Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023

Subject Code:3140708 Date:17-07-2023

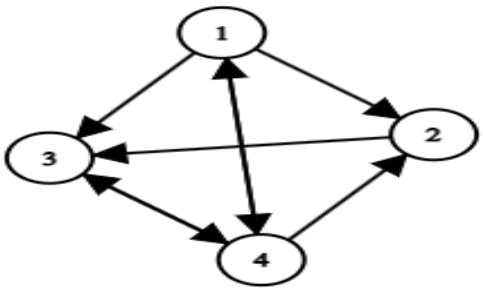
Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	If $A=\{x\|x=3^n,\ n\le 6, n\in N\}$ $B=\{x\|x=9^n,\ n\le 4, n\in N\}$ then find $A\cup B$, $A\cap B$ and $A-B$.	**03**
	(b)	Check whether the relation R defined in the $\{1,2,3,4,5,6\}$ as $R=\{(a,b):b=a+1\}$ is reflexive, symmetric or transitive.	**04**
	(c)	**(I)** Prove that $p\to q$ and $(\neg p\vee q)$ are logically equivalent.	**03**
		(II) Let $Y=\{n^2:n\in N\}$ and $f:N\to Y$ defined as $f(n)=n^2$. Show that f is invertible and find the inverse of f.	**04**
Q.2	**(a)**	In a survey of 400 students in a school, 100 were found as drinking apple juice, 150 as drinking orange juice and 75 drinking both apple and orange juice. Find how many students drink neither apple nor orange juice.	**03**
	(b)	Let $f:R\to R$ be function defined by $f(x)=ax+b$. Find values of a and b for which $f\circ f=I_R$	**04**
	(c)	If $R=\{(a,b):\|a-b\|=1\}$ and $S=\{(a,b):a-b \text{ is even}\}$ be two relations on $A=\{1,2,3,4\}$. Then (I) Find matrices of R and S, (II) Find digraph of R and S (III) Find the relation RS	**07**
		OR	
	(c)	**(I)** Find n if ${}^nC_3+{}^{n+2}C_3={}^nP_3$	**03**
		(II) From 4 professors and 6 students, a committee of 3 is to be formed. In how many ways, this can be done, if the committee contains **(1)** at most 1 professor **(2)** at least 2 professors.	**04**
Q.3	**(a)**	Find reachable set of all the vertices and node base of following graph.	**03**



(b) Show that in a lattice if $a \le b \le c$, then **04**
(I) $a \oplus b = b * c$ and (II) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

(c) Solve the recurrence relation which represents the Fibonacci sequence **07**
$F_n = F_{n-1} + F_{n-2}$ with $F_0 = F_1 = 1$.

OR

Q.3 **(a)** The Indian cricket team consist of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways eleven player can be selected if we have to select on wicket keeper and at-least 4 bowlers? **03**

(b) Prove that any graph has even number of odd vertices. **04**

(c) Find the generating function of $a_{n+2} = a_{n+1} + a_n$, where $a_0 = a_1 = 1$ for $n \ge 1$ **07**

Q.4 **(a)** If G is an abelian group with n elements $g_1, g_2, ..., g_n$ then show that $(g_1 g_2 ..... g_n)^2 = e$, where e is the identity element of G **03**

(b) Draw a binary tree whose post-order produced the string **04**
$d - g - e - b - i - j - h - f - c - a$ and pre-order produces the string
$j - b - a - c - d - i - h - e - g - f$

(c) Define Lattice. And draw the Hasse diagram representing the partial ordering $\{(A, B) : A \subseteq B\}$ on the power set $P(S)$ where $S = \{1, 2, 3\}$. Find the maximal, minimal, greatest and least elements of the poset. **07**

OR

Q.4 **(a)** Define Isolated vertex, Pendent vertex and Size of a graph **03**

(b) Let G be a graph with n vertices and m edges such that vertices have degree k or $k+1$. Prove that if G has N_k vertices of degree k and N_{k+1} vertices of degree $k+1$ then $N_k = (k+1)n - 2m$ **04**

(c) Show that G is an abelian group under usual matrix addition, where **07**
$G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a, b, c, d \in R \right\}$

Q.5 **(a)** Show that the only right coset of a subgroup H in a group G that is also subgroup of G is H itself. **03**

(b) Dose there exists a 4- regular graph with 6 vertices? If so, construct the graphs. **04**

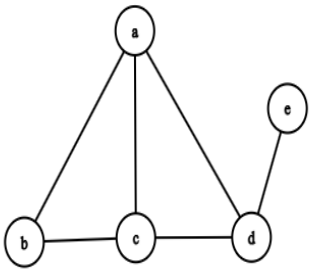
(c) Show that $(R, +, \times)$ is an integral domain, where $R = \left\{ a + b\sqrt{5} \middle| a, b \in I \right\}$ **07**

OR

Q.5 **(a)** Define cycle, walk and tree. **03**

(b) Find transitive closure by Warshall's Algorithm if $A = \{1, 2, 3, 4, 5\}$ and **04**
$R = \{(1, 2), (3, 4), (4, 5), (4, 1), (1, 1)\}$

(c) Define Isomorphic Graphs. Determine whether the following graphs are isomorphic or not. **07**



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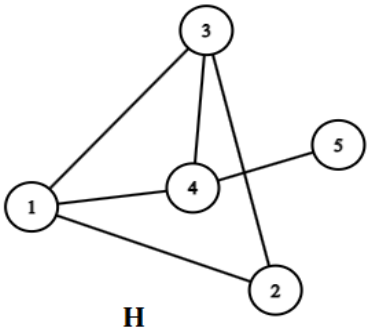

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