Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2024

Subject Code:3130908 **Date:16-07-2024**

Subject Name: Applied Mathematics for Electrical Engineering

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Marks

Q.1 **(a)** Write Newton's iteration formula for finding square root of a positive number N. Hence obtain the approximate value of $\sqrt{8}$ correct upto four decimal places. **03**

(b) Using Gaussian three points quadrature formula, evaluate $\int_{-2}^{2} e^{-\frac{x}{2}}\, dx$. **04**

(c) Write any two differences between Bisection method and Newton-Raphson method. Using Bisection method, find an approximate root of equation $x - \cos x = 0$ taking $a = 0.5$ and $b = 1$ correct upto three decimal places. **07**

Q.2 **(a)** Evaluate $\int_{0}^{6} \frac{1}{1+x}\, dx$ using the Simpson's $\frac{1}{3}$ rule taking $h = 1$. **03**

(b) The table below gives the x and y. If $y = a + bx$ then find the values of a and b by the least squares method. **04**

x	1	2	3	4	5
y	3	4	5	6	8

(c) Apply Runge−Kutta fourth order method to calculate approximate values of $y(0.1)$ and $y(0.2)$ for the differential equation $\frac{dy}{dx} = x + y\,;\, y(0) = 1$ (Take h = 0.1). **07**

OR

(c) Using Euler's method find an approximate value of $y(1.5)$ for the differential equation $\frac{dy}{dx} = xy\ ;\ y(1) = 5$ (Take h = 0.1). **07**

Q.3 **(a)** Write normal equations to fit a parabola $y = ax^2 + bx + c$ by the least squares method. **03**

(b) Define (i) Shift operator (E) (ii) Central difference operator (δ) (iii) Forward difference operator (Δ). Hence prove that $\delta = \Delta E^{-\frac{1}{2}}$. **04**

(c) State Newton's divided difference interpolation formula. Apply it to find an approximate value of $y(2.75)$ using the following data. **07**

x	2.5	3.0	4.5	4.75	6.0	7.0
$y(x)$	8.85	11.45	20.66	22.85	38.6	55.6

OR

Q.3 **(a)** Write normal equations to fit the curve $y = ae^{bx}$ by the least squares method. **03**

(b) Use the following data to obtain approximate value of $\sin 52^0$ using Newton's forward interpolation formula: **04**

x (in degree)	45	50	55	60
$\sin x^0$	0.7071	0.7660	0.8192	0.8660

	(c)	Given $f(x) = 1/x$ and nodes $x_0 = 2, x_1 = 2.75, x_2 = 4$. Using Lagrange's interpolation formula, fit a second degree polynomial to approximate the given function $f(x)$ using Lagrange's interpolation. Hence obtain the approximate value of $f(3)$ using it.	07

Q.4 **(a)** A box contains 8 items out of which 2 are defective. Let X denote the number of defective items. If a person selects 3 items from the box at random, find the expected number of defective items he has drawn. **03**

(b) A discrete random variable *X* has the following probability distribution: **04**

X	*0*	*1*	*2*	*3*	*4*	*5*
P(X)	*0*	*k*	*0.2*	*2k*	*0.3*	*2k*

Find the value of *k* and the compute *P(X<5).*

(c) In each of 4 races, the players have a 60 % chance of winning. Assuming that the races are independent of each other and using Binomial distribution, find the probability that **07**

(i) The player will win 0 race, 1 race, 2 races, 3 races or all 4 races.
(ii) The player will win at least 1 race.
(iii) The player will win majority of the races.

OR

Q.4 **(a)** Define a Discrete probability density function. Write its properties. **03**

(b) The probability density function of a continuous random variable X is **04**

$$f(x) = \begin{cases} \dfrac{2}{x^3} \; ; \; 1 < x < \infty \\ 0 \; ; \; elsewhere \end{cases}$$

Find the cumulative distribution function (cdf) *F(x).*

(c) Let *X* be a continuous random variable with the probability density function (pdf) $f(x) = kx(1 - x), \; 0 \le X \le 1$ **07**

Find *k* and a number *b* such that $P(X < b) = P(X \ge b)$

Q.5 **(a)** A random variable X has the following distribution: **03**

X=*x*	0	1	2	3	4
P(x)	*0.6561*	*0.2916*	*0.0486*	*0.0036*	*0.0001*

Find mean, variance and standard deviation.

(b) A continuous random variable *X* is distributed over the interval [1, 3] with the probability distribution function (pdf) $f(x) = 4k(x-1)^4$, where k is a constants. Find the value of *k* and mean. **04**

(c) Define Kurtosis. The following data is given for analysis: **07**

$N = 100, \Sigma fd = 50, \Sigma fd^2 = 1970, \Sigma fd^3 = 2948, \Sigma fd^4 = 86752$ in which $d = x - 48$. Do you think that the distribution is platykurtic? Justify your answer.

OR

Q.5 **(a)** A random variable X with unknown probability distribution has a mean 8 and standard deviation 3. Find the bound of $P\{\,|X-8| \ge 6\,\}$ using Chebyshev's inequality. **03**

(b) Compute the coefficient of skewness from the data: **04**

25, 15, 23, 40, 27, 25, 23, 25, 20

(c) Calculate the first four moments about the mean of the following data. **07**

X=*x*	0	1	2	3	4	5	6	7	8
P(x)	1	8	28	56	70	56	28	8	1

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