Seat No./Enrollment No.: ________

GUJARAT TECHNOLOGICAL UNIVERSITY
BACHELOR OF ENGINEERING – SEMESTER 1 – EXAMINATION – SUMMER 2026

Subject Code: 3110014 **Date: 13/07/2026**

Subject Name: Mathematics - 1

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Evaluate $\lim_{x\to 0} \frac{x-\sin x}{x^3}$ **03**

(b) Discuss the convergence of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots \ldots \ldots \ldots \ldots$ If convergent then find it's sum. **04**

(c) (i) Check the convergence of $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$. **07**

(ii) Find the volume of the solid of revolution of the area bounded by the curve $y = xe^x$ and lines $x = 0 \; to \; x = 1$.

Q.2 **(a)** State Sandwich theorem for the sequences and show that $\lim_{n\to\infty} \frac{\sin^2 n}{n} = 0$ **03**

(b) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{5^n+6^n}{7^n}$ **04**

(c) Find Fourier series of $f(x) = \frac{(\pi-x)}{2}$ in the interval $(0,2\pi)$. **07**

Hence deduce that $\frac{\pi}{4} = 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$..

OR

Q.2 **(c)** Find the Fourier series of $f(x) = x^2$ in the interval $-\pi \le x \le \pi$. **07**

Hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots \ldots . = \frac{\pi^2}{12}$

Q.3 **(a)** If $u = \log(x^2+y^2)$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **03**

(b) Find the rate of change of xyz in the direction normal to the surface $x^2y+xy^2+yz^2 = 3$ at point P (1,1,1). **04**

(c) If $\frac{4}{x}+\frac{9}{y}+\frac{16}{z} = 25$ then by Lagrange's method find the values of x, y and z which makes $x+y+z$ maximum. **07**

OR

Q.3 **(a)** If $z = e^{xy}, x = t\cos t, \; y = t\sin t$ then find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$. **03**

(b) Discuss the maxima and minima of the function $3x^2 - y^2 + x^3$. **04**

(c) (i) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ then show that $u_x + u_y + u_z = 0$. **07**

(ii) Find the equations of the tangent plane and the normal line to the surface $x^2y^2 + xz - 2y^3 = 10$ at (2,1,4).

Q.4 **(a)** Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$ **03**

(b) Evaluate the integral $\int_0^1 \int_0^{2x} e^{x^2} dydx$ by changing the order of Integration. **04**

(c) Evaluate $\iint_R xydA$ where R is the region bounded by x-axis, x=2a and the curve $x^2 = 4ay$. **07**

OR

Q.4 **(a)** Evaluate $\iint r^3 sin2\theta dr d\theta$ over the area bounded in the first quadrant between two circles $r = 2$ and $r = 4$. **03**

(b) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by transforming into Polar coordinates. **04**

(c) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\ dzdydx$. **07**

Q.5 **(a)** Find the rank of the matrix A by reducing it to row echelon form **03**

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

(b) State Cauchy's Integral Test and discuss the convergence of $\sum_{n=1}^{\infty} \frac{1}{n(1+(logn)^2)}$ **04**

(c) Solve the following system of equations using Gauss Jordan Method **07**

$x + 2y - z = 1, x + y + 2z = 9, 2x + y - z = 2$.

OR

Q.5 **(a)** Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **03**

(b) Expand $e^x sinhx$ in powers of x upto x^4. **04**

(c) Find the eigen values and eigen vectors for the matrix **07**

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

*****