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Harmonic Progression: Introduction & Solved Examples

Harmonic Progression (H.P.)

1. A series of quantities is said to be in a harmonic progression when their reciprocals are in arithmetic progression.
2. e.g. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ and $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots\ldots$ are in HP as their reciprocals
3. 3, 5, 7, ..., and a, a + d, a + 2d........ are in AP.

nth term of HP

1. Find the nth term of the corresponding AP and then take its reciprocal.
2. If the HP be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} \ldots$
3. Then the corresponding AP is a, a + d, a + 2d, ...
4. T_n of the AP is a + (n - 1) d
5. T_{nth} of the HP is ...... $\frac{1}{a+(n-1)d}$
6. In order to solve a question on HP, one should form the corresponding AP.

A comparison between AP and GP

Description	AP	GP
Principal Characteristic	Common Difference (d)	Common Ratio (r)
n^{th} Term	$T_n = a + (n - 1) d$	$T_n = ar^{(n-1)}$
Mean	$A = (a + b) / 2$	$G = (ab)^{1/2}$
Sum of First n Terms	$S_n = n/2 [2a + (n - 1) d] = n/2 [a + \ell]$	$S_n = a (1 - r^n) / (1 - r)$
'm'th mean	$a + [m (b - a) / (n + 1)]$	$a (b / a)^{m/(n+1)}$

Arithmetic - Geometric progression

$a + (a + d)r + (a + 2d)r^2 + (a + 3d)r^3 + \ldots\ldots\ldots$ Is the form of Arithmetic geometric progression (A.G.P).

One part of the series is in Arithmetic progression and other part is a Geometric progression.

The sum of n terms series is $S_n = \dfrac{a}{1-r} + dr\dfrac{(1-r^{n-1})}{(1-r)^2} - \dfrac{[a+(n-1)d]r^n}{(1-r)}$.

The infinite term series sum is $S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$

Arithmetic geometric series can be solved as explained in the example below:

Relation between AM, GM and HM:

For two positive numbers a and b

A = Arithmetic mean = $\dfrac{a+b}{2}$

G = Geometric Mean = $\sqrt{ab}$

H = Harmonic Mean = $\dfrac{2ab}{a+b}$

Multiplying A and H, we get AH = $\dfrac{a+b}{2} \times \dfrac{2ab}{a+b} = ab = G^2$

Do you know?
AM $\geq$ GM $\geq$ HM (always for positive numbers) and G^2 = AH

This mean A, G, H are in G.P.

Verifying for numbers 1, 2

$$AM = \frac{1+2}{2} = 1.5 = \frac{3}{2},\ GM = \sqrt{2} \text{ and } HM = \frac{2\times1\times2}{3} = \frac{4}{3}$$

Hence $\boxed{\mathbf{AM \geq GM \geq HM}}$

$\mathbf{G^2} = 2$, and $\mathbf{AH} = \frac{3}{2}\times\frac{4}{3} = 2$

$\boxed{\mathbf{G^2 = AH}}$

Toolkit

$$\sum n = 1+2+3+\ldots\ldots n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2+2^2+3^2+\ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = 1^3+2^3+3^3+\ldots\ldots n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Ex.1 Find the sum of $1 + 2x + 3x^2 + 4x^3 + \ldots \infty$

Sol. The given series in an arithmetic-geometric series whose corresponding A.P. and G.P. are 1, 2, 3, 4,...

and 1, x, x^2, x^3, ... respectively. The common ratio of the G.P. is x. Let S_∞ denote the sum of the given

series.

Then, $S_\infty = 1 + 2x + 3x^2 + 4x^3 + \dots \infty$ ....(i)

$\Rightarrow x\,S_\infty = x + 2x^2 + 3x^3 + \dots \infty$ ....(ii)

Subtracting (ii) from (i), we get

$S_\infty - x\,S_\infty = 1 + [x + x^2 + x^3 + \dots \infty]$

$$\Rightarrow \quad S_\infty (1 - x) = 1 + \frac{x}{1 - x}$$

$$\Rightarrow \quad S_\infty = \frac{1}{1 - x} + \frac{x}{(1 - x)^2} = \frac{1}{(1 - x)^2}$$

Ex.2 If the first item of an A.P is 12, and 6th term is 27. What is the sum of first 10 terms?

Sol.

$a = 12,\ t_6 = a + 5d = 27 \Rightarrow d = 3$

$$\therefore S_{10} = \frac{10}{2}[2 \times 12 + (10 - 1)3] = 255.$$

Ex.3 If the fourth & sixth terms of an A.P are 6.5 and 9.5. What is the 9th term of that A.P?

Sol.

$a + 3d = 6.5$ & $a + 5d = 9.5$

$\Rightarrow a = 2,$ & $d = 1.5$

$\therefore \mathbf{t_9} = a + 8d = \mathbf{14}$

Ex.4 What is the arithmetic mean of first 20 terms of an A.P. whose first term is 5 and 4th term is 20?

Sol.

$a = 5$, $\mathbf{t_4} = a + 3d = 20 \Rightarrow d = 5$

A.M is the middle number = average of 10^{th} & 11^{th} number = $\frac{50+55}{2} = 52.5$

(or) $A.M = \frac{Sn}{n} = \frac{1}{2}[2a + (n-1)d]$, where $a = 5$, $n = 20$, $d = 5 \Rightarrow A.M =$ **52.5**

Ex.5 The first term of a G.P is half of its fourth term. What is the 12th term of that G.P, if its sixth term

is 6

Sol.

$t_1 = \frac{1}{2}t_4$

$\Rightarrow a = \frac{1}{2}ar^3 \Rightarrow r^3 = 2$

$t_6 = ar^5 = 6$

$t_{12} = ar^{11} = ar^5 \times r^6 = 6(2)^2 = \mathbf{24}$

Ex.6 If the first and fifth terms of a G.P are 2 and 162. What is the sum of these five terms?

Sol.

$a = 2$

$ar^4 = 162 \Rightarrow r = 3$

$S_5 = \frac{2(3^5 - 1)}{3-1} = 242$

Ex.7 What is the value of $r + 3r^2 + 5r^3 + \text{- - - - -}$

Sol.

Assume $S = r + 3r^2 + 5r^3 + - - - - -$ ....(1)

$$r \times S = r^2 + 3r^3 + 5r^4 + \ldots\ldots \quad \ldots.(2)$$

(1) – (2)

$s(1-r) = r + 2r^2 + 2r^3 + \ldots\ldots$

$$= r + \frac{2r^2}{1-r} = \frac{1+r^2}{1-r} \Rightarrow s = \frac{1+r^2}{(1-r)^2}$$

Ex.8 The first term of a G.P. 2 and common ratio is 3. If the sum of first n terms of this G.P is greater

than 243 then the minimum value of 'n' is

Sol.

$$\frac{a(r^n - 1)}{r - 1} > 243$$

$$\Rightarrow \frac{2(3^n - 1)}{3 - 1} > 243$$

$\Rightarrow 3^n > 244$

$\Rightarrow n > 5$ So, min possible value of n is **6.**

Ex.9

$$\frac{1}{2\times 5} + \frac{1}{3\times 6} + \frac{1}{4\times 7} + - - - - - - + \frac{1}{11\times 14} \text{ is}$$

Sol.

$$= \frac{1}{3}\left[\frac{5-2}{2\times5}+\frac{6-3}{3\times6}+\frac{7-4}{4\times7}+----+\frac{14-11}{11\times14}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}-\frac{1}{5}+\frac{1}{3}-\frac{1}{6}+\frac{1}{4}-\frac{1}{7}+----+\frac{1}{11}-\frac{1}{14}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{12}-\frac{1}{13}-\frac{1}{14}\right]$$

$$= \frac{1}{3}\left[\frac{546+364+273-91-84-78}{12\times13\times7}\right] = \frac{155}{546}$$

Ex.10 $a_n = 2^n + 1$ **then** $(a_1 + a_2 + ---- + a_{20}) - a_{21}$ **is:**

Sol.

$a_1 + a_2 + ---- + a_{20} = 2^1 + 1 + 2^2 + 1 + ----- + 2^{20} + 1$

$= 2(2^{20} - 1) + 20$

$= 2^{21} + 18$

$\therefore$ Ans = **18**

Up Next

Monotonic Sequence - Bounded Sequence

Video | 13:26 min

(https://edurev.in/v/353913/Monotonic-Sequence-Bounded-Sequence)

Sequence

Video | 09:39 min

(https://edurev.in/v/353914/Sequence)

Sequence - Definition & Examples

Video | 09:39 min

(https://edurev.in/v/334027/Sequence-Definition-Examples)

Other Options