Important Formulas & Tips: Percentages

Table of Contents

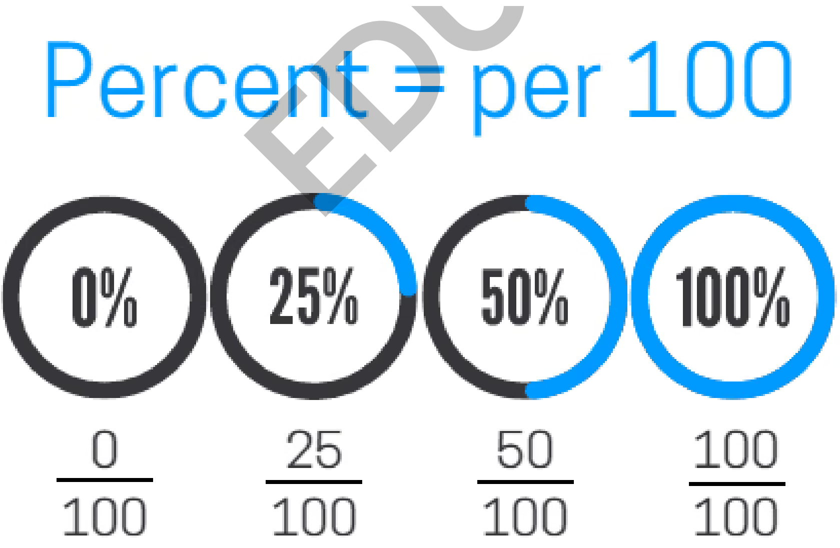
1. What are Percentages?
2. How to Calculate a Percentage
3. Concept of Percentage Change
4. Percentage Point Change vs. Percentage Change
5. Increase / Decrease in Percentage

View More

Percentages are a cornerstone of the Quantitative Ability (QA) section in many competitive exams, appearing in problems related to **profit and loss**, **discounts**, **population growth**, and **data interpretation**. Mastering percentage concepts enables students to solve questions quickly and accurately, a critical skill given the exam's time constraints. This document provides essential **formulas**, **tips**, and **shortcuts** for percentage calculations, including conversions, percentage changes, and successive changes.

What are Percentages?

- The term "per cent" comes from the Latin word *centum*, meaning "per hundred."
- A percentage is the representation of a number, ratio, fraction, or decimal as a **fraction of 100**.
- To calculate the percentage of a number, divide the part by the whole and multiply by 100; in formula form: $\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\%$.



How to Calculate a Percentage

1. Number: To represent a number N as a percentage, multiply the number by 100.

Example: Convert '4' into a percentage.

Sol:

$4 \times 100 = 400\%$

2. Fraction: To represent a fraction as a percentage, multiply the fraction by 100. To convert a percentage into a fraction divide the percentage by 100.

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\%$$

Example: Convert the fraction $\frac{3}{5}$ into a percentage.

Sol:

$\frac{3}{5} \times 100\% = 60\%$

3. Ratio: Convert the ratio into a fraction and then multiply by 100.

Example: Convert the ratio 2 : 5 into a percentage.

Sol:

$\frac{2}{5} \times 100\% = 40\%$

4. Decimal: To represent a decimal as a percentage, multiply the decimal by 100; to convert back divide by 100.

Example: Convert 0.773 into a percentage.

Sol:

$0.773 \times 100\% = 77.3\%$

Concept of Percentage Change

Whenever the value of a measured quantity changes, the change can be described using absolute change or percentage change.

- **Absolute change** is the simple difference between the new and the old value. Example: sales increase from ₹5000 crore to ₹6000 crore ⇒ absolute change = ₹1000 crore.
- **Percentage change** compares the absolute change to a chosen base (usually the original value). The formula is $\frac{\text{Absolute Change}}{\text{Base}} \times 100\%$.

$$\text{Percentage Change} = \left(\frac{\text{New Value} - \text{Old Value}}{\text{Old Value}}\right) \times 100$$

$\frac{1000}{5000} \times 100\% = 20\%$

- **Percentage change of A from B** (how much more is A relative to B): $\frac{A - B}{B} \times 100\%$
- **Percentage change from A to B**: $\frac{B - A}{A} \times 100\%$

Example: The population grew from 20 lakhs to 22 lakhs. Find (a) percentage change (base original) and (b) percentage change using the final value as base.

Sol:

$\text{Absolute Change} = 22 - 20 = 2$ lakhs.

$\text{(a) Percentage change (base original)} = \frac{2}{20} \times 100\% = 10\%$

$\text{(b) Percentage change (base final)} = \frac{2}{22} \times 100\% \approx 9.09\%$

Percentage Point Change vs. Percentage Change

- **Percentage point change** measures the absolute difference between two percentages. Example: 25% to 30% is a 5 percentage point change.

Percentage change measures relative change against the original percentage. Using the example above, the percentage change is $\frac{5}{25} \times 100\% = 20\%$.

Example: Savings rate was 25% first year and 30% second year; GDP unchanged. Calculate percentage point change and percentage change.

Sol:

Percentage point change = $30\% - 25\% = 5$ percentage points.

Percentage change = $\frac{5}{25} \times 100\% = 20\%$

Increase / Decrease in Percentage

Percentage Increase

When comparing the increase in a quantity over time, compute the difference and divide by the original value: $\frac{\text{New}-\text{Original}}{\text{Original}} \times 100\%$.

$$\text{Percentage Increase} = \left(\frac{\text{Increased Value} - \text{Original Value}}{\text{Original Value}}\right) \times 100$$

- When A is increased by $x\%$, the new value B becomes:

$$B = A \times \left(1 + \frac{x}{100}\right)$$

- To revert B back to A, we would need to reduce B by:

$$\frac{x}{100 + x} \times 100\%$$

Percentage Decrease

When a quantity decreases, compute the difference and divide by the original value: $\frac{\text{Original}-\text{New}}{\text{Original}} \times 100\%$.

$$\text{Percentage Decrease} = \left(\frac{\text{Original Value} - \text{Decreased Value}}{\text{Original Value}}\right) \times 100$$

- When A is decreased by $y\%$, the new value B becomes:

$$B = A \times \left(1 - \frac{y}{100}\right)$$

- To revert B back to A, we would need to increase B by:

$$\frac{y}{100 - y} \times 100\%$$

MULTIPLE CHOICE QUESTION

Try yourself: What is the percentage increase if the price of a product increased from $50 to $70?

A 20%

B 40%

C 30%

D 50%

View Solution

Solved Examples

Example 1: Raghu has 20% more toffees than Pinki. How much percentage less toffees does Pinki have compared to Raghu?

Sol:

Raghu has 20% more than Pinki ⇒ if Pinki has 100, Raghu has 120.

Percentage less for Pinki relative to Raghu = $\frac{120 - 100}{120} \times 100\% = \frac{20}{120} \times 100\% = 16.\bar{6}\%.$

$$\implies \left(\frac{x}{100 + x}\right) \times 100\%$$

$$\implies \left(\frac{20}{100 + 20}\right) \times 100\% = 16.66\%$$

Note: When a quantity is "increased/decreased by X%" the multiplier becomes $1 \pm \frac{X}{100}$. When a quantity is "increased/decreased to X%" the final value is $\frac{X}{100}$ of original.

Example 2: A TV cost \$100 last year but now costs \$125. Find the price increase.

Sol:

New price - Old price = $125 - 100 = 25$.

Divide by old price: $\frac{25}{100} = 0.25$.

As percentage: 0.25 \times 100\% = 25\%\).

Example 3: Salary increased from Rs 18,00,000 to Rs 22,00,000. Find percentage increase.

Sol:

Increase = $22{,}00{,}000 - 18{,}00{,}000 = 4{,}00{,}000$.

Percentage increase = $\frac{4{,}00{,}000}{18{,}00{,}000} \times 100\% = 22.22\%$ (approx.).

Example 4: A TV cost \$100 last year but now costs \$75. Determine the price decrease.

Sol:

Decrease = $100 - 75 = 25$.

Percentage decrease = $\frac{25}{100} \times 100\% = 25\%$.

Concept of Product Constancy (Inverse Proportionality)

Product constancy means keeping a product of two quantities constant (inverse proportionality). If price × consumption = expenditure and the expenditure is fixed, an increase in price must be offset by decrease in consumption.

Common relations: $\text{Expenditure} = \text{Price} \times \text{Consumption}, \text{Distance} = \text{Speed} \times \text{Time}, \text{Work} = \text{Efficiency} \times \text{Time}, \text{Area} = \text{Length} \times \text{Breadth}.$

Solved Examples

Example 1: Price of sugar increases by 25%. To keep expenditure constant, by what percent should consumption decrease?

Sol:

Let original price be P and original consumption be C; expenditure $E = PC$.

New price = $P \times (1 + \frac{25}{100}) = 1.25P$.

To keep E constant, new consumption C' must satisfy $1.25P \times C' = PC$.

So $C' = \frac{C}{1.25} = 0.8C$.

Consumption decrease = $\frac{C - 0.8C}{C} \times 100\% = 20\%$.

$$\implies \left(\frac{x}{100 + x}\right) \times 100\%$$

$$\implies \left(\frac{25}{100 + 25}\right) \times 100 = \frac{1}{5} * 100 = 20\%$$

Example 2: A usually does a job. B takes 6 hours more than A. C is 1.5 times more efficient than B and finishes the job 4 hours earlier than A. Find the usual time taken by A.

Sol:

Let B's efficiency = x units/hour, so time B takes = T_B. Then A's time $T_A = T_B - 6$.

C's efficiency = $1.5x$. Time C takes = $T_C = \frac{T_B}{1.5} = \frac{2}{3}T_B$.

Given $T_C = T_A - 4$. Substitute $T_A = T_B - 6$:

$\frac{2}{3}T_B = T_B - 6 - 4 = T_B - 10$.

Rearrange: $T_B - \frac{2}{3}T_B = 10 \Rightarrow \frac{1}{3}T_B = 10 \Rightarrow T_B = 30$ hours.

Thus $T_A = 30 - 6 = 24$ hours.

Relationship C = A + B and Maintaining C Constant

If $C = A + B$ and A increases by $x\%$, to keep C constant, B must decrease accordingly so that the sum remains unchanged.

$$= (x \; of \; \frac{A}{B})\%$$

Example: A person's saving is 20% of income. If expenditure increases by 10% with income unchanged, find the percentage decrease in savings.

Sol:

Assume income = 100. Initial saving = 20, expenditure = 80.

Final expenditure = $80 \times 1.10 = 88$.

Final saving = $100 - 88 = 12$.

Savings decreased from 20 to 12 ⇒ decrease = 8 on base 20 ⇒ $\frac{8}{20} \times 100\% = 40\%$.

$$Percentage\ decrease\ in\ savings = \frac{(Initial\ saving - Final\ saving)}{(Initial\ saving)} * 100$$

$$\implies \frac{20-12}{20} * 100 = \frac{8}{20} * 100 = 40\%\ decrease$$

Successive Percentage Increase / Decrease

When two or more percentage changes are applied consecutively, each change applies to the result of the previous change. The net change is multiplicative, not additive.

1. Successive Increment

If an initial value x is increased by $a\%$, then by $b\%$, final value = $x(1+\frac{a}{100})(1+\frac{b}{100})$.

Net percent increase = $[(1+\frac{a}{100})(1+\frac{b}{100})-1] \times 100\%$.

Sol (generic): After three successive increases $a\%, b\%, c\%$, final value = $x(1+\frac{a}{100})(1+\frac{b}{100})(1+\frac{c}{100})$.

2. Successive Decrement

If a value x is decreased by $a\%$ then by $b\%$, final value = $x(1-\frac{a}{100})(1-\frac{b}{100})$.

Sol (generic): For successive decrements $a\%, b\%, c\%$: final value = $x(1-\frac{a}{100})(1-\frac{b}{100})(1-\frac{c}{100})$.

3. Mixed Successive Changes

If changes include both increases and decreases, use the same multiplicative rule with plus or minus signs accordingly: final value = $x\prod_i (1 \pm \frac{p_i}{100})$, where each p_i is the percentage for that step.

4. Application to Population Growth

If population grows at rate $r\%$ per year, population after n years = $P(1+\frac{r}{100})^n$.

If population n years ago was P_0, the present population after growth rate $r\%$ per year is $P_0\,(1+\frac{r}{100})^n$. Conversely, population n years ago given present population P is $P(1+\frac{r}{100})^{-n}$.

$$= P\left(1 \pm \frac{r}{100}\right)^n$$

$$= \frac{P}{\left(1 \pm \frac{r}{100}\right)^n}$$

MULTIPLE CHOICE QUESTION

Try yourself: A TV originally cost $500, but now it costs $400. What is the percentage decrease in the price of the TV?

A 10%

B 20%

C 30%

D 15%

View Solution

Percentage Change Graphic (PCG) and Its Applications

The Percentage Change Graphic (PCG) is a visual / structured method to track successive percentage changes and avoid errors. For example, increase 20 by 20% can be tracked using PCG.

$$20 \xrightarrow[=+4]{20\% \uparrow} 24$$

> **EduRev Tip:** Sketch PCG on rough paper for complex successive change problems to avoid errors.

The PCG is especially useful in the following six applications.

1. Successive Changes

Example 1: Start with 30, increase by 20% then by 10% and find final value.

> **Sol:**
>
> After first increase: $30 \times 1.20 = 36$.
>
> After second increase: $36 \times 1.10 = 39.6$.
>
> $$30 \xrightarrow[+6]{20\% \text{ increase}} 36 \xrightarrow[+3.6]{10\% \text{ increase}} 39.6$$

Example 2: Salary increases by 20% then decreases by 20%. Net change?

> **Sol:**
>
> Multiplier = $1.2 \times 0.8 = 0.96$.
>
> Net change = $0.96 - 1 = -0.04$ ⇒ -4%.
>
> $$100 \xrightarrow[+20]{20\% \text{ increase}} 120 \xrightarrow[-24]{20\% \text{ decrease}} 96$$
>
> Hence the salary falls by 4% overall.

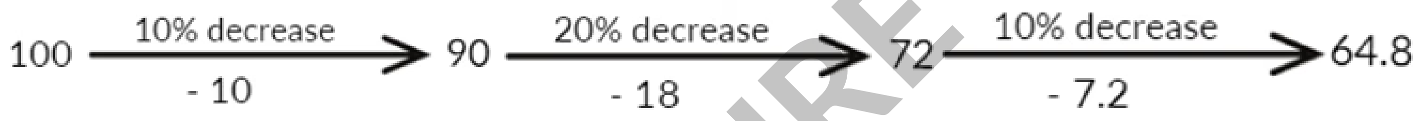
Example 3: Trader marks up by 20% then gives 10% discount on marked price. Net profit percent?

Sol:

Marked price multiplier = 1.20.

After 10% discount multiplier = $1.20 \times 0.90 = 1.08$.

Net gain = 8%.

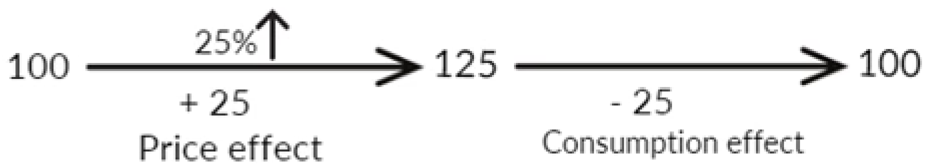


2. Product Change Application

If two multiplicative components change by $A\%$ and $B\%$, percentage change in product ≈ $A + B + \frac{AB}{100}$ (exact formula via multiplicative factors).
Example: one part increases 10%, another increases 20% ⇒ product change = $10 + 20 + \frac{10 \times 20}{100} = 32\%$.

3. Product Constancy Application

If price increases by 25% and expenditure must remain the same, reduce consumption by 20% (see earlier worked example).

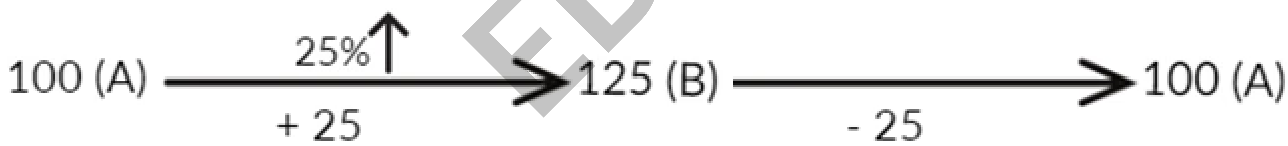


Hence consumption must drop by 20% to offset a 25% price rise.

4. A → B → A (Reverse Relationship)

If B's salary is 25% more than A's (B = 125% of A), then A is less than B by $\frac{25}{125} \times 100\% = 20\%$. So A's salary is 20% less than B's.

Sol:



A drop of 25 on 125 gives a 20% drop.

5. Denominator Change → Ratio Change

The denominator inversely affects a ratio. Example: ratio changes from 10/20 to 10/25. Denominator increases 25%, ratio falls from 0.5 to 0.4 ⇒ percentage drop =
$\frac{0.1}{0.5} \times 100\% = 20\%.$
PCG can produce the same result by treating denominator change in reverse (from 25 to 20 gives a 20% effect on ratio value).

$$100 \longrightarrow 125 \longrightarrow 100$$

6. PCG for Changes in Both Numerator and Denominator

When both numerator and denominator change, split the effect into **numerator effect** (direct) and **denominator effect** (inverse) and combine.

Example: Ratio 1 = 10/20 → 0.5. Ratio 2 = 15/25 → 0.6. Numerator increases 50% and denominator increases 25% (which acts as a 20% decrease on the ratio). Net effect ≈ 50% - 20% + (interaction term) ≈ 20% increase (exact calculation yields 20%).

Did You Know

If numerator ↑ and denominator ↓ ⇒ ratio ↑.

If numerator ↓ and denominator ↑ ⇒ ratio ↓.

Solved Examples

Q 1. If 5/9 is multiplied instead of 2/3 in a number, then what will be the percentage error in the calculation?

Sol:

Let a number be a common multiple to illustrate; calculations proceed by comparing the two multipliers: $\frac{5}{9}$ vs. $\frac{2}{3} = \frac{6}{9}$. The relative error =
$\frac{6/9 - 5/9}{6/9} \times 100\% = \frac{1/9}{6/9} \times 100\% = \frac{1}{6} \times 100\% = 16.\bar{6}\%$.

$$Now,\ Error\ number = \frac{5}{9} * 9 = 5$$

$$Correct\ number = \frac{2}{3} * 9 = 6$$

$$Percentage\ error = \frac{Difference\ between\ correct\ and\ error\ no.}{Correct\ no.} * 100$$

$$\implies \frac{6-5}{6} * 100 = 16.66\%$$

Q 2. In an examination, out of 400 students, 54% of boys and 70% of girls pass. If the total pass percentage was 60%, find the total number of girls.

Sol:

Let number of girls = x. Then number of boys = $400 - x$.

Number passed = $0.54(400 - x) + 0.70x = 0.60 \times 400$.

Compute: $216 - 0.54x + 0.70x = 240$.

Solve: $0.16x = 24 \Rightarrow x = 150$.

Q 3. Population of a town increases by 20% each year. If population 3 years ago was 2500, what is present population?

Sol:

Population after 3 years with annual growth 20%:

$P = 2500 \times (1 + \frac{20}{100})^3 = 2500 \times (1.2)^3$.

Compute: $1.2^3 = 1.728$, so $P = 2500 \times 1.728 = 4320$.

Q 4. If A earns 20% more than B, B earns 25% more than C, C earns 16.67% less than D, then A earns how much percent of D?

Sol:

Let D = 100. Then C = 100 × (1 - 16.67%) = 100 × (1 - 1/6) = 100 × 5/6 = 83.333...

B = C × 1.25 = $83.333\ldots \times 1.25 = 104.1666\ldots.$

A = B × 1.20 = $104.1666\ldots \times 1.20 = 125.$

Thus A is 125% of D (i.e., A = 125 when D = 100). The detailed image steps are shown below.

$20\% = \frac{1}{5}, 25\% = \frac{1}{4},\ and\ 16.67\% = 16$

$A = \frac{6}{5}B$

$B = \frac{5}{4}C$

$C = \frac{5}{6}D$

$A = \frac{6}{5} * \frac{5}{4} * \frac{5}{6}D$

$\implies A = \frac{5}{4}D$

We know, $\frac{5}{4} = 125\%$

So, we can say that A is 125% of D.

Q 5. A 20% ethanol solution is mixed with another ethanol solution, say, S of unknown concentration in the proportion 1:3 by volume. This mixture is then mixed with an equal volume of 20% ethanol solution. If the resultant mixture is a 31.25% ethanol solution, then the unknown concentration of S is

A) 30% B) 40% C) 50% D) 60%

Ans. Option 'c' is correct.

Sol:

Let volumes of first solution (20%) and second solution S be 100 and 300 respectively (ratio 1:3). Ethanol in their mixture = $20 + 300S$ (percent-equivalent units).

When mixed with an equal volume (400) of 20% solution, total volume = 800 and ethanol quantity = $20 + 300S + 80 = 300S + 100$.

Given final strength 31.25% of 800 = 250. So 300S + 100 = 250.

Solve: 300S = 150 \Rightarrow S = \dfrac{150}{300} = \dfrac{1}{2} = 50\%\).

Hence answer = 50% (Option C).

Q 6. The salaries of Ramesh, Ganesh and Rajesh were in the ratio 6:5:7 in 2010, and in the ratio 3:4:3 in 2015. If Ramesh's salary increased by 25% during 2010-2015, then the percentage increase in Rajesh's salary during this period is closest to

A) 10 B) 7 C) 9 D) 8

Ans: Option 'b' is correct.

Sol:

Let 2010 salaries be $6x, 5x, 7x$ for Ramesh, Ganesh, Rajesh respectively.

Let 2015 salaries be $3y, 4y, 3y$ respectively (given ratios).

Ramesh's salary increased by 25%: $3y = 1.25 \times 6x$.

Solve for y: $3y = 7.5x \Rightarrow y = 2.5x$.

Rajesh's salary in 2010 = $7x$. In 2015 = $3y = 3 \times 2.5x = 7.5x$.

Percentage increase = $\dfrac{7.5x - 7x}{7x} \times 100\% = \dfrac{0.5x}{7x} \times 100\% \approx 7.14\%$.

Closest option = 7% (Option B).

MULTIPLE CHOICE QUESTION

Try yourself: **A shopkeeper increases the price of a product by 25% and then offers a discount of 10%. What is the net percentage change in the price of the product?**

A 10%

B 12.5%

C 13%

D 15%

View Solution

Quick Reference: Percentage Formulas

- **Basic Percentage:**

$$\text{Basic Percentage} = \left(\frac{\text{Part}}{\text{Whole}}\right) \times 100$$

- **Number to Percentage:**

$$\text{Number to Percentage} = N \times 100\%$$

- **Fraction to Percentage:**

$$\text{Fraction to Percentage} = \text{Fraction} \times 100$$

- **Ratio to Percentage:**

$$\text{Ratio to Percentage} = \frac{a}{a+b} \times 100 \quad \text{for ratio } a : b$$

- **Decimal to Percentage:**

$$\text{Decimal to Percentage} = \text{Decimal} \times 100$$

- **Percentage Change:**

$$\text{Percentage Change} = \frac{\text{New Value} - \text{Original Value}}{\text{Original Value}} \times 100$$

- **Percentage Increase:**

$$\text{New Value} = \text{Original} \times \left(1 + \frac{x}{100}\right)$$

- **Percentage Decrease:**

$$\text{New Value} = \text{Original} \times \left(1 - \frac{y}{100}\right)$$

- **Revert Increase:**

$$\text{Decrease by } \frac{x}{100+x} \times 100\%$$

- **Revert Decrease:**

$$\text{Increase by } \frac{y}{100-y} \times 100\%$$

- **Successive Change:**

$$\text{Final Value} = \text{Initial} \times \left(1 \pm \frac{a}{100}\right)\left(1 \pm \frac{b}{100}\right)\ldots$$

- **Product Constancy:**

$$\text{Decrease by } \frac{x}{100+x} \times 100\% \quad \text{to maintain constant product}$$

- **Population Growth:**

$$P \times \left(1 \pm \frac{r}{100}\right)^n \quad \text{(n years later)}$$

- **Population n Years Ago:**

$$\frac{P}{\left(1 \pm \frac{r}{100}\right)^n}$$

Common Mistakes in Percentage Problems

Confusing percentage with percentage points: Treating a 5 percentage point rise (25% → 30%) as a 5% relative change instead of $\frac{5}{25} \times 100\% = 20\%$.

Incorrect base value: Using the new value instead of the original as base for percentage change.

- **Adding successive changes:** Assuming $10\% + 10\% = 20\%$ rather than the multiplicative result $(1.1)^2 - 1 = 0.21 = 21\%$.
- **Misapplying product constancy:** Forgetting to use the correct inverse multiplier (e.g., use $\frac{x}{100+x}$ to find consumption decrease when price increases by $x\%$).
- **Rounding errors:** Rounding intermediate values early in successive-change problems can give wrong final answers.

Avoidance Tips:

- Always identify and fix the correct base before calculating percentage change.
- Use PCG (or write multipliers) for successive changes to track each step clearly.
- Double-check product constancy and population growth formulas before applying them.
- Perform calculations fully and round only at the final step when high precision (e.g., CAT-level) is required.

CAT Strategy Tips for Percentage Questions

- **Prioritise high-weightage topics:** Successive changes, product constancy, and ratio-change problems appear frequently in QA and Data Interpretation.
- **Use approximations:** For MCQs, rough estimation often eliminates wrong options quickly.
- **Practice DI sets:** Work on tables and charts from past papers to become comfortable with multi-step percentage reasoning.
- **Time management:** Aim to solve standard percentage questions in 1-2 minutes by using multipliers and PCG shortcuts.
- **Mock tests:** Take timed mocks to simulate exam pressure and refine shortcuts.