Geometric Progression: Introduction & Solved Examples

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Geometric Progression (GP) is a type of sequence where each succeeding term is produced by multiplying each preceding term by a fixed number, which is called a common ratio. This progression is also known as a geometric sequence of numbers that follow a pattern.

Geometric Progression is an important topic in CAT Quant, frequently appearing in questions involving sequences, growth and decay, infinite series, and algebraic manipulation.

A clear understanding of GP formulas and their applications is essential to solve CAT-level problems efficiently.

- A series in which each preceding term is formed by multiplying it by a constant factor is called a Geometric Progression G. P. The constant factor is called the common ratio and is formed by dividing any term by the term which precedes it.
- In other words, a sequence, $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called a geometric progression
- If $\frac{a_{n+1}}{a_n}$ = constant for all n ∈ N.
- The General form of a G. P. with n terms is a, ar, ar^2,...ar^{n-1}
  Thus if a = the first term
  r = the common ratio
  T_n = nth term and
  S_n = sum of n terms
- General term of GP = $\boxed{T_n = ar^{n-1}}$

Example.1: Find the 9th term and the general term of the progression.

$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots\ldots\ldots\ldots$

- The given sequence is clearly a G. P. with first term a = 1 and common ratio = r = $\left(-\frac{1}{2}\right)$.
- Now $T_9 = ar^8 = 1\left(-\frac{1}{2}\right)^8 = \frac{1}{2^8} = \frac{1}{256}$ and $T_n = ar^{n-1} = 1.\left(-\frac{1}{2}\right)^{n-1}$
  $= (-1)^{n-1} \cdot \frac{1}{2^{n-1}}$

MULTIPLE CHOICE QUESTION

Try yourself: How many terms are there in the GP 5, 20, 80, 320, ..., 20480?

A 5

B 6

C 7

D 8

View Solution

Common Ratio of G.P.

Consider the sequence a, ar, ar^2, ar^3,.....

First term = a

Second term = ar

Third term = ar^2

Similarly, nth term, $t_n = ar^{n-1}$

Thus, the common ratio of geometric progression formula is given as:

Common ratio = (Any term) / (Preceding term)

$= t_n / t_{n-1}$

$= (ar^{n-1}) / (ar^{n-2})$

= r

Thus, the general term of a GP is given by ar^{n-1} and the general form of a GP is a, ar, ar^2,.....

For Example: $r = t_2 / t_1 = ar / a = r$

Types of Geometric Progression

Geometric progression can be divided into two types based on the number of terms it has. They are:

- Finite geometric progression (Finite GP)
- Infinite geometric progression (Infinite GP)

(a) Finite Geometric Progression (Finite G.P.)

Suppose a, ar, ar^2, ar^3,.....ar^{n-1} is the given Geometric Progression.

Sum of n terms of a G.P.

The sum of n terms of GP is given by:

$S_n = a + ar + ar^2 + ar^3 + ... + ar^{n-1}$

The formula to find the sum of n terms of GP is:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{where } r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{where } r < 1$$

$$S_n = an \quad \text{where } r = 1$$

Where,

a is the first term

r is the common ratio

n is the number of terms

(b) Infinite geometric progression (Infinite GP)

Terms of an infinite G.P. can be written as a, ar, ar^2, ar^3, ...ar^{n-1},.....

a, ar, ar^2, ar^3, ...ar^{n-1},..... is called infinite geometric series.

Sum of Infinite G.P.

The sum of infinite geometric series is given by:

If a G.P. has infinite terms and -1 < r < 1 or |x| < 1, then sum of infinite G.P. is $S_\infty = \frac{a}{1-r}$.

Example.2: The inventor of the chess board suggested a reward of one grain of wheat for the first square, 2 grains for the second, 4 grains for the third and so on, doubling the number of the grains for subsequent squares. How many grains would have to be given to inventor? (There are 64 squares in the chess board).

- Required number of grains = 1 + 2 + 2^2 + 2^3 + ... To 64 terms = $1\left(\frac{2^{64}-1}{2-1}\right) = 2^{64} - 1.$

Recurring Decimals as Fractions

This concept is occasionally useful in CAT when recurring decimals appear as part of GP-based or infinite series questions

- If in the decimal representation a number occurs again and again, then we place a dot (.) on the number and read it as that the number is recurring.
- Example: 0.5 (read as decimal 5 recurring).

  This mean $0.\overline{5}$ = 0.55555.....∞

  $0.4\overline{7}$ = 0.477777.....∞

  These can be converted into fractions as shown in the example given below

Example.3 Find the value in fractions which is same as of $0.\overline{437}$

We have $0.\overline{437} = .4373737.....\infty$

$= 0.4 + 0.037 + 0.00037 + 0.0000037 + ...\infty$

$= \frac{4}{10}+\frac{37}{10^3}+\frac{37}{10^5}+\frac{37}{10^7}.....\infty$

$= \frac{4}{10}+\frac{37/10^3}{1-\frac{1}{10^2}}$ $\left[\text{Here } a=\frac{37}{10^3}; r=\frac{1}{10^2}\right]$

$= \frac{4}{10}+\frac{37}{1000}\times\frac{100}{99}=\frac{4}{10}+\frac{37}{990}=\frac{396+37}{990}=\frac{433}{990}$

Properties of G.P.

1. If each term of a GP is multiplied or divided by the same non-zero quantity, then the resulting sequence is also a GP.

   Example: For G.P. is 2, 4, 8, 16, 32...

If you multiply each term by constant let say 2,you get	4, 8, 16, 32, 64..	This is a G.P.
If you divide each term by constant let say 2,you get	1, 2, 4, 8, 16 ..	This is a G.P.

2. Selection of terms in G.P.

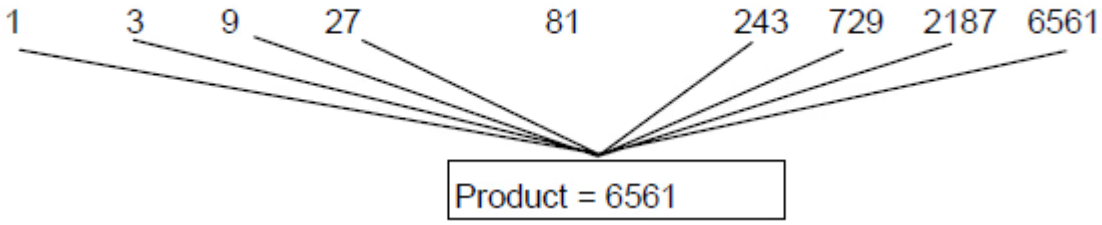
   Sometimes it is required to select a finite number of terms in G.P. It is always convenient if we select the terms in the following manner:

No. of terms	*Terms*	*Common ratio*
3	$\frac{a}{r}, a, ar$	r
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	r^2
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	r

If the product of the numbers is not given, then the numbers are taken as a, ar, ar^2, ar^3, ...

3. Three non-zero numbers a, b, c are in G.P. if and only if

   $b^2 = ac$ or $b = \sqrt{ac}$ b is called the geometric mean of a & c
4. In a GP, the product of terms equidistant from the beginning and end is always same and equal to the product of first and last terms as shown in the next example.



Geometric Progression Formulas

The list of formulas related to GP is given below which will help in solving different types of problems.

- The general form of terms of a GP is a, ar, ar^2, ar^3, and so on. Here, a is the first term and r is the common ratio.
- The nth term of a GP is $T_n = ar^{n-1}$
- Common ratio = $r = T_n / T_{n-1}$
- The formula to calculate the sum of the first n terms of a GP is given by:

  $S_n = a[(r^n - 1)/(r - 1)]$ if $r \neq 1$and $r > 1$

  $S_n = a[(1 - r^n)/(1 - r)]$ if $r \neq 1$ and $r < 1$
- The nth term from the end of the GP with the last term l and common ratio r = l/ [r(n - 1)].
- The sum of infinite, i.e. the sum of a GP with infinite terms is $S_\infty = a/(1 - r)$ such that $0 < r < 1$.
- If three quantities are in GP, then the middle one is called the geometric mean of the other two terms.

- If a, b and c are three quantities in GP, then and b is the geometric mean of a and c. This can be written as $b^2 = ac$ or $b = \sqrt{ac}$
- Suppose a and r be the first term and common ratio respectively of a finite GP with n terms. Thus, the kth term from the end of the GP will be = ar^{n-k}.

Solved Examples of Geometric Progression

Q.1: If the first term of a G.P. is 20 and the common ratio is 4. Find the 5th term.

Solution: Given,

First term, a = 20

Common ratio, r = 4

We know, Nth term of G.P.,

$a_n = ar_{n-1}$

$\Rightarrow a_5 = 20 \times 4_4$

$= 20 \times 256$

$= 5120$

Q.2: The sum of the first three terms of a G.P. is 21/2 and their product is 27. Find the common ratio.

Solution: Let three terms of G.P. be a/r, a, ar.

Given,

Product of first three terms = 27

$\Rightarrow (a/r)\ (a)\ (ar) = 27$

$\Rightarrow a^3 = 27$

$\Rightarrow a = 3.$

Sum of first three terms = 21/2

$\Rightarrow (a / r + a + ar) = 21/2$

$\Rightarrow a\ (1 / r + 1 + 1r) = 21/2$

$\Rightarrow (1 / r + 1 + 1r) = (21/2)/3 = 7/2$

$\Rightarrow (r^2 + r + 1) = (7/2)\, r$

$\Rightarrow r^2 - (5/2)\, r + 1 = 0$

$\Rightarrow r = 2$ and ½

Q.3: Find a Geometric Progress for which the sum of first two terms is -4 and the fifth term is 4 times the third term.

Solution: Let the first term of the geometric series be a and the common ratio is r.

Sum of the first two terms = -4

$a + ar = -4$ .....(i)

Fifth term is 4 times the third term.

$ar^4 = 4ar^2$

$r^2 = 4$

$r = \pm 2$

If we consider r = 2, then putting value of r in eq.(i)

$a(1+2) = -4$

$a = -4/3$

$ar = -8/3$

$ar^2 = -16/3$

Thus, the G.P. is -4/3, -8/3, -16/3, ...

Q.4: The number 2048 is which term in the following Geometric sequence 2, 8, 32, 128, ...

Solution: Here a = 2 and r = 4

nth term G.P is $a_n = ar^{n-1}$

$\Rightarrow 2048 = 2 \times (4)^{n-1}$

$\Rightarrow 1024 = (4)^{n-1}$

$\Rightarrow (4)^5 = (4)^{n-1}$

$\Rightarrow n = 6$

Q.5: In a G.P, the 6th term is 24 and the 13th term is 3/16 then find the 20th term of the sequence.

Solution: Let first term be 'a' and common ratio is 'r'

Given,

$a_6 = 24$ ----- (i)

$a_{13} = 3/16$ ----- (ii)

$\Rightarrow a_6 = a\, r^{6-1}$

$a_{13} = a\, r^{13-1}$

$\Rightarrow 24 = a\, r^5$

$3/16 = a\, r^{12}$

$\Rightarrow r^7 = 3 / 24 \times 16 = 1 / (2)^7$

$\Rightarrow r = 1/2$ ---- (iii)

Thus,

$\Rightarrow a_6 = 24 = a\, (1/2)^5$

$\Rightarrow a = 3 \times 2^8$

Now $a_{20} = a\, r^{20-1}$

$a_{20} = 3 \times 2^8 \times (1/2)^{19} = 3 / 2^{11}$

Q.6: Find the sum of the geometric series:

4 - 12 + 36 - 108 + ... to 10 terms

Solution: The first term of the given Geometric Progression = a = 4 and its common ratio = r = -12/4 = -3.

Sum of the first 10 terms of geometric series:

$S_{10} = a. (r^n - 1/r-1)$

$= 4. ((-3)^{10} - 1)/(-3-1)$

$= - (-3)^{10} - 1$

$= - 59048$

Q.7: 'x' and 'y' are two numbers whose AM is 25 and GM is 7. Find the numbers.

Solution: Here x' and 'y' are two numbers then

Arithmetic mean = AM = (x+y)/2

25 = (x+y)/2

x+y = 50 .....(i)

Geometric mean, GM = $\sqrt{(xy)}$

$7 = \sqrt{(xy}$

$7^2 = xy$

$xy = 49$ .....(ii)

Solving equation (i) and (ii), we get;

$x = 1$ and $y = 49$.

Q.8: Determine the common ratio r of a geometric progression with the first term is 5 and fourth term is 40.

Solution: Given,

First term, $a_1 = 5$

Fourth term, $a_4 = 40$

$a_4/a_1 = 40/5$

$a_1r^3/a_1 = 40/5$

$r^3 = 8$

$r = 2$

Q.9: If the nth term of a GP is 128 and both the first term a and the common ratio 'r' are 2. Find the number of terms in the GP.

Solution: nth term of a GP, $a_n = 128$

First term of GP, a = 2

Common ratio, r = 2

Nth term of G.P., $an = a.r^{n-1}$

$128 = 2.2^{n-1}$

$64 = 2^{n-1}$

$2^6 = 2^{n-1}$

$n- 1 = 6$

$n = 7$

Therefore, there are 7 terms in GP.

Q.10: What is the sum of infinite geometric series with first term equal to 1 and common ratio is ½?

Solution: By the formula of sum of infinite geometric series, we have;

$S = a_1. 1/(1 - r)$

$S = 1. 1/(1-½)$

$S = 1/(½)$

$S = 2$

Hence, the required sum is 2.

CAT Application Note:

The following examples range from easy to moderate level and are representative of foundational CAT GP problems.

Frequently Asked Questions on Geometric Progression

1. What is a Geometric Progression?

Answer: Geometric Progression (GP) is a type of sequence where each succeeding term is produced by multiplying each preceding term by a fixed number, which is called a common ratio.

2. Give an example of Geometric Progression.

Answer: The example of GP is: 3, 6, 12, 24, 48, 96,...

3. What is the general form of GP?

Answer: The general form of a Geometric Progression (GP) is given by a, ar, ar^2, ar^3, ar^4,...,ar^{n-1}

a = First term

r = common ratio

ar^{n-1} = nth term

4. What is the common ratio in GP?

Answer: The common multiple between each successive term and preceding term in a GP is the common ratio. It is a constant value that is multiplied by each term to get the next term in the Geometric series. If a is the first term and ar is the next term, then the common ratio is equal to:

ar/a = r

5. What is not a geometric progression?

Answer: If the common ratio between each term of a geometric progression is not equal then it is not a GP.

6. What is the sum of a geometric series?

Answer: If a, ar, ar^2, ar^3,.....ar^{n-1} is the given Geometric Progression, then the formula to find sum of GP is:

$S_n = a + ar + ar^2 + ar^3 + ... + ar^{n-1}$

Or

$S_n = a[(r^n - 1)/(r - 1)]$ where $r \neq 1$ and $r > 1$

Harmonic Progression: Introduction & Solved Examples

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Monotonic Sequence - Bounded Sequence

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