

Seat No./Enrollment No.: _____

GUJARAT TECHNOLOGICAL UNIVERSITY
BACHELOR OF ENGINEERING – SEMESTER 4 – EXAMINATION – SUMMER 2026

Subject Code: 3140610

Date: 16/06/2026

Subject Name: Complex Variables and Partial Differential Equations

Time: 10:30 AM TO 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) Find the complete integral of $q = pq + p^2$ 03
- (b) State 1) Liouville's Theorem 2) Maximum-modulus theorem 04
- (c) Define Harmonic Function. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate of $u(x, y)$ 07
- Q.2** (a) Define Residue at simple pole. Discuss $\text{Res}[f(z), 0]$ where $f(z) = \frac{1 - e^z}{z^3}$ 03
- (b) Solve $(D^3 - 4D^2D' + 4DD'^2)z = 0$ 04
- (c) State Cauchy's Residue theorem. Evaluate $\int_c \frac{5z - 2}{z(z - 1)} dz, c: |z| = 2$ by using Cauchy's Residue theorem 07
- Q.2** (c) Define Laurent's Series. Expand $f(z) = \frac{\text{OR } 1}{(z + 1)(z + 3)}$ as a Laurent's series in regions 07
i) $|z| < 1$, ii) $|z| > 3$
- Q.3** (a) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{z^n}{2^n + 1}$ 03
- (b) Describe how to construct the Mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively 04
- (c) Evaluate $\int_c \frac{4 - 3z}{z(z - 1)(z - 2)} dz$ where c is the circle $|z| = \frac{3}{2}$ 07
- OR
- Q.3** (a) Evaluate $\int_c \frac{z}{z - 3} dz$ where c is the unit circle $|z| = 1$ 03
- (b) Describe how to construct the Mobius transformation that maps $z_1 = -1, z_2 = 0, z_3 = 1$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively 04
- (c) Evaluate $\int_c \frac{z^2 e^z}{(z - 1)^3} dz$ where $c: |z| = 2$ 07

- Q.4** (a) Show that $f(z) = \bar{z}$ is not analytic **03**
- (b) Find the fifth root of unity **04**
- (c) Solve $x^2p + y^2q = (x + y)z$ **07**
- OR**
- Q.4** (a) Show that $f(z) = 3x + y + i(3y - x)$ is analytic **03**
- (b) Evaluate $(1 + i\sqrt{3})^{100} + (1 - i\sqrt{3})^{100}$ **04**
- (c) Solve $zpq = p + q$ by using Charpit's method **07**
- Q.5** (a) Express the complex number $z = \sqrt{3} + i$ in polar form **03**
- (b) Find the particular integral of $(D^3 - 3D^2D' + 4D'^3)z = e^{x+2y}$ **04**
- (c) Using method of separation of variables, solve $u_x + u_y = 2(x + y)u$ **07**
- OR**
- Q.5** (a) Find the principal argument of $z = \frac{-2}{1 + i\sqrt{3}}$ **03**
- (b) Find the particular integral of $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$ **04**
- (c) Using method of separation of variables, solve $u_x = 4u_y, u(0, y) = 8e^{-3y}$ **07**
