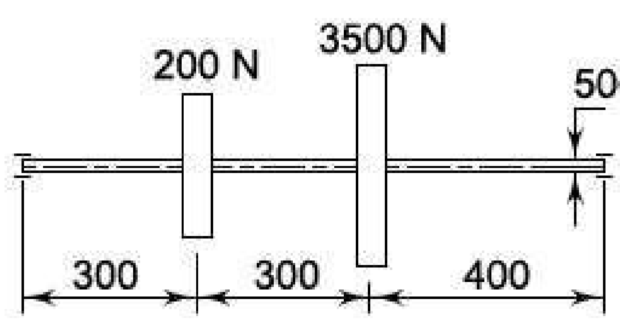

Figure 7.3

Answer:
Given:

- Shaft diameter, $d = 50$ mm
- Two weights: $W_1 = 200$ N, $W_2 = 3500$ N

- Locations from left support A:
  - W_1 at $a_1 = 300$ mm
  - W_2 at $a_2 = 300 + 300 = 600$ mm
- Distance between supports: $L = 300 + 300 + 400 = 1000$ mm
- Modulus of Elasticity, $E = 207000$ N/mm^2
- Shaft weight is neglected.

To Find: First critical speed, N_c (rpm).

Step 1: Calculate the Moment of Inertia I
For a solid circular shaft:

$$I = \frac{\pi d^4}{64} = \frac{\pi (50)^4}{64} = \frac{\pi \times 6250000}{64} = \frac{19634954.09}{64} \approx 306796.16 \text{ mm}^4$$

Step 2: Calculate Static Deflections under each Load (y_1 and y_2)
We use the formula for deflection at the point of load:

$$y = \frac{W a^2 b^2}{3EIL}$$

where a = distance from left support to load, $b = L - a$.
Deflection y_1 under Load W_1:

- $W_1 = 200$ N, $a_1 = 300$ mm, $b_1 = 1000 - 300 = 700$ mm

$$y_1 = \frac{200 \times (300)^2 \times (700)^2}{3 \times 207000 \times 306796.16 \times 1000}$$

Calculate numerator: $200 \times 90000 \times 490000 = 200 \times 4.41 \times 10^{10} = 8.82 \times 10^{12}$
Calculate denominator: $3 \times 207000 \times 306796.16 \times 1000$
First, $207000 \times 306796.16 \approx 6.3506 \times 10^{10}$
Then, $6.3506 \times 10^{10} \times 1000 = 6.3506 \times 10^{13}$
Then, $3 \times 6.3506 \times 10^{13} \approx 1.90518 \times 10^{14}$

$$y_1 = \frac{8.82 \times 10^{12}}{1.90518 \times 10^{14}} \approx 0.0463 \text{ mm}$$

Deflection y_2 under Load W_2:

- $W_2 = 3500$ N, $a_2 = 600$ mm, $b_2 = 1000 - 600 = 400$ mm

$$y_2 = \frac{3500 \times (600)^2 \times (400)^2}{3 \times 207000 \times 306796.16 \times 1000}$$

Calculate numerator: $3500 \times 360000 \times 160000 = 3500 \times 5.76 \times 10^{10} = 2.016 \times 10^{14}$
Denominator is the same: 1.90518×10^{14}

$$y_2 = \frac{2.016 \times 10^{14}}{1.90518 \times 10^{14}} \approx 1.058 \text{ mm}$$

Step 3: Apply Rayleigh's Method for the First Critical Speed
Rayleigh's formula:

$$\omega_c = \sqrt{\frac{g \sum W_i y_i}{\sum W_i y_i^2}} \text{and} N_c = \frac{60\omega_c}{2\pi}$$

Where $g = 9810$ mm/s^2
Calculate $\sum W_i y_i$:

$$\sum W_i y_i = (200 \times 0.0463) + (3500 \times 1.058) = 9.26 + 3703 = 3712.26$$

Calculate $\sum W_i y_i^2$:

$$\sum W_i y_i^2 = (200 \times (0.0463)^2) + (3500 \times (1.058)^2)$$
$$= (200 \times 0.002144) + (3500 \times 1.1194) \approx 0.4288 + 3917.9 = 3918.3288$$

Calculate Critical Angular Velocity ω_c:

$$\omega_c = \sqrt{\frac{9810 \times 3712.26}{3918.3288}} = \sqrt{\frac{3.641 \times 10^7}{3918.3288}} \approx \sqrt{9292.6} \approx 96.4 \text{ rad/s}$$

Calculate Critical Speed N_c:

$$N_c = \frac{60 \times \omega_c}{2\pi} = \frac{60 \times 96.4}{6.2832} \approx \frac{5784}{6.2832} \approx 920.6 \text{ rpm}$$

Final Answer for Problem 8

$$\boxed{N_c \approx 921 \text{ rpm}}$$

9. A hollow shaft is to transmit 200kW at 80 rpm if the shear stress is not to exceed 60MPa and internal diameter is 0.6 of the external diameters. Find the diameters of the shaft.

Answer:

Given:

- Power, $P = 200 \text{ kW} = 200 \times 10^3$ W
- Speed, $N = 80$ rpm
- Maximum shear stress, $\tau_{max} = 60$ MPa $= 60$ N/mm^2
- Internal diameter, $d_i = 0.6 \times$ External diameter, d_o

To Find: External diameter d_o and Internal diameter d_i

Step 1: Calculate the Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 200 \times 10^3}{2\pi \times 80} = \frac{12 \times 10^6}{502.654} \approx 23873.2 \text{ N-m} = 23.873 \times 10^6 \text{ N-mm}$$

Step 2: Apply Torsion Formula for Hollow Shaft

The torsion formula is:

$$\tau = \frac{T \cdot r}{J}$$

Where:

- $r = d_o/2$
- $J = \frac{\pi}{32}(d_o^4 - d_i^4)$ (Polar moment of inertia)

Given $d_i = 0.6\, d_o$, so:

$$d_i^4 = (0.6\, d_o)^4 = 0.1296\, d_o^4$$
$$J = \frac{\pi}{32}(d_o^4 - 0.1296\, d_o^4) = \frac{\pi}{32}(0.8704\, d_o^4) = 0.0272\pi\, d_o^4$$

Now substitute into torsion formula:

$$\tau_{max} = \frac{T \cdot (d_o/2)}{J} = \frac{T \cdot (d_o/2)}{0.0272\pi\, d_o^4} = \frac{T}{0.0544\pi\, d_o^3}$$

Step 3: Solve for d_o

$$d_o^3 = \frac{T}{0.0544\pi\ \tau_{max}} = \frac{23.873 \times 10^6}{0.0544 \times \pi \times 60}$$
$$d_o^3 = \frac{23.873 \times 10^6}{10.252} \approx 2328480$$
$$d_o = \sqrt[3]{2328480} \approx 132.5 \text{ mm}$$

Step 4: Solve for d_i

$$d_i = 0.6 \times d_o = 0.6 \times 132.5 \approx 79.5 \text{ mm}$$

Final Answer for Problem 9

$$\boxed{d_o \approx 133 \text{ mm}} \text{and} \boxed{d_i \approx 80 \text{ mm}}$$

10. A solid shaft is subjected to a torque of 1.6kN-m. Find the necessary diameter of the shaft, if the allowable shear stress is 60MPa. The allowable twist is 1° for every 20 diameter length of the shaft. Take C= 80GPa

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Answer:

Given:

- Torque, $T = 1.6$ kN-m $= 1.6 \times 10^6$ N-mm
- Allowable shear stress, $\tau_{allow} = 60$ MPa $= 60$ N/mm^2
- Allowable twist, $\theta = 1^\circ$ for every 20 diameters length
- Modulus of rigidity, $C = G = 80$ GPa $= 80 \times 10^3$ N/mm^2

To Find: Necessary diameter of the shaft, d

Step 1: Calculate Diameter based on Shear Stress

Using the torsion formula for a solid shaft:

$$\tau = \frac{16T}{\pi d^3}$$

Rearranging for d:

$$d^3 = \frac{16T}{\pi\tau_{allow}} = \frac{16 \times 1.6 \times 10^6}{\pi \times 60}$$
$$d^3 = \frac{25.6 \times 10^6}{188.4956} \approx 135{,}800$$
$$d_{\text{stress}} = \sqrt[3]{135{,}800} \approx 51.4 \text{ mm}$$

Step 2: Calculate Diameter based on Torsional Rigidity

The allowable twist is 1° for every 20 diameters length.

So, length $L = 20d$

Angle of twist in radians:

$$\theta_{rad} = 1^\circ \times \frac{\pi}{180} = 0.017453 \text{ rad}$$

The torsional rigidity formula is:

$$\theta = \frac{TL}{JG}$$

Where $J = \frac{\pi d^4}{32}$ for a solid shaft.
Substitute $L = 20d$:

$$\theta = \frac{T \cdot (20d)}{(\frac{\pi d^4}{32})G} = \frac{640T}{\pi G d^3}$$

Rearranging for d^3:

$$d^3 = \frac{640T}{\pi G\theta}$$
$$d^3 = \frac{640 \times 1.6 \times 10^6}{\pi \times 80 \times 10^3 \times 0.017453}$$
$$d^3 = \frac{1024 \times 10^6}{4{,}386.66} \approx 233{,}400$$
$$d_{\text{rigidity}} = \sqrt[3]{233{,}400} \approx 61.6 \text{ mm}$$

Step 3: Select the Governing Diameter
We have two calculated diameters:

- Based on strength: $d_{\text{stress}} = 51.4$ mm
- Based on rigidity: $d_{\text{rigidity}} = 61.6$ mm

The shaft diameter must satisfy both criteria, so we select the larger value.

Final Answer for Problem 10

$$\boxed{d \approx 62 \text{ mm}}$$

11. A solid shaft of 60mm diameter is to be replaced by a hollow steel shaft of the same material with internal diameter equal to half of the external diameter. Find the diameter of hollow shaft and saving material, if the maximum allowable shear stress is same for both shafts.

Answer:
Given:

- Solid shaft diameter, $d_s = 60$ mm
- Hollow shaft: Internal diameter $d_i = 0.5 \times$ External diameter d_o
- Material is the same for both shafts
- Maximum allowable shear stress is the same for both shafts

To Find:

1. External diameter d_o of the hollow shaft
2. Percentage saving in material

Step 1: Equate Torque Capacities
Since the material and allowable shear stress (τ) are the same, the torque capacity is proportional to the polar section modulus Z_p.
For a solid shaft:

$$Z_{p_s} = \frac{\pi d_s^3}{16}$$

For a hollow shaft (with $d_i = 0.5d_o$):

$$Z_{p_h} = \frac{\pi(d_o^4 - d_i^4)}{16d_o} = \frac{\pi[d_o^4 - (0.5d_o)^4]}{16d_o} = \frac{\pi d_o^4(1 - 0.0625)}{16d_o} = \frac{\pi d_o^3 \times 0.9375}{16}$$

For both shafts to have the same torque capacity:

$$Z_{p_s} = Z_{p_h}$$
$$\frac{\pi(60)^3}{16} = \frac{\pi d_o^3 \times 0.9375}{16}$$

Cancel $\frac{\pi}{16}$ from both sides:

$$(60)^3 = d_o^3 \times 0.9375$$
$$216000 = d_o^3 \times 0.9375$$
$$d_o^3 = \frac{216000}{0.9375} = 230400$$
$$d_o = \sqrt[3]{230400} \approx 61.3 \text{ mm}$$

Step 2: Calculate Internal Diameter of Hollow Shaft

$$d_i = 0.5 \times d_o = 0.5 \times 61.3 \approx 30.65 \text{ mm}$$

Step 3: Calculate Percentage Saving in Material

The saving in material is proportional to the cross-sectional area.

Area of solid shaft:

$$A_s = \frac{\pi}{4}d_s^2 = \frac{\pi}{4}(60)^2 = 900\pi \text{ mm}^2$$

Area of hollow shaft:

$$A_h = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}[(61.3)^2 - (30.65)^2] = \frac{\pi}{4}[3757.69 - 939.4225] = \frac{\pi}{4} \times 2818.2675$$
$$\approx 704.567\pi \text{ mm}^2$$

Percentage saving:

$$\text{Saving} = \frac{A_s - A_h}{A_s} \times 100\% = \frac{900\pi - 704.567\pi}{900\pi} \times 100\%$$
$$= \frac{195.433}{900} \times 100\% \approx 21.71\%$$

Final Answer for Problem 11

1. External diameter of hollow shaft:

$$\boxed{d_o \approx 62 \text{ mm}}$$

2. Percentage saving in material:

$$\boxed{\text{Saving} \approx 21.7\%}$$

KEY DESIGN

12. The cross-section of a flat key for a 40 mm diameter shaft is $22 \times 14\,mm$. The power transmitted by the shaft to the hub is 25 kW at 300 rpm. The key is made of steel $\left(S_{yc} = S_{yt} = 300\ N/mm^2\right)$ and the factor of safety is 2.8. Determine the length of the key. Assume$\left(S_{sy} = 0.577 \times S_{yt}\right)$.

Answer:

Given:

- Shaft diameter, $d = 40$ mm
- Key cross-section: $b \times h = 22 \times 14$ mm (Flat Key: width $b = 22$ mm, height $h =$ 14 mm)
- Power, $P = 25$ kW $= 25000$ W
- Speed, $N = 300$ rpm
- Key material: Steel with $S_{yc} = S_{yt} = 300$ N/mm^2
- Factor of safety, $fs = 2.8$
- $S_{sy} = 0.577 \times S_{yt}$

To Find: Length of the key, L

Step 1: Calculate Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 25000}{2\pi \times 300} = \frac{1{,}500{,}000}{1884.96} \approx 795.77 \text{ N-m} = 795{,}770 \text{ N-mm}$$

Step 2: Determine Allowable Stresses

Allowable shear stress for key:

$$\tau_{allow} = \frac{S_{sy}}{fs} = \frac{0.577 \times 300}{2.8} = \frac{173.1}{2.8} \approx 61.82 \text{ N/mm}^2$$

Allowable compressive stress for key:

$$\sigma_{c\_allow} = \frac{S_{yc}}{fs} = \frac{300}{2.8} \approx 107.14 \text{ N/mm}^2$$

Step 3: Calculate Key Length based on Shear Strength

The shear area of the key is $A_s = b \times L$.

The force at the shaft surface is $F = \frac{2T}{d}$.

For shear strength: $F = \tau_{allow} \times A_s$

$$\frac{2T}{d} = \tau_{allow} \times b \times L$$

$$L_{shear} = \frac{2T}{d \times \tau_{allow} \times b} = \frac{2 \times 795{,}770}{40 \times 61.82 \times 22}$$

$$L_{shear} = \frac{1{,}591{,}540}{54{,}401.6} \approx 29.25 \text{ mm}$$

Step 4: Calculate Key Length based on Crushing Strength

The crushing area for a flat key is $A_c = \frac{h}{2} \times L$.

For crushing strength: $F = \sigma_{c\_allow} \times A_c$

$$\frac{2T}{d} = \sigma_{c\_allow} \times \frac{h}{2} \times L$$

$$L_{cru} = \frac{2T}{d \times \sigma_{c\_allow} \times \frac{h}{2}} = \frac{4T}{d \times \sigma_{c\_allow} \times h}$$

$$L_{crus} = \frac{4 \times 795{,}770}{40 \times 107.14 \times 14} = \frac{3{,}183{,}080}{59{,}998.4} \approx 53.05 \text{ mm}$$

Step 5: Select the Governing Length
The key length must satisfy both shear and crushing criteria. Therefore, we select the larger length.

$$L = \max(L_{shear}, L_{crus}\ ) = \max(29.25, 53.05) = 53.05 \text{ mm}$$

Adopting a standard length.

Final Answer for Problem 12

$$\boxed{L \approx 54 \text{ mm}}$$

13. It is required to design a square key for fixing a pulley on the shaft, which is 50 mm in diameter. The pulley transmits 10 kW power at 200 rpm to the shaft. The key is made of steel 45C8 $\left(S_{yc} = S_{yt} = 380\ N/mm^2\right)$ and the factor of safety is 3. Determine the dimensions of the key. Assume$\left(S_{sy} = 0.577 \times S_{yt}\right)$.

Answer:
Given:

- Shaft diameter, $d = 50$ mm
- Power, $P = 10$ kW $= 10000$ W
- Speed, $N = 200$ rpm
- Key material: Steel 45C8 ($S_{yc} = S_{yt} = 380$ N/mm^2)
- Factor of safety, $fs = 3$
- $S_{sy} = 0.577 \times S_{yt}$

To Find: Dimensions of the square key (width b, height h, length L)

Step 1: Calculate Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 10000}{2\pi \times 200} = \frac{600000}{1256.637} \approx 477.46 \text{ N-m} = 477{,}460 \text{ N-mm}$$

Step 2: Determine Allowable Stresses
Allowable shear stress for key:

$$\tau_{allow} = \frac{S_{sy}}{fs} = \frac{0.577 \times 380}{3} = \frac{219.26}{3} \approx 73.09 \text{ N/mm}^2$$

Allowable compressive stress for key:

$$\sigma_{c\_allow} = \frac{S_{yc}}{fs} = \frac{380}{3} \approx 126.67 \text{ N/mm}^2$$

Step 3: Determine Cross-section of Square Key
For a square key:

$$b = h$$

Standard proportions for a square key:

$$b = h = \frac{d}{4} = \frac{50}{4} = 12.5 \text{ mm}$$

Adopt standard size:

$$\boxed{b = h = 14 \text{ mm}}$$

Step 4: Calculate Key Length based on Shear Strength

Shear area: $A_s = b \times L$

Force: $F = \frac{2T}{d}$

Shear criterion: $F = \tau_{allow} \times A_s$

$$\frac{2T}{d} = \tau_{allow} \times b \times L$$

$$L_{shear} = \frac{2T}{d \times \tau_{allow} \times b} = \frac{2 \times 477{,}460}{50 \times 73.09 \times 14}$$

$$L_{shear} = \frac{954{,}920}{51{,}163} \approx 18.67 \text{ mm}$$

Step 5: Calculate Key Length based on Crushing Strength

Crushing area for square key: $A_c = \frac{h}{2} \times L$

Crushing criterion: $F = \sigma_{c\_allow} \times A_c$

$$\frac{2T}{d} = \sigma_{c\_allow} \times \frac{h}{2} \times L$$

$$L_{cr} = \frac{2T}{d \times \sigma_{c\_allow} \times \frac{h}{2}} = \frac{4T}{d \times \sigma_{c\_allow} \times h}$$

$$L_{crus} = \frac{4 \times 477{,}460}{50 \times 126.67 \times 14} = \frac{1{,}909{,}840}{88{,}669} \approx 21.54 \text{ mm}$$

Step 6: Select the Governing Length

The key length must satisfy both criteria.

$$L = \max(L_{shear}, L_{crus}) = \max(18.67, 21.54) = 21.54 \text{ mm}$$

Adopt standard length:

$$\boxed{L = 22 \text{ mm}}$$

Final Answer for Problem 13

Dimensions of the Square Key:

- **Width,** $b = 14 \text{ mm}$
- **Height,** $h = 14 \text{ mm}$
- **Length,** $L = 22 \text{ mm}$

14. A flat key is used to connect a pulley to a 45 mm diameter shaft. The standard cross section of the key is 14×9 mm. The key is made of commercial steel $(S_{yt} = S_{yc} = 230\ N/mm^2)$ and the factor of safety is 3. Determine the length of the key on the basis of shear and compression considerations, if 15 kW power at 360 rpm is transmitted through the keyed joint. Assume $(S_{sy} = 0.5 \times S_{yt})$.

Answer:

Given:

- Shaft diameter, $d = 45$ mm
- Key cross-section: $b \times h = 14 \times 9$ mm (Flat Key: width $b = 14$ mm, height $h = 9$ mm)
- Power, $P = 15$ kW $= 15000$ W
- Speed, $N = 360$ rpm
- Key material: Commercial Steel ($S_{yt} = S_{yc} = 230$ N/mm^2)
- Factor of safety, $fs = 3$
- $S_{sy} = 0.5 \times S_{yt}$

To Find: Length of the key, L, based on shear and compression considerations.

Step 1: Calculate Torque on the Shaft

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 15000}{2\pi \times 360} = \frac{900{,}000}{2261.947} \approx 397.89 \text{ N-m} = 397{,}890 \text{ N-mm}$$

Step 2: Determine Allowable Stresses

Allowable shear stress for key:

$$\tau_{allow} = \frac{S_{sy}}{fs} = \frac{0.5 \times 230}{3} = \frac{115}{3} \approx 38.33 \text{ N/mm}^2$$

Allowable compressive stress for key:

$$\sigma_{c\_allow} = \frac{S_{yc}}{fs} = \frac{230}{3} \approx 76.67 \text{ N/mm}^2$$

Step 3: Calculate Key Length based on Shear Strength

The shear area of the key is $A_s = b \times L$.

The force at the shaft surface is $F = \frac{2T}{d}$.

For shear strength: $F = \tau_{allow} \times A_s$

$$\frac{2T}{d} = \tau_{allow} \times b \times L$$

$$L_{shear} = \frac{2T}{d \times \tau_{allow} \times b} = \frac{2 \times 397{,}890}{45 \times 38.33 \times 14}$$

$$L_{shear} = \frac{795{,}780}{24{,}147.9} \approx 32.95 \text{ mm}$$

Step 4: Calculate Key Length based on Crushing Strength

The crushing area for a flat key is $A_c = \frac{h}{2} \times L$.

For crushing strength: $F = \sigma_{c\_allow} \times A_c$

$$\frac{2T}{d} = \sigma_{c\_allow} \times \frac{h}{2} \times L$$

$$L_{crus} = \frac{2T}{d \times \sigma_{c\_allow} \times \frac{h}{2}} = \frac{4T}{d \times \sigma_{c\_allow} \times h}$$

$$L_{crus} = \frac{4 \times 397{,}890}{45 \times 76.67 \times 9} = \frac{1{,}591{,}560}{31{,}051.35} \approx 51.26 \text{ mm}$$

Step 5: Select the Governing Length

The key length must satisfy both shear and crushing criteria. Therefore, we select the larger length.

$$L = \max(L_{she} \quad , L_{crush}) = \max(32.95, 51.26) = 51.26 \text{ mm}$$

Adopting a standard length.

Final Answer for Problem 14

$$\boxed{L \approx 52 \text{ mm}}$$

15. A standard splined connection $8 \times 36 \times 40$ is used for a gear and shaft assembly rotating at 700 rpm. The dimensions of the splines are as follows:

Major diameter = 40 mm

Minor diameter = 36 mm

Number of splines = 8

The length of the gear hub is 50 mm and the normal pressure on the splines is limited to 6.5 N/mm^2. Calculate the power that can be transmitted from the gear to the shaft.

Answer:

Given:

- Splined connection specification: $8 \times 36 \times 40$
  - Number of splines, $n = 8$
  - Minor diameter, $d_m = 36$ mm
  - Major diameter, $d_M = 40$ mm
- Speed, $N = 700$ rpm
- Length of gear hub, $L = 50$ mm
- Permissible normal pressure, $p = 6.5$ N/mm^2

To Find: Power that can be transmitted from the gear to the shaft.

Step 1: Calculate the Mean Diameter of the Spline

$$d = \frac{d_M + d_m}{2} = \frac{40 + 36}{2} = 38 \text{ mm}$$

Step 2: Calculate the Total Torque Capacity based on Bearing Pressure

The torque transmitted by a splined connection is given by:

$$T = p \times A \times \frac{d}{2} \times n$$

Where:

- p = permissible normal pressure (N/mm^2)
- A = bearing area per spline = $h \times L$
- h = depth of engagement = $\frac{d_M - d_m}{2}$
- L = length of engagement
- n = number of splines

First, calculate the depth of engagement h:

$$h = \frac{d_M - d_m}{2} = \frac{40 - 36}{2} = 2\ \text{mm}$$

Now, calculate the bearing area per spline A:

$$A = h \times L = 2 \times 50 = 100\ \text{mm}^2$$

Now, calculate the total torque T:

$$T = p \times A \times \frac{d}{2} \times n = 6.5 \times 100 \times \frac{38}{2} \times 8$$

$$T = 6.5 \times 100 \times 19 \times 8 = 6.5 \times 15{,}200 = 98{,}800\ \text{N-mm} = 98.8\ \text{N-m}$$

Step 3: Calculate the Power Transmitted

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 700 \times 98.8}{60}$$

$$P = \frac{2 \times 3.1416 \times 700 \times 98.8}{60} = \frac{434{,}500}{60} \approx 7241.7\ \text{W}$$

Final Answer for Problem 15

$$\boxed{P \approx 7.24\ \text{kW}}$$

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