Tips & Tricks: Ratio & Proportion

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Introduction

Ratio is a comparison of two quantities by division. It shows how many times one quantity contains another and is written as **a:b** or as the fraction $\frac{a}{b}$. In the ratio **a:b**, **antecedent** denotes **a** and **consequent** denotes **b**. A ratio is dimensionless (it has no units). A ratio remains unchanged if both terms are multiplied or divided by the same non-zero number; for example, $\frac{4}{3} = \frac{40}{30}$.

Proportion is a statement that two ratios are equal. If $\frac{a}{b} = \frac{c}{d}$, then the four numbers **a, b, c, d** are said to be in proportion and we write **a:b :: c:d**. In a proportion the outer terms (**a** and **d**) are called the **extremes** and the inner terms (**b** and **c**) are called the **means**. The product of the means equals the product of the extremes: $bc = ad$.

Theory

Ratio expresses how one quantity relates to another of the same kind. It can be seen as the factor by which one quantity is larger or smaller than another. Ratios are often written as **m:n** or as the fraction $\frac{m}{n}$. The first term is the **antecedent** and the second term is the **consequent**. When the antecedent is greater than the consequent the ratio is sometimes informally called an **improper ratio**, and when the antecedent is less it is called a **proper ratio**.

Different Types of Ratios

Duplicate ratio: the duplicate ratio of **a:b** is $a^2 : b^2$.

Triplicate ratio: the triplicate ratio of **a:b** is $a^3 : b^3$.

Sub-duplicate ratio: the sub-duplicate ratio of **a:b** is $\sqrt{a}$: $\sqrt{b}$.

Sub-triplicate ratio: the sub-triplicate ratio of **a:b** is $\sqrt[3]{a}$: $\sqrt[3]{b}$.

Compound ratio: the compound ratio of two ratios **a:c** and **b:d** is the ratio of the products of antecedents and consequents, i.e. ab: cd. For several ratios multiply all antecedents and multiply all consequents.

Inverse ratio: the inverse ratio of **a:b** is $\frac{1}{a}$: $\frac{1}{b}$; inverse ratios are used when quantities vary inversely (for example, speed and time for a fixed distance).

Componendo: from $\frac{a}{b} = \frac{c}{d}$ we get $\frac{a+b}{b} = \frac{c+d}{d}$.

Dividendo: from $\frac{a}{b} = \frac{c}{d}$ we get $\frac{a-b}{b} = \frac{c-d}{d}$.

Componendo-Dividendo: from $\frac{a}{b} = \frac{c}{d}$ we get $\frac{a+b}{a-b} = \frac{c+d}{c-d}$, provided denominators are non-zero.

Different Types of Proportion

Continued proportion: if three quantities **a, b, c** satisfy **a:b :: b:c**, then $b^2 = ac$. Here **b** is the second (or mean) proportional between **a** and **c**.

Fourth proportion: if **a, b, c** and **x** satisfy **a:b :: c:x**, then **ax = bc** and **x = \dfrac{bc}{a}**. The value **x** is the fourth proportional to **a, b, c**.

Mean (second) proportion: if **a:x :: x:b** then $x^2 = ab$ and **x = \sqrt{ab}**; **x** is the geometric mean of **a** and **b**.

- **Invertendo**: from **a:b :: c:d** we obtain **b:a :: d:c**.
- **Alternendo**: from **a:b :: c:d** we obtain **a:c :: b:d**.
- **Componendo**: from **a:b :: c:d** we obtain **(a+b):b :: (c+d):d**.
- **Dividendo**: from **a:b :: c:d** we obtain **(a-b):b :: (c-d):d**.
- **Componendo-Dividendo**: from **a:b :: c:d** we obtain $\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$, when the differences are non-zero.

Exam-Oriented Tips & Tricks (Quick Methods and Shortcuts)

- **Cross-multiplication** is the fastest way to test or solve a proportion: if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$. Use this to find the unknown term directly.
- **Reduce first**: before multiplying, cancel common factors to keep numbers small and avoid arithmetic mistakes.
- **Scale ratios** to convenient numbers when splitting or combining quantities; always keep ratios in simplest form unless a specific scale is required.
- **Alligation (mixture problems)**: use the difference method to find parts of mixtures quickly. This is especially useful for problems involving concentrations, prices or mix proportions (common in competitive exams and engineering mix design).
- **Use equivalent fractions** to compare ratios-convert to the same denominator or same antecedent for quick comparison.

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Programming note: when implementing ratio calculations, ensure you use floating-point division where required (avoid integer division that truncates). Check for zero denominators and normalise ratios if you must compare them.

Civil engineering note: ratios are used in concrete/cement mortar mix design (for example, 1:2:4 is cement : sand : aggregate). When designing, convert ratios to parts by volume or mass consistently and scale to required batch sizes.

Mental shortcuts: recognise common proportional patterns (e.g., direct proportionality, inverse proportionality, doubling/halving). For small numbers, use simple factor pairs to rapid-fire answers.

Worked Short Examples (Method shown step-wise)

Example 1. If $\frac{3}{4} = \frac{x}{28}$, find **x**.

Multiply both sides by the denominators to use cross-multiplication.

$3 \times 28 = 4 \times x$

$84 = 4x$

$x = \frac{84}{4} = 21$

Example 2. Two quantities are in the ratio **5:7**. If their sum is **240**, find the quantities.

Let the quantities be **5k** and **7k**.

Their sum is **5k + 7k = 12k = 240**.

Therefore **k = 240/12 = 20**.

The quantities are **5k = 100** and **7k = 140**.

Solved Examples

MULTIPLE CHOICE QUESTION

Try yourself: If x:y = 4:7, the value of (x + y) /(x – y) = ?

A -33/65

B 65/33

C 33/65

D -11/3

View Solution

MULTIPLE CHOICE QUESTION

Try yourself: **Two numbers are in the ratio of 5:8. If 4 is subtracted from each then the ratio become 1:2. The small number is**

A 16

B 4

C 10

D 2

View Solution

MULTIPLE CHOICE QUESTION

Try yourself: x:y = 7:4 then (4x+7y) : (7x+4y)

A 65:56

B 56:65

C 34:45

D 13:18

View Solution