Important Formulae: Progressions

1. Arithmetic Progression Formulae

- $a_n = a_1 + (n - 1)d$

  $S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$
- Number of terms = $\frac{a_n - a_1}{d} + 1$

  Sum of first n natural numbers

  $\Rightarrow 1 + 2 + 3 ... + n = \frac{n(n+1)}{2}$

  Sum of squares of first n natural numbers

  $\Rightarrow 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

  Sum of cubes of first n natural numbers

  $\Rightarrow 1^3 + 2^3 + 3^3 ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$
- Sum of first n odd numbers

  $\Rightarrow 1 + 3 + 5 ... + (2n - 1) = n^2$
- Sum of first n even numbers

  $\Rightarrow 2 + 4 + 6 ... 2n = n(n - 1)$
- If you have to consider 3 terms in an AP, consider {a-d, a, a+d}. If you have to consider 4 terms, consider {a-3d, a-d, a+d, a+3d}
- If all terms of an AP are multiplied with k or divided with k, the resultant series will also be an AP with the common difference dk or d/k respectively.

Arithmetic Progression

Arithmetic Progression(AP)

1. It is a sequence in which the difference between any two consecutive terms is constant.
2. This constant difference is called as the common difference of the AP.

Finite AP

An AP with countable number of terms. For e.g. 2, 4, 6, and 8.

Infinite AP

An AP with uncountable number of terms. For e.g. 2, 4, 6, 8, 10, 12,...

n^{th} Term Of an AP

$$\boxed{t_n = a + (n - 1)d}$$

Where,

$t_n = n^{th}$ term
$a = 1^{st}$ term
$d =$ common difference

Sum Of first n Terms Of an AP

$$\boxed{\begin{aligned} S_n &= \frac{n}{2}[\,2a + (n - 1)d\,] \\ S_n &= \frac{n}{2}[\,a + t_n\,] \end{aligned}}$$

Where,

$S_n =$ Sum of first n terms
$t_n = n^{th}$ term
$a = 1^{st}$ term
$d =$ common difference

Consecutive Terms In AP

1. 3 terms: $a - d, a, a + d$
2. 4 terms: $a - 3d, a - d, a + d, a + 3d$
3. 5 terms: $a - 2d, a - d, a, a + d, a + 3d$

Arithmetic Mean

For a set of positive numbers $a_1, a_2, a_3, ... a_n$, the arithmetic mean is

$$\boxed{AM = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}}$$

So, AM for two positive numbers

$$\boxed{AM = \frac{a + b}{2}}$$

2. Geometric Progression Formulae

The list of formulas related to GP is given below which will help in solving different types of problems.

- The general form of terms of a GP is a, ar, ar^2, ar^3, and so on. Here, a is the first term and r is the common ratio.
- The nth term of a GP is $T_n = ar^{n-1}$
- Common ratio = $r = T_n / T_{n-1}$
- The formula to calculate the sum of the first n terms of a GP is given by:
  $S_n = a[(r^n - 1)/(r - 1)]$ if $r \neq 1$and $r > 1$
  $S_n = a[(1 - r^n)/(1 - r)]$ if $r \neq 1$ and $r < 1$
- The nth term from the end of the GP with the last term l and common ratio r = l/ [r(n - 1)].

MULTIPLE CHOICE QUESTION

Try yourself: What is the formula to find the sum of the first n terms of an arithmetic progression?

A Sn = n(a1 + an)/2

B Sn = n(a1 + d)/2

C Sn = n/2(a1 + an)

D Sn = n(a1 + d)

View Solution

- The sum of infinite, i.e. the sum of a GP with infinite terms is $S_\infty = a/(1 - r)$ such that $0 < r < 1$.
- If three quantities are in GP, then the middle one is called the **geometric mean** of the other two terms.
- If a, b and c are three quantities in GP, then and b is the geometric mean of a and c. This can be written as $\mathbf{b^2 = ac}$ or $\mathbf{b = \sqrt{ac}}$
- Suppose a and r be the first term and common ratio respectively of a finite GP with n terms. Thus, the kth term from the end of the GP will be = ar^{n-k}.

Geometric Progression

1. It is the sequence in which ratio of two consecutive terms is constant.
2. This constant ratio is called common ratio.
3. Sequence a, ar, ar^2, ar^3...

n^{th} Term Of GP

$$t_n = ar^{n-1}$$

where

a = first term
t_n = n^{th} term
r = common ratio

Sum Of First n Terms Of AP

$$s_n = \frac{a(r^n - 1)}{r - 1}, |r| > 1$$
$$= \frac{a(1 - r^n)}{1 - r}, |r| < 1$$
$$= an, r = 1$$

where

s_n = sum of first n terms
a = first term
r = common ratio

Sum To Infinite

$$s_\infty = \frac{a}{1 - r}, |r| < 1$$

where

s_∞ = sum of infinite terms in GP
a = first term
r = common ratio

Properties Of GP

1. If every term of GP is multiplied or divided by a non-zero number, then the resulting terms are also in GP.
2. If all terms of GP are raised to the same powers, then the resulting terms are also in GP.

Means

1. **Arithmetic mean:** For two numbers a and b arithmetic mean AM $= \frac{a + b}{2}$

   For n numbers,
   $$AM = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$$

2. **Geometric mean:** For two numbers a and b, Geometric mean
   $$GM = \sqrt{ab}$$
   For n numbers,
   $$GM = \sqrt[n]{a_1 . a_2 . a_3 ... a_n}$$

3. **Relation between AM and GM:**
   For positive numbers,
   $$AM \geq GM$$

Multiple means Between Two Numbers

Let a and b two numbers

1. n Arithmetic means: If $A_1, A_2, A_3 ... A_n$ are AM between a and b then,
   $$A_n = a + \frac{n(b - a)}{n + 1}$$
2. n Geometric means: If $G_1, G_2, G_3 ... G_n$ are GM between a and b then,
   $$G_n = a\left(\frac{b}{a}\right)^{n/n+1}$$

Sum Of Special Series

1. $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
2. $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$

(https://edurev.in/courses/5594_General-Aptitude-...

General Aptitude for GATE (https://edurev.in/courses/5594_General-Aptitude-for-...

Go to Course (https://edurev.in/cou

Up Next

Arithmetic Progression and Geometric Progression

Video | 02:56 min

(https://edurev.in/v/72974/Arithmetic-Progression-and-Geometric-Progression)

Geometric Progression: Introduction & Solved Examples

Doc | 9 pages

(https://edurev.in/t/73614/Geometric-Progression-Introduction-Solved-Examples)

Test: Progression (AP And GP)- 1

Test | 10 ques

(https://edurev.in/test/1794/Test-Progression-AP-And-GP-1)

Other Options

Download this Doc

Share with a friend

Report a Problem

Practice this topic

Refer & Earn ₹50