Permutation & Combination

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Introduction

- Permutation and combination questions are frequent in competitive examinations and are a core part of the counting methods in the General Aptitude syllabus.
- Most problems rely on the **fundamental principle of counting**; advanced techniques are seldom required for standard competitive questions.
- For reliable performance, focus on mastering basic principles, factorial notation, permutation and combination formulae, and typical arrangements (linear, circular, with/without repetition).

Fundamental Principle of Counting

- If an operation A can occur in **m** different ways and, after A, an operation B can occur in **n** different ways, then the two operations in sequence can occur in **m × n** ways.
- This principle generalises: if there are a finite sequence of events with numbers of ways **m**, **n**, **p**, ... then the total number of ways to perform them in that order is the

product **m × n × p × ...**

What is a Permutation?

- A **permutation** is an arrangement of objects in a definite order. When order matters, we count permutations.
- The conventional notation for the number of permutations of **n** distinct objects taken **r** at a time is $^{n}P_{r}$.
- The standard formula for permutations (no repetition) is given below.

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Factorial notation

- The symbol **n!** denotes the product of the first **n** positive integers.

Permutation formula (no repetition)

When 0 < r ≤ n, the number of permutations of **n** distinct objects taken **r** at a time is

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Permutation - useful formulae and cases

- **All objects distinct, no repetition:** the number of arrangements of **n** distinct objects taken **r** at a time is $^{n}P_{r} = \frac{n!}{(n-r)!}$.
- **Repetition allowed:** number of arrangements of **n** different symbols taken **r** at a time when repetition is allowed is n^{r}.
- **Some objects identical:** if among **n** objects there are **p** identical objects and the rest are distinct, the number of distinct linear arrangements of all **n** objects is $\frac{n!}{p!}$.
- **Multiple groups of identical objects:** if there are **p₁, p₂, ..., p_k** objects of k identical kinds (with $p_1 + p_2 + \cdots + p_k = n$), then the number of distinct linear

arrangements of all n objects is $\frac{n!}{p_1!p_2!\ldots p_k!}$.

What is a Combination?

- A **combination** is a selection of objects where order does not matter. The number of ways to choose **r** objects from **n** distinct objects is denoted $^{n}C_{r}$.
- The standard formula is given below.

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

OR

$$^{n}C_{r} = \frac{^{n}P_{r}}{r!}$$

Relationship between Permutation and Combination

- **Permutation-combination link:** the number of permutations equals the number of combinations multiplied by the number of orders for each combination.
- The standard identity is

$$^{n}P_{r} = {}^{n}C_{r} \times r!$$

- Binomial identity useful for combinations (Vandermonde/adjacency form):

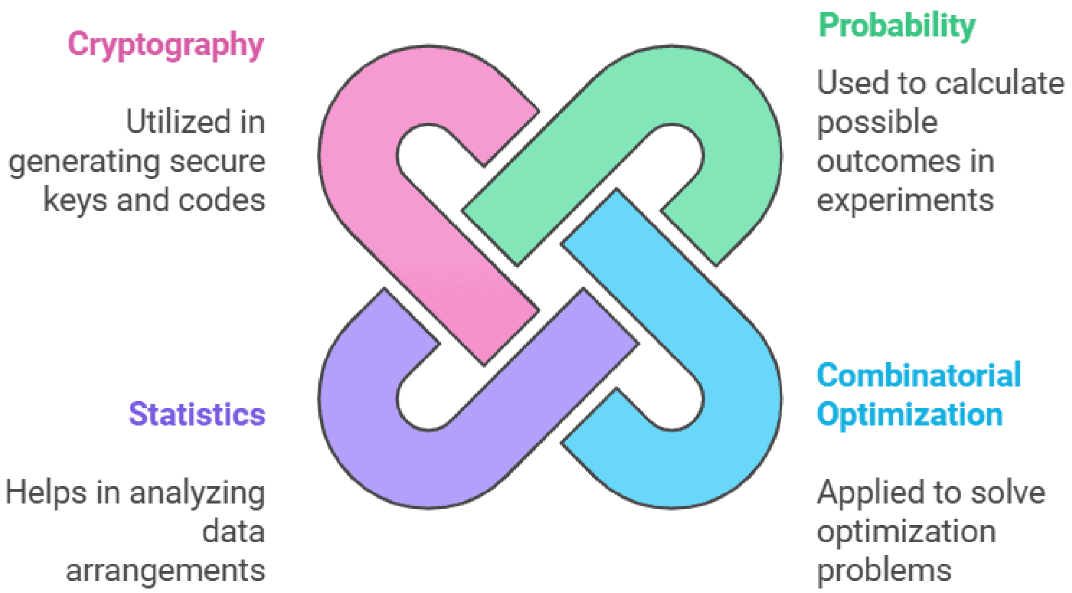
$$^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

Difference between Permutation and Combination

- In a **permutation**, the order of selection matters; in a **combination**, the order does not matter.
- Permutation formula: $^{n}P_{r} = \frac{n!}{(n-r)!}$.
- Combination formula: $^{n}C_{r} = \frac{n!}{r!(n-r)!}$.

Applications of Permutation and Combination

Counting arrangements (seating, line-ups), selecting committees, distributing distinct objects, code/number formation, probability computations, and problems in discrete structures.



Circular Permutations

1. Number of circular permutations of **n** different objects taken all at a time (arrangements up to rotation) is $(n-1)!$.
2. If arrangements that are mirror images are considered identical (i.e., rotation and reflection considered same), the number of distinct circular arrangements of all n different objects in one direction is $\frac{(n-1)!}{2}$.
3. Number of circular permutations of **n** different objects taken **r** at a time is $\frac{{}^{n}P_{r}}{r}$.
4. If rotations and reflections both reduce distinctness for the r-object circular arrangements, the count becomes $\frac{{}^{n}P_{r}}{2r}$.

MULTIPLE CHOICE QUESTION

Try yourself: Find the number of words, with or without meaning, that can be formed with the letters of the word 'INDIA'.

A 60

B 65

C 70

D 75

View Solution

MNP Rule

- If three tasks are to be done sequentially with **M**, **N**, and **P** possible ways respectively, then there are **M × N × P** total ways to perform all three tasks.
- If the tasks are mutually exclusive choices (select one of them), the total number of ways is **M + N + P**.

Example: The numbers 1, 2, 3, 4 and 5 are to be used for making three-digit numbers without repetition In how many ways, can this be done?

Sol:

There are three digit places to fill: ___ ___ ___.

Number of choices for the first place = 5.

After filling the first place, number of choices for the second place = 4.

After filling first two places, choices for the third place = 3.

Total number of three-digit numbers without repetition = 5 × 4 × 3 = 60.

Important Results

The following standard results are frequently used in problem solving and should be memorised and practised.

1. Number of permutations of n different things taken at a single time
   $= n!$
2. Number of permutations of n things, of which P_1 are alike of one kind, P_2 alike of another, P_3 alike of a third, and the rest all different
   $= \dfrac{n!}{P_1!\,P_2!\,P_3!\,\cdots}$
3. Number of permutations of n different things taken r times when repetition is allowed
   $= n^r$
4. Number of selections of r things out of b identical things
   $= 1$
5. Total number of selections of zero or more things out of k identical things
   $= k + 1$
6. Total number of selections of zero or more things out of n different things
   $= {}^nC_0 + {}^nC_1 + \cdots + {}^nC_n = 2^n$
7. Total number of selections of one or more things out of n different things
   $= 2^n - 1$
8. Number of ways of distributing n identical things among r persons (each may get any number)
   $= {}^{n+r-1}C_{r-1}$

9. Number of ways of dividing n non-distinct things into r distinct non-empty groups = ${}^{n-1}C_{r-1}$

10. Number of ways in which n distinct things can be distributed to r different people = r^n

11. Number of ways of defining $m+n$ different things in two groups containing m and n things respectively = ${}^{m+n}C_n \times {}^{m}C_m = \frac{(m+n)!}{m!\,n!}$

12. Number of ways of dividing $2n$ different things in two groups containing n things = $\frac{(2n)!}{n!\,n!\,2!}$

13. ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

14. ${}^{n}C_x = {}^{n}C_y \implies x = y \text{ or } x + y = n$

15. ${}^{n}C_r = {}^{n}C_{n-r}$

16. $r\ {}^{n}C_r = n\ {}^{n-1}C_{r-1}$

17. $\dfrac{{}^{n}C_r}{r+1} = \dfrac{{}^{n+1}C_{r+1}}{n+1}$

18. For ${}^nC_r$ to be the greatest

$$r = \frac{n}{2} \quad (\text{if } n \text{ is even}), \qquad r = \frac{n+1}{2} \text{ or } \frac{n-1}{2} \quad (\text{if } n \text{ is odd})$$

19. Number of selections of r things out of n different things

a. When k particular things are always included

= ${}^{n-k}C_{r-k}$

b. When k particular things are excluded

= ${}^{n-k}C_r$

c. When all the k particular things are not together in any selection

= ${}^nC_r - {}^{n-k}C_{r-k}$

20. Number of ways in which 0 to t things can be selected out of n things such that p are of one type, q of another type and the balance r all different

= $(p+1)(q+1)(2^r - 1)$

21. Total number of ways of taking some or all out of $p+q+r$ things such that p are of one type, q of another type and the balance r all different

= $(p+1)(q+1)(r+1) - 1$

Solved Examples

Example 1: From a pack of cards in how many ways can you select-

a) A king and a queen

b) A king or a queen

Sol:

There are 52 cards in a pack, with 4 kings and 4 queens.

Number of ways to select a king AND a queen: number of choices for king × number of choices for queen = 4 × 4 = 16.

Number of ways to select a king OR a queen: number of kings + number of queens = 4 + 4 = 8.

Example 2: In how many ways can 3 people A, B, C be arranged in 2 places?Sol:

Consider two ordered positions. For the first position, there are 3 choices (A, B or C).

After choosing the first person, the second position can be filled by any of the remaining 2 people.

Total arrangements = 3 × 2 = 6.

Enumerating: AB, BA, AC, CA, BC, CB.

Example 3: In how many ways can 4 people be seated in 3 chairs?

Sol:

First chair: 4 choices.

Second chair: 3 choices.

Third chair: 2 choices.

Total arrangements = 4 × 3 × 2 = 24.

MULTIPLE CHOICE QUESTION

Try yourself: **In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?**

A 360

B 480

C 720

D 5040

View Solution

Example 4: In how many ways can you select a President and a Vice-President from 3 people?

Sol:

Choose the President: 3 choices.

Choose the Vice-President from the remaining people: 2 choices.

Total = 3 × 2 = 6 ways.

Example 5: a, b, c are three distinct integers from 2 to 10 (both inclusive). Exactly one of ab, bc and ca is odd. abc is a multiple of 4. The arithmetic mean of a and b is an integer and so is the arithmetic mean of a, b and c. How many such triplets are possible (unordered triplets)?

Sol:

Exactly one of ab, bc, ca is odd implies exactly one of the pairwise products is odd.

A product of two integers is odd only if both factors are odd, so exactly two of the numbers must be odd and one even.

abc is divisible by 4, so the even number must be a multiple of 4.

Arithmetic mean of a and b is an integer ⇒ a and b are both odd or both even. Since exactly two numbers are odd, a and b must be the two odd numbers.

Arithmetic mean of a, b, c is an integer ⇒ a + b + c is divisible by 3.

Possible even choices that are multiples of 4 in [2,10] are 4 and 8; analyse each:

If c = 4, choose odd a and b from {3,5,7,9} so that a + b + 4 is divisible by 3 and a ≠ b. Valid unordered pairs (a,b): (3,5) and (5,9).

If c = 8, valid unordered pairs (a,b): (3,7) and (7,9).

Total unordered triplets = 4.

Example 6: If we listed all numbers from 100 to 10,000, how many times would the digit 3 be printed?

Sol:

Count occurrences of digit 3 in each place separately, then sum.

For 3-digit numbers (100-999):

Hundreds place: numbers 300-399 contribute 100 occurrences.

Tens place: for each hundred (1xx through 9xx) there are 10 numbers where tens digit is 3, so 9 × 10 = 90 occurrences in tens place.

Units place: similarly 9 × 10 = 90 occurrences in units place.

Total for 3-digit numbers = 100 + 90 + 90 = 280.

For 4-digit numbers (1000-9999):

Thousands place: 3000-3999 contribute 1000 occurrences.

Hundreds place: for each thousand block there are 100 numbers with hundreds digit 3; there are 9 such thousand blocks (1000-9000), so 9 × 100 = 900.

Tens place: 9 × 100 = 900 occurrences.

Units place: 9 × 100 = 900 occurrences.

Total for 4-digit numbers = 1000 + 900 + 900 + 900 = 3700.

Total occurrences from 100 to 9,999 = 280 + 3700 = 3980.

Example 7: All numbers from 1 to 150 (in decimal system) are written in base 6 notation. How many of these will contain zero's?

Sol:

Any number divisible by 6 ends with digit 0 in base 6. There are ⌊150/6⌋ = 25 such numbers.

Additionally, three-digit base-6 numbers in the range 1-150 can have a middle digit 0. Identify base-6 ranges that produce a zero in the middle digit and count them. The input counts these as 20 additional numbers (explicitly enumerated ranges: 37-41, 73-77, 109-113, 145-149 in decimal correspond to those base-6 patterns).

Total numbers containing at least one zero = 25 + 20 = 45.

MULTIPLE CHOICE QUESTION

Try yourself: Find the number of words, with or without meaning, that can be formed with the letters of the word 'CHAIR'.

A 110

B 120

C 130

D 140

View Solution

Summary

- Master the factorial notation, permutation formula $^{n}P_{r}$, and combination formula $^{n}C_{r}$.
- Use the fundamental counting principle (product rule) to break multi-stage choices into independent counts.
- Recognise common variants: repetition allowed, identical objects, circular arrangements, and symmetry (reflection) reductions.
- Practice standard counting patterns and worked examples to build speed and accuracy for competitive examinations.

Up Next

Tips & Tricks: Permutation & Combination
Doc | 1 page

(https://edurev.in/t/255951/Tips-Tricks-Permutation-Combination)

Important Formulae: Permutations & Combinations
Doc | 3 pages

(https://edurev.in/t/182213/Important-Formulae-Permutations-Combinations)

Forming Numbers: Permutations & Combinations- 1
Video | 06:50 min

(https://edurev.in/v/353895/Forming-Numbers-Permutations-Combinations-1)

Other Options