Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV EXAMINATION – SUMMER 2025

Subject Code:3140708 Date:17-05-2025

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Prove that: $(A\cap B)\cup(A-B)=A$ and $A\cup(B-A)=A\cup B$	**03**
	(b)	Check whether the relation R defined in the $\{1,2,3,4,5,6\}$ as $R=\{(a,b):b=a+2\}$ is reflexive, symmetric or transitive.	**04**
	(c)	**(I)** Define connected graph, Boolean matrix and Euler path.	**03**
		(II) If $f(x)=\frac{x+1}{x-1}$ and $g(x)=\frac{x-1}{x+1}$ then find $g\circ f(x)$ and $f\circ g(x)$	**04**
Q.2	**(a)**	Find total number of diagonals in a polygon with n sides.	**03**
	(b)	If R be the relation defined in Q -the set of rational numbers by $R=\left\{\left(\frac{a}{b},\frac{c}{d}\right)\in Q\times Q:ad=bc\right\}$, then show that R is an equivalence relation.	**04**
	(c)	Solve the recurrence relation $S_n-6S_{n-1}+9S_{n-2}=3^{n+1}$	**07**
		OR	
	(c)	Solve the recurrence relation $S_n+3nS_{n-1}=0$, $S_0=1$.	**07**
Q.3	**(a)**	Define complete graph and find order and size of the graph K_{2025}	**03**
	(b)	Show that the proposition $\lfloor(p\to q)\wedge(q\to r)\rfloor\to(p\to r)$ is tautology.	**04**
	(c)	Show that $(R,+,\times)$ is an integral domain, where $R=\left\{a+b\sqrt{11}\middle\vert a,b\in I\right\}$	**07**
		OR	
Q.3	**(a)**	Draw Hasse diagram for $P=\{1,2,3,5,11,13,17,19,23\}$ and $\leq$ is a relation such that $x\leq y\Leftrightarrow x\mid y$.	**03**
	(b)	Check whether the relation R defined by $R=\{(a,b):a\leq b^3, a \text{ \& } b \textit{ are real numbers}\}$ is reflexive, symmetric or transitive.	**04**
	(c)	**(I)** Does there exist a graph with 20 edges and each vertex of degree 3?	**03**

(II) Show that the function $f:R\to R$ such that $f(x)=x^3+x$ is a bijection. **04**

Q.4 **(a)** Prove that $(1\times 2)+(2\times 3)+....+(n\times(n+1))=\frac{n(n+1)(n+2)}{3}$ **03**

(b) If G is an abelian group with n elements $g_1,g_2,.....,g_n$ then show that $(g_1g_2.....g_n)^2=e$, where e is the identity element of G **04**

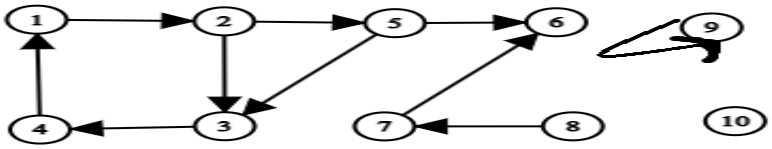
(c) Define Lattice. And consider a poset $(\{3,\ 5,\ 9,\ 15,\ 24,\ 45\},\ |\ )$, where $|$ denotes "divides" is a lattice. Then **07**

(i) Draw its Hasse Diagram. **(ii)** Find its maxima, minima, greatest and least elements when they exist. (iii) Find maxima, minima, greatest and least elements of the set M = {3, 9, 15}, when they exist.

OR

Q.4 **(a)** Find total number of vertices of a full ternary tree with n levels. **03**

(b) Find a Node base for the following graph: **04**



(c) Show that arbitrary intersection of subgroups of a group is a subgroup of a group. Will union of two subgroups be also subgroup of group? **07**

Q.5 **(a)** Show that the set $\{1,\ -1,\ i,\ -i\}$ is a group with respect to multiplication. **03**

(b) Show that in a lattice if $a\le b\le c$, then **04**
(I) $a\oplus b=b*c$ and (II) $(a*b)\oplus(b*c)=b=(a\oplus b)*(a\oplus c)$

(c) Find transitive closure by Wars hall's Algorithm if $A=\{1,2,3,4,5\}$ and $R=\{(1,4),(2,1),(2,5),(2,4),(4,3),(5,3),(3,2)\}$ **07**

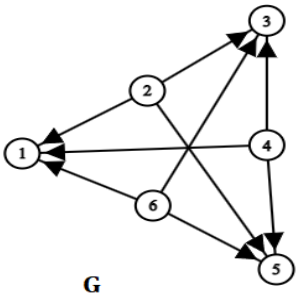
OR

Q.5 **(a)** Show that $H=\left\{\begin{bmatrix}1 & 0\\ a & 1\end{bmatrix}:a\in R\right\}$ is a subgroup of G, where G is the group of all non-singular lower triangular matrices of order 2×2 over R with usual matrix multiplication. **03**

(b) **(I)** Draw a complete bipartite graph which is not regular. **04**

(II) draw a graph which is regular but not bipartite.

(c) Define Isomorphic graphs. Check whether the following graphs are isomorphic? **07**



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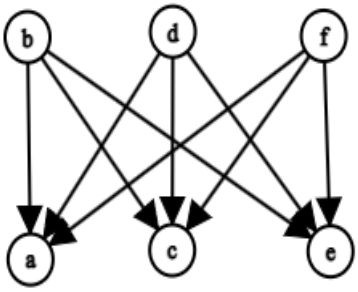

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